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FIXED POINT THEOREMS FOR $(\alpha - \psi)$-TYPE CONTRACTION MAPPINGS IN PERTURBED METRIC SPACES

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Abstract

This paper presents fixed point theorems for an $\alpha-\psi$ contractive mapping in complete perturbed metric spaces. As applications of our findings, we obtain fixed point theorems for perturbed metric spaces enriched with graphs. Furthermore, examples are provided to illustrate the usability of the obtained results.
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1 Introduction and Preliminaries

In 2025, Jleli and Samet [6] introduced Perturbed metric spaces, which are among the most intriguing of all abstract spaces.
The definitions below introduces the recently introduced concept of perturbed metric spaces:

Definition 1.1 ([6]). *Let $\mathcal{D}, \mathcal{P} : \Omega \times \Omega \to [0, \infty)$ be two given mappings. Then $\mathcal{D}$ constitutes a perturbed metric on Ω with respect to $\mathcal{P}$, if $\mathcal{D} - \mathcal{P} : \Omega \times \Omega \to [0, \infty)$ defined as $(\mathcal{D} - \mathcal{P})(\zeta, \eta) = \mathcal{D}(\zeta, \eta) - \mathcal{P}(\zeta, \eta)$ forms a metric on Ω, that is, for all $\zeta, \eta, \delta \in \Omega$,*
(i) $(\mathcal{D} - \mathcal{P})(\zeta, \eta) \geq 0$,
(ii) $(\mathcal{D} - \mathcal{P})(\zeta, \eta) = 0 \Leftrightarrow \zeta = \eta$,
(iii) $(\mathcal{D} - \mathcal{P})(\zeta, \eta) = (\mathcal{D} - \mathcal{P})(\zeta, \eta)$,
(iv) $(\mathcal{D} - \mathcal{P})(\zeta, \eta) \leq (\mathcal{D} - \mathcal{P})(\zeta, \delta) + (\mathcal{D} - \mathcal{P})(\delta, \eta)$. *The mapping $\mathcal{P}$ is called a perturbed mapping, with $\Gamma = \mathcal{D} - \mathcal{P}$ forming a standard metric, and ($\Omega, \mathcal{D}, \mathcal{P}$) is called a perturbed metric space. We use (Ω, Γ) to represent the standard metric space.*

Note that a perturbed metric on Ω is not necessarily a metric on Ω. For some examples and further preliminaries of perturbed metric space one can see Jleli and Samet[6]. We now present several topological notions in perturbed metric spaces :

Definition 1.2 ([6]). *Let the triple ($\Omega, \mathcal{D}, \mathcal{P}$) form a perturbed metric space. Consider a sequence $\{p_n\}$ in Ω, and a self-mapping Λ defined on Ω.*
(i) If $\{p_n\}$ is a convergent sequence in an standard metric space (Ω, Γ), where the metric Γ is defined as $\Gamma = \mathcal{D} - \mathcal{P}$, then $\{p_n\}$ is a convergent sequence in the "perturbed sense in $(\Omega, \mathcal{D}, \mathcal{P})$.
(ii) If a sequence $\{p_n\}$ is a Cauchy in the context of a standard metric space (Ω, Γ), then $\{p_n\}$ is a perturbed Cauchy sequence in ($\Omega, \mathcal{D}, \mathcal{P}$).

(iii) ($\Omega, \mathcal{D}, \mathcal{P}$) is named a complete perturbed metric space, , when (Ω, Γ) is a standard complete metric space, that is, each perturbed Cauchy sequence in ($\Omega, \mathcal{D}, \mathcal{P}$) converges in the "perturbed sense" in ($\Omega, \mathcal{D}, \mathcal{P}$).
(iv) A mapping Λ is a perturbed continuous whenever the mapping Λ is continuous in the setting of the corresponding standard metric space (Ω, Γ)

Samet *et al.* [16] introduced $\alpha-\psi$ contraction maps to study the existence and uniqueness of fixed points. The fundamental finding of this interesting study led to the conclusion of several well-known fixed point theorems, including the Banach mapping principle. A number of authors have investigated and improved the strategies described in this study; see, for example, $[1-4, 8-17]$ and the references therein.
We will now review some essential concepts, notations, and basic results that will be used throughout this study.

The family of non-decreasing continuous functions Ψ is represented by $\psi : [0, +\infty) \rightarrow [0, +\infty)$ such that for any t $> 0, \sum_{n=1}^{\infty} \psi^n < +\infty$, where ψ^n is the n^{th} iterate of ψ.

Definition 1.3 ([16])**.** *Let Λ be a self-map on Ω and (Ω, Γ) be a metric space. If two functions $\alpha : \Omega \times \Omega \rightarrow [0, \infty)$ and $\psi \in \Psi$ exist such that $\alpha(\zeta, \eta)\Gamma(\Lambda\zeta, \Lambda\eta) \leq \psi(\Gamma(\zeta, \eta))$, for all $\zeta, \eta \in \Omega$, then Λ is said to be $\alpha - \psi$ contractive mapping*

Definition 1.4. *Consider a perturbed metric space ($\Omega, \mathcal{D}, \mathcal{P}$) with Λ as a self-map on Ω. Then Λ is said to be perturbed $\alpha - \psi$ contractive mapping if two functions $\alpha : \Omega \times \Omega \rightarrow [0, \infty)$ and $\psi \in \Psi$ exist such that, for any $\zeta, \eta \in \Omega, \alpha(\zeta, \eta)\mathcal{D}(\Lambda\zeta, \Lambda\eta) \leq \psi(\mathcal{D}(\zeta, \eta))$.*

Lemma 1.1 ([14] *also see* [16])**.** *. For every function $\psi : [0, +\infty) \rightarrow [0, +\infty)$ the following holds:*
(i) If ψ is non decreasing, then for each $t > 0$
(ii) $\lim_{t\rightarrow\infty} \psi^n(t) = 0$ *implies $\psi(t) < t$.*

Remark 1.1. *Taking $\alpha(\zeta, \eta) = 1$ for any $\zeta, \eta \in \Omega$ and $\psi(t) = kt$ for some $k \in [0, 1], \Lambda$ is a perturbed $\alpha - \psi$ contractive mapping if $\Lambda : \Omega \rightarrow \Omega$ fulfils the Banach contraction principle in the context of perturbed metric space.*

Definition 1.5 ([16])**.** *Let $\alpha : \Omega \times \Omega \rightarrow [0, \infty)$ and $\Lambda : \Omega \rightarrow \Omega$. If $\zeta, \eta \in \Omega$, then $(\zeta, \eta) \geq 1$ implies $\alpha(\Lambda\zeta, \Lambda\eta) \geq 1$, and we say that Λ is α-admissible.*

Example 1.1. Let $\Omega = [0, \infty)$. Define $\Lambda : \Omega \rightarrow \Omega$ be defined by $\Lambda\zeta = \frac{\zeta}{4}$. Also, define $\alpha : \Omega \times \Omega \rightarrow [0, +\infty)$ by $\alpha(\zeta, \eta) = 3$. Then Λ is α-admissible mapping.

Example 1.2. Let $\Omega = [0, \infty)$. Define $\Lambda : \Omega \rightarrow \Omega$ be defined by

$$\Lambda\zeta = \begin{cases} \frac{\zeta^2}{4} \text{ if } \zeta \in [0, 1] \\ 2\zeta^3 + 1 \text{ if } \zeta \in (1, \infty) \end{cases}.$$

Also, define $\alpha : \Omega \times \Omega \rightarrow [0, +\infty)$ by $\alpha(\zeta, \eta) = \begin{cases} 1 & \text{if } \zeta, \eta \in [0, 1] \\ 4 & \text{if otherwise .} \end{cases}$
Then Λ is α-admissible mapping.

2 Main results

Now, we generalize the result established by Samet and Vetro[16] in the context of perturbed metric space as follows:

Theorem 2.1. *Let ($\Omega, \mathcal{D}, \mathcal{P}$) be a complete perturbed metric space. $\Lambda : \Omega \rightarrow \Omega$ must meet the following requirements.*
$(C_1)\, \Lambda$ *is $\alpha - \psi$ contractive mapping,*
$(C_2)\, \Lambda$ *is α-admissible mapping,*
(C_3) *there exists, $\zeta_0 \in \Omega$ such that $\alpha(\zeta_0,\ \Lambda\zeta_0) \geq 1$ and $\alpha\left(\zeta_0, \Lambda^2\zeta_0\right) \geq 1$,*
$(C_4)\, \Lambda$ *is continuous,*
(C_5) *the value of the perturbed function $\mathcal{P}(\zeta, \eta) = 0$ when $\zeta = \eta$.*
Then Λ possesses a fixed point.

Proof. According to (C_3), there is a point ζ_0 in Ω where $\alpha(\zeta_0, \Lambda\zeta_0) \geq 1$ and $\alpha\left(\zeta_0, \Lambda^2\zeta_0\right) \geq 1$.
We develop a sequence $\{\zeta_n\}$ in Ω by $\zeta_n = \Lambda^n\zeta_0 = \Lambda\zeta_{n-1}$, for all $n \in \mathbb{N}$.
It is obvious that if $\zeta_{n_0} = \zeta_{n_0+1}$ for some $n_0 \in \mathbb{N}$, then ζ_{n_0} is a fixed point of Λ.
Assume that $\zeta_n \neq \zeta_{n+1}$ for all n $\in \mathbb{N} \cup \{0\}$.
Since Λ is an α-admissible mapping, we have
$\alpha(\zeta_0, \zeta_1) = \alpha(\zeta_0, \Lambda\zeta_0) \geq 1$ implies that $\alpha(\zeta_1, \zeta_2) = \alpha(\Lambda\zeta_0, \Lambda\zeta_1) \geq 1$,
implies that $\alpha(\zeta_2, \zeta_3) = \alpha(\Lambda\zeta_1, \Lambda\zeta_2) \geq 1$,
implies that $\alpha(\zeta_3, \zeta_4) = \alpha(\Lambda\zeta_2, \Lambda\zeta_3) \geq 1$.
Keeping going in this manner, we have

$$\alpha(\zeta_n, \zeta_{n+1}) \geq 1 \text{ for all } n \in \mathbb{N} \cup \{0\} \tag{2.1}$$

In the same way, we have

$$\alpha(\zeta_n, \zeta_{n+2}) \geq 1 \text{ for all } n \in \mathbb{N} \cup \{0\} \tag{2.2}$$

When we look at (C_1) and (2.1), we obtain

$$\begin{aligned} \mathcal{D}(\zeta_{n+1}, \zeta_n) = \mathcal{D}(\Lambda\zeta_n, \Lambda\zeta_{n-1}) \\ &\leq \alpha(\zeta_n, \zeta_{n-1}) \mathcal{D}(\Lambda\zeta_n, \Lambda\zeta_{n-1}) \\ &\leq \psi(\mathcal{D}(\zeta_n, \zeta_{n-1})), \text{ for all } n \in \mathbb{N}. \end{aligned}$$

By doing the previous procedure again, we get that

$$\mathcal{D}(\zeta_{n+1}, \zeta_n) \leq \psi^n(\Gamma(\zeta_1, \zeta_0)) \text{ for all } n \in \mathbb{N} \tag{2.3}$$

The exact metric is $\Gamma = \mathcal{D} - \mathcal{P}$ by definition. Considering (2.3), we conclude that

$$\Gamma(\zeta_{n+1}, \zeta_n) \leq \Gamma(\zeta_{n+1}, \zeta_n) + \mathcal{P}(\zeta_{n+1}, \zeta_n) \leq \psi^n(\Gamma(\zeta_1, \zeta_0)). \tag{2.4}$$

Taking $n \to \infty$, we have $\lim_{n\to\infty} \Gamma(\zeta_{n+1}, \zeta_n) = 0$.
Examining (C_1) and (2.2), we obtain

$$\begin{aligned} \mathcal{D}(\zeta_{n+2}, \zeta_n) =&\mathcal{D}(\Lambda\zeta_{n+1}, \Lambda\zeta_{n-1}) \\ &\leq \alpha(\zeta_{n+1}, \zeta_{n-1}) \mathcal{D}(\Lambda\zeta_{n+1}, \Lambda\zeta_{n-1}) \\ &\leq \psi(\mathcal{D}(\zeta_{n+1}, \zeta_{n-1})), \text{ for all } n \in \mathbb{N} \end{aligned}$$

Continuing the above process, we have

$$\mathcal{D}(\zeta_{n+2}, \zeta_n) \leq \psi^n(\mathcal{D}(\zeta_2, \zeta_0)) \text{ for all } n \in \mathbb{N} \tag{2.5}$$

The exact metric is $\Gamma = \mathcal{D} - \mathcal{P}$ by definition. Considering (2.2), we conclude that

$$\Gamma(\zeta_{n+2}, \zeta_n) \leq \Gamma(\zeta_{n+2}, \zeta_n) + \mathcal{P}(\zeta_{n+2}, \zeta_n) \leq \psi^n(\Gamma(\zeta_2, \zeta_0)). \tag{2.6}$$

Taking $n \to \infty$, we have $\lim_{n\to\infty} \Gamma(\zeta_{n+2}, \zeta_n) = 0$.
Let $\zeta_n = \zeta_m$ for some $m, n \in \mathbb{N}$ with $m \neq n$.
Without loss of generality, we will assume that $m > n$.
Since $\alpha(\zeta_m, \zeta_{m-1}) \geq 1$, we get

$$\begin{aligned} \mathcal{D}(\zeta_{n+1}, \zeta_n) &= \mathcal{D}(\Lambda\zeta_n, \zeta_n) \\ &= \mathcal{D}(\Lambda\zeta_m, \zeta_m) \\ &= \mathcal{D}(\Lambda\zeta_m, \Lambda\zeta_{m-1}) \\ &\leq \psi(\mathcal{D}(\zeta_m, \zeta_{m-1})) \\ &\leq \psi^{m-n}(\mathcal{D}(\zeta_{n+1}, \zeta_n)), \end{aligned}$$

for all $m, n \in \mathbb{N}$.
By Lemma 1.1, we have

$$\mathcal{D}(\zeta_{n+1}, \zeta_n) \leq \psi^{m-n}(\mathcal{D}(\zeta_{n+1}, \zeta_n)) < \mathcal{D}(\zeta_{n+1}, \zeta_n),$$

which is a contradiction.
Thus, $\zeta_n \neq \zeta_m$ for all $m, n \in \mathbb{N}$ with $m \neq n$.
We now demonstrate that $\{\zeta_n\}$ is a Cauchy sequence, meaning that for any $k \in \mathbb{N}$, $\lim_{n\to\infty} \Gamma(\zeta_n, \zeta_{n+k}) = 0$. The situations for $k = 1$ and $k = 2$ have already been established in (2.4) and (2.6), respectively. Let k be arbitrarily greater than or equal to 3 .

We discuss two cases.
Case 1. Suppose that $k = 2m + 1$, where $m \geq 1$. Using the triangle inequality, we have

$$\begin{aligned}
\Gamma(\zeta_n, \zeta_{n+k}) &= \Gamma(\zeta_n, \zeta_{n+2m+1}) \\
&\leq \Gamma(\zeta_n, \zeta_{n+1}) + \Gamma(\zeta_{n+1}, \zeta_{n+2}) + \cdots + \Gamma(\zeta_{n+2m}, \zeta_{n+2m+1}) \\
&\leq \sum_{p=n}^{n+2m} \psi^p(\mathcal{D}(\zeta_0, \zeta_1)) \\
&\leq \sum_{p=n}^{\infty} \psi^p(\mathcal{D}(\zeta_0, \zeta_1)) \to 0 \text{ as } n \to \infty.
\end{aligned}$$

Case 2. Suppose that $k = 2m$, where $m \geq 2$. Using the triangle inequality, we have

$$\begin{aligned}
\Gamma(\zeta_n, \zeta_{n+k}) &= \Gamma(\zeta_n, \zeta_{n+2m}) \\
&\leq \Gamma(\zeta_n, \zeta_{n+2}) + \Gamma(\zeta_{n+2}, \zeta_{n+3}) + \cdots + \Gamma(\zeta_{n+2m-1}, \zeta_{n+2m}) \\
&\leq \Gamma(\zeta_n, \zeta_{n+2}) + \sum_{p=n}^{n+2m-1} \psi^p(\mathcal{D}(\zeta_0, \zeta_1)) \\
&\leq \Gamma(\zeta_n, \zeta_{n+2}) + \sum_{p=n}^{\infty} \psi^p(\mathcal{D}(\zeta_0, \zeta_1)) \to 0 \text{ as } n \to \infty.
\end{aligned}$$

By (2.6), we have $\lim_{n\to\infty} \Gamma(\zeta_{n+2}, \zeta_n) = 0$, therefore $\lim_{n\to\infty} \Gamma(\zeta_n, \zeta_{n+k}) = 1$ for all $k \geq 3$. The above inequality shows that the sequence $\{\zeta_n\}$ is Cauchy in the standard metric spaces (Ω, Γ). The constructed sequence $\{\zeta_n\}$ is Cauchy in perturbed metric spaces $(\Omega, \mathcal{D}, \mathcal{P})$. Therefore, we conclude that $\{\zeta_n\}$ is a Cauchy sequence in $(\Omega, \mathcal{D}, \mathcal{P})$. As $(\Omega, \mathcal{D}, \mathcal{P})$ is complete, there exists $\zeta^* \in \Omega$ such that

$$\lim_{n\to\infty} \Gamma(\zeta_n, \zeta^*) = 0.$$

Finally, we show that the limit ζ^* of the sequence $\{\zeta_n\}$ is a fixed point of Λ.
Since Λ is continuous. Then we have

$$\lim_{n\to\infty} \Gamma(\Lambda\zeta_n, \Lambda\zeta^*) = \lim_{n\to\infty} \Gamma(\zeta_{n+1}, \Lambda\zeta^*) = 0.$$

By uniqueness of limit, ζ^* is a fixed point of Λ.
This completes the proof. □

Theorem 2.2. *Given a complete perturbed metric space $(\Omega, \mathcal{D}, \mathcal{P})$ and $\Lambda : \Omega \to \Omega$ satisfying the following requirements: $(C_1), (C_2), (C_3), (C_5)$*
(C_6) if $\{\zeta_n\}$ is sequence in Ω such that $\zeta_n \to \zeta$ as $n \to \infty$ for some $\zeta \in \Omega$ and $\alpha(\zeta_n, \zeta_{n+1}) \geq 1$ for all $n \in \mathbb{N}$, then $\alpha(\zeta_n, \zeta) \geq 1$ for all $n \in \mathbb{N}$. Then there is a fixed point for Λ .

Proof. Based on the proof of Theorem 2.1, we know that the sequence $\{\zeta_n\}$ given by $\zeta_{n+1} = \Lambda\zeta_n$ for all $n \in \mathbb{N} \cup \{0\}$ converges for some $u \in \Omega$. For any $n \in \mathbb{N}$, we obtain $\alpha(\zeta_n, u) \geq 1$ from (2.1) and (C_6). By using (3.1), we get

$$\begin{aligned}
\mathcal{D}(\zeta_{n+1}, \Lambda u) &\leq \alpha(\zeta_n, \mu)\, \mathcal{D}(\Lambda\zeta_n, \Lambda u) \\
&\leq \psi(\mathcal{D}(\zeta_n, u)) \text{ for all } n \in \mathbb{N}
\end{aligned}$$

Letting $n \to \infty$ in the above inequality and using (C_5), we obtain that

$$\lim_{k\to\infty} \mathcal{D}(\zeta_{n+1}, \Lambda u) = 0.$$

By definition, $\Gamma = \mathcal{D} - \mathcal{P}$ is the exact metric. In view of (2.2), we deduce that $\lim_{k\to\infty} \Gamma(\zeta_{n+1}, \Lambda u) \leq \lim_{k\to\infty} \Gamma(\zeta_{n+1}, \Lambda u) + \lim_{k\to\infty} \mathcal{P}(\zeta_{n+1}, \Lambda u)$.
We get
$\lim_{k\to\infty} \Gamma(\zeta_{n+1}, \Lambda u) = 0$. i.e., $\Lambda u = u$.
Hence u is a fixed point of Λ.
To assure the uniqueness of fixed points, we consider the following hypothesis:
(P) If for all $\zeta, \eta \in F(\Lambda)$, where $F(\Lambda)$ is a set of fixed points of Λ, we have $\alpha(\zeta, \eta) \geq 1$. □

Theorem 2.3. *The uniqueness of fixed points of Λ is obtained by inserting condition (**P**) to the hypothesis of Theorem* 2.1 *(resp. Theorem* 2.2*).*

Proof. Suppose that ζ^* and η^* $(\zeta^* \neq \eta^*)$ are two fixed points of Λ. Then by the hypothesis $(H), \alpha(\zeta^*, \eta^*) \geq 1$. Hence, from (C_1) with $\zeta = \zeta^*$ and $\eta = \eta^*$, we have

$$\begin{aligned}\mathcal{D}(\zeta^*, \eta^*) &= \mathcal{D}(\Lambda\zeta^*, \Lambda\eta^*)\\ &\leq \alpha(\zeta^*, \eta^*) \cdot \mathcal{D}(\Lambda\zeta^*, \Lambda\eta^*)\\ &\leq \psi(\mathcal{D}(\zeta^*, \eta^*)) < \mathcal{D}(\zeta^*, \eta^*)\end{aligned}$$

which is impossible, that is, $\zeta^* = \eta^*$.
Hence Λ has a unique fixed point.
This completes the proof. □

Example 2.1. For an interval $\Omega = [0, \infty)$, we shall define $\mathcal{D} : \Omega \times \Omega \to [0, \infty)$ be the mapping defined by

$$\mathcal{D}(u, t) = (u + t)^2 + |u - t|, \quad \text{for all } u, t \in \Omega.$$

Then $\mathcal{D}$ is a perturbed metric on Ω , where the perturbed mapping $\mathcal{P}$ is given by

$$\mathcal{P}(u, t) = (u + t)^2$$

$u, t \in \Omega$, and the exact metric Γ is given by

$$\Gamma(u, t) = |u - t|, u, t \in \Omega$$

Clearly, ($\Omega, \mathcal{D}, \mathcal{P}$) is a complete perturbed metric space.
Define the mapping Λ: $\Omega \to \Omega$ by

$$\Lambda\zeta = \begin{cases} \frac{\zeta}{4}, & \zeta \in [0, 1] \\ 2\zeta - \frac{7}{4}, & \zeta \in (1, \infty). \end{cases}$$

We define the mapping $\alpha : \Omega \times \Omega \to [0, \infty)$ by

$$\alpha(\zeta, \eta) = \begin{cases} e^{|\zeta - \eta|}, & \text{if } \zeta, \eta \in \left(0, \frac{1}{4}\right] \\ e^{-|\zeta - \eta|}, & \text{otherwise.} \end{cases}$$

Clearly, Λ is a triangular α-admissible and $\alpha - \psi$ contractive mapping with $\psi(\mathrm{t}) = \frac{\mathrm{t}}{4}$ for all $\mathrm{t} \in [0, \infty)$.
For all $\zeta, \eta \in \Omega$, we have

$$\alpha(\zeta, \eta)\mathcal{D}(\Lambda\zeta, \Lambda\eta) \leqslant \psi(\mathcal{D}(\zeta, \eta)).$$

Moreover, there exists $\zeta_0 = \frac{1}{4} \in \Omega$ such that

$$\alpha(\zeta_0, \Lambda\zeta_0) = \alpha\left(\frac{1}{4}, \frac{1}{16}\right) = e^{\frac{3}{16}} \geqslant 1.$$

Obviously, Λ is continuous, Theorem 2.1's hypotheses are now completely satisfied; Λ has two fixed points, 0 and $\frac{7}{4}$

3 Application

The existence of fixed point theorems on a perturbed metric space with a graph is provided in this section. Some fixed point findings were established on a metric space with a graph by Jachymski [7] in 2008. Before presenting our results, we give some definitions and notions in setting of perturbed metric spaces which are useful for our results as follow:

Consider the perturbed metric space $(\Omega, \mathcal{D}, \mathcal{P})$. A set $\{(\zeta, \zeta) : \zeta \in \Omega\}$ is represented as Δ and is referred to as a diagonal of the Cartesian product $\Omega \times \Omega$. Suppose that a graph G has all loops (i.e., $\Delta \subseteq E(G)$) in its edges and that the set $V(G)$ of its vertices matches with Ω. The pair ($V(G), E(G)$) can be used to identify G since we believe it has no parallel edges. Furthermore, we can treat G as a weighted graph by allocating the distance between its vertices to each edge.

Definition 3.1. *Let Ω have a graph G and be a nonempty set. The following condition must be satisfied for the mapping $\Lambda : \Omega \to \Omega$ to be referred to be preserve edge:*

$$\zeta, \eta \in \Omega \text{ with } (\zeta, \eta) \in E(G) \Rightarrow (\Lambda\zeta, \Lambda\eta) \in E(G).$$

Definition 3.2. *Let $(\Omega, \mathcal{D}, \mathcal{P})$ be a perturbed metric space endowed with a graph G. The mapping $\Lambda : \Omega \to \Omega$ is called a ψ contractive mapping with respect to $E(G)$ if there exists a function $\psi \in \Psi$ such that the following condition hold:*

$$\zeta, \eta \in \Omega \text{ with } (\zeta, \eta) \in E(G) \Rightarrow \mathcal{D}(\Lambda\zeta, \Lambda\eta) \leq \psi(\mathcal{D}(\zeta, \eta)).$$

Theorem 3.1. *Consider $(\Omega, \mathcal{D}, \mathcal{P})$ be a perturbed metric space endowed with a graph G and $\Lambda : \Omega \to \Omega$ be a self-mapping satisfying (C_4) and the following conditions:*
(C_7) Λ be a ψ contractive mapping with respect to $E(G)$,
(C_8) Λ is preserve edge,
(C_9) there exists $\zeta_0 \in \Omega$ such that $(\zeta_0, \Lambda\zeta_0) \in E(G)$ and $\left(\zeta_0, \Lambda^2\zeta_0\right) \in E(G)$.
Then Λ has a fixed point.

Proof. Consider a mapping $\alpha : \Omega \times \Omega \to [0, \infty)$ defined by

$$\alpha(\zeta, \eta) = \begin{cases} 1, & \zeta, \eta \in E(G) \\ 0, & \text{otherwise} \end{cases}$$

From (C_9), we have $\alpha\left(\zeta_0, \Lambda\zeta_0\right) = 1$ and $\alpha\left(\zeta_0, \Lambda^2\zeta_0\right) = 1$.
It follows from Λ is preserve edge that Λ is an α-admissible mapping.
From (C_7), we have, for all $\zeta, \eta \in \Omega$,

$$(\zeta, \eta) \in E(G) \Rightarrow \mathcal{D}(\Lambda\zeta, \Lambda\eta) \leq \psi(\mathcal{D}(\zeta, \eta)),$$

and thus

$$\alpha(\zeta, \eta)\mathcal{D}(\Lambda\zeta, \Lambda\eta) \leq \psi(\mathcal{D}(\zeta, \eta)).$$

Therefore, Λ is a $\alpha - \psi$ contractive mapping.
Now all the hypotheses of Theorem 2.1 are satisfied.
Therefore, Λ has a fixed point. □

Theorem 3.2. *Consider ($\Omega, \mathcal{D}, \mathcal{P}$) be a complete perturbed metric space and $\Lambda : \Omega \to \Omega$ be a self mapping satisfying $(C_7), (C_8), (C_9)$ and the following conditions:*
(C_{10}) if $\{\zeta_n\}$ is sequence in Ω such that $\zeta_n \to u$ as $n \to \infty$ for some $\zeta \in \Omega$ and $(\zeta_n, \zeta_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$, then $(\zeta_n, u) \in E(G)$ for all $n \in \mathbb{N}$.
Then Λ has a fixed point.

Proof. Consider a mapping $\alpha : \Omega \times \Omega \to [0, \infty)$ defined by

$$\alpha(\zeta, \eta) = \begin{cases} 1, & \zeta, \eta \in E(G) \\ 0, & \text{otherwise} \end{cases}$$

From (C_9), we have $\alpha\left(\zeta_0, \Lambda\zeta_0\right) = 1$ and $\alpha\left(\zeta_0, \Lambda^2\zeta_0\right) = 1$.
It follows from Λ is preserve edge that Λ is an α-admissible mapping.
From (C_7), we have, for all $\zeta, \eta \in \Omega$,

$$(\zeta,\eta)\in E(G) \text{ implies that } \mathcal{D}(\Lambda\zeta,\Lambda\eta)\leq\psi(\mathcal{D}(\zeta,\eta)),$$

and thus $\alpha(\zeta,\eta)\mathcal{D}(\Lambda\zeta,\Lambda\eta)\leq\psi(\mathcal{D}(\zeta,\eta))$.
Therefore, Λ is a $\alpha-\psi$ contractive mapping.
From condition (C_{10}), we have $(\zeta_n,\zeta_{n+1})\in E(G)$ implies that $(\zeta_n,u)\in E(G)\alpha(\zeta_n,\zeta_{n+1})\geq 1$ implies that $\alpha(\zeta_n,u)\geq 1$ for all $n\in\mathbb{N}$.
Then by Theorem 2.2, u is a fixed point of Λ. □

4 Conculsion

In the context of perturbed metric spaces, we define the $\alpha-\psi$ contraction mapping and the $\alpha-\psi$ admissible mapping . In the complete perturbed metric spaces, which generalized the metric spaces, we establish the Banach fixed point theorem. Additionally, an application scenario and an illustrative example are shown. It is important to note that these findings generalize the findings described in [16].

References

[1] H. H.Alsulami, S. Chandok, M. A. Taoudi, and I. M. Erhan, Some fixed-point theorems for (α,ψ)-rational type contractive mappings, *Fixed Point Theory Appl.*, **2015**(1)(2015), Article no. 97 .

[2] P. Amiri, S. Rezapour, N. Shahzad, Fixed points of generalized $\alpha-\psi$-contractions, *Rev. R. Acad. Cienc. Exactas Fs. Nat. Ser. A Math. RACSAM*, **108**(2014), 519-526.

[3] J. H. Asl, S. Rezapour, N. Shahzad, On fixed points of $\alpha-\psi$-contractive multifunctions, *Fixed Point Theory Appl.*,**2012**(1)(2012), Article no. 212 .

[4] M. Berzig,E. Karapnar,Note on Modified $\alpha-\psi$-contractive mappings with applications, *Thai J. Math.*, **13** (2015), 147-152.

[5] A. Ghaziyah ,E. Karapinar and P. Shahi , On rational contractions in perturbed metric spaces,*Research in Mathematics*,**12**(1)(2025), 1-8 .

[6] M. Jleli, and B. Samet, On Banach's fixed-point theorem in perturbed metric spaces, *Journal of Applied Analysis and Computation*, **15**(2)(2025), 993-1001.

[7] J. Jachymski, The contraction principle for mappings on a metric space with a graph, *Proc. Amer. Math. Soc.*,**136**(2008), 1359-1373.

[8] M. Jleli, , E. Karapnar, and B.Samet, Best proximity points for generalized (α,ψ) proximal contractive type mappings, *Journal of Applied Mathematics*, **2013**(3)(2013), Article ID 534127.

[9] M. Jleli, E. Karapnar, and B. Samet, Fixed point results for $\alpha-\psi\lambda$ contractions on gauge spaces and applications, *Abstract and Applied Analysis*, **2013**(2013), Article ID 730825.

[10] E. Karapinar, Discussion on $\alpha-\psi$ contractions on generalized metric spaces, *Abstr. Appl. Anal.*,**2014**(2014), Article ID 962784 .

[11] E. Karapnar, B. Samet, Generalized $\alpha-\psi$ contractive type mappings and related fixed point theorems with applications, *Abstr. Appl. Anal.*, **2012**(2012), Article ID 793486 .

[12] A. Latif, M. Eshaghi Gordji, E. Karapnar, and W. Sintunavarat, Fixed point results for generalized (α,ψ)-Meir-Keeler contractive mappings and applications, *J. Inequal. Appl.*, **2014**(2014), Article no. 68.

[13] B. Mohammadi,S. Rezapour, On modified $\alpha-\varphi$-contractions, *J. Adv. Math. Stud.*, **6** (2013), 162-166.

[14] J. Matkowski, Integrable Solutions of Functional Equations, *Dissertationes Mathematicae*, **127** (1975),40-45.

[15] Ozturk, M., Girgin, E. " On some fixed-point theorems for ψ-contraction on metric space involving a graph, *J. Inequal. Appl.*, **2014**(2014), Article no. 39.

[16] B. Samet, C. Vetro, and P. Vetro, Fixed point theorems for $\alpha-\psi$-contractive type mappings, *Nonlinear Anal.*, **75** (2012), 2154-2165.

[17] P. Salimi, N. Hussain, A. Latif, Modified $\alpha-\psi$-contractive mappings with applications, *Fixed Point Theory Appl.*, **2013**(2013), Article no. 151.