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ON SOFT M^*-OPEN SET IN SOFT TOPOLGICAL SPACES

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Abstract

In this paper we introduce a new class of generalized soft open sets called soft M^*-open ($\tilde{M}^*O$) sets in soft topological space. Also we discuss some of their properties. We hope that the result findings in this paper will help researcher enhance and promote the further study on soft M^*-open set in soft topological spaces to carry out a general framework for their applications in practical life.
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Keywords and Phrases: soft M^*-open sets, soft M^*-closed sets, soft M^*-interior, soft M^*-closure, soft M^*-limit point, soft M^*-derived set, soft M^*-neighborhood, soft M^*-bases and soft M^*-subbases.

1 Introduction

In 1999, Molodtsov [7] developed the concept of a soft set to handle difficult problem in economics, engineering and the environment, where no mathematical methods could effectively deal with the many types of uncertainty. The notion of soft topological spaces was introduced by Shabir and Naz [9] on initial universe with a fixed set of parameters and they investigated some properties of soft topological spaces. The concept of generalzed closed set was introduced by N. Levine [6].Many mathematicians extended the results of generalized closed sets and open sets in many directions. Weak forms of open set were first studied by Chen [4]. He investigate soft semi-open sets in soft topological spaces and studied some properties of it. Akdag and Ozkan [1] defined soft α-open (soft α-closed) sets. Alzahran, EL-Maghrabi, AL-Juhani and Badr [2] introduce soft M-open sets. M^*-open sets were introduced into general topology by A. Devika and Thilagavathi [5]. The purpose of this article is to conduct a theoretical investigation of the new set termed $\tilde{M}^*O$ and $\tilde{M}^*C$ sets over soft topological space (U, E, τ) and to analysis some of their properties.

2 Preliminaries

In this segment, we recall some necessary definitions and notations. Also in this research, we express soft operations by ‘ $\sim$ ’, soft M^*-open ($\tilde{M}^*O$), soft M^*-closed ($\tilde{M}^*C$). Throughout this entire article, we will refer to U as an initial universal set, (U, E, τ) is a soft topological space and (F, E) is a soft set over U.

Definition 2.1 ([7]). *Molodtsov Soft Set Let U be an initial universe set and let E be a set of parameters. Let $P(U)$ denote the power set of U and let $A \subseteq E$. A pair (F, E) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

Definition 2.2 ([7]). *For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if*
(i) $A \subseteq B$ and
(ii) for all $e \in A, F(e)$ and $G(e)$ are identical approximations. We write $(F, A)\tilde{\subset}(G, B)$.$(F, A)$ is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supset}\ (G, B)$.

Definition 2.3 ([7]). *The complement of a soft set (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', /A)$ where, $F' : /A \to P(U)$ is a mapping given by $F'(/\alpha) = U\backslash F(\alpha)$, for all $\alpha \in A$.*

Let us call F' the soft complement function of F. Clearly $(F')'$ is the same as F and $((F, A)')' = (F,\ A)$.

Definition 2.4 ([7]). *A soft set (F, A) over U is said to be a NULL soft set denoted by Φ if for all e $\in A, F(e) = \Phi$ (null set).*

Definition 2.5 ([8]). *Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if*
(i) $\Phi, \tilde{X}$ belong to τ.
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.6 ([9]). *Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.*

Definition 2.7 ([8]). *Let (X, τ, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ.*

Definition 2.8. *A soft set (F, A) in a soft topological space X is called:*

(i) *Soft regular open (resp. soft regular closed) set* [3] *if $(F, A) = \mathrm{int}(cl\ (F, A))$ [resp. $(F,\ A) = cl(\mathrm{int}(F, A))$].*
(ii) *Soft semi-open (resp. soft semi-closed) set* [4] *if $(F, A)\tilde{\subseteq}\,\mathrm{cl}(int(F, A))$[resp. int $(cl(F,\ A))\tilde{\subseteq}(F, A)$].*
(iii) *Soft pre-open (resp. soft pre-closed)* [3] *if $(F, A)\tilde{\subseteq}\,\mathrm{int}(\mathrm{cl}(F, A))$ [resp. $cl(int\ (F,\ A))\tilde{\subseteq}(F, A)$].*
(iv) *Soft α-open (resp. soft-α closed)* [3] *if $(F, A)\tilde{\subseteq}$ int $(cl(int\ (\ F, A)\ ))$ [resp. $cl(int\ (cl(F,\ A)))\ \tilde{\subseteq}(F, A)$].*
(v) *Soft β-open (resp. soft-β closed) set* [3] *if $(F, A)\tilde{\subseteq}\,\mathrm{cl}(\mathrm{int}(\mathrm{cl}(F, A)))$ [resp. int $(cl(int\ (F, A)))\tilde{\subseteq}(F, A)$].*
(vi) *A soft (F_1, H) in a soft topological space (U_1, τ, S) is called:*
    (i) *Soft θ-semi open set iff $(F_1, H)\tilde{\subseteq}\ cl(\mathrm{int}_\theta(F_1, H))$.*
    (ii) *Soft θ-semi closed set iff $(F_1, H)\tilde{\supseteq}\ int(\mathrm{cl}_\theta(F_1, H))$.*
(vii) *A soft (F_1, H) in a soft topological space (U_1, τ, S) is called:*
    (i) *Soft δ-pre open set iff $(F_1, H)\tilde{\subseteq}\mathrm{int}(\mathrm{cl}_\delta(F_1, H)$..*
    (ii) *Soft δ-pre closed set iff $cl(int_\delta(F_1, H))\tilde{\subseteq}(F_1, H)$.*

The union of all Soft δ-pre open sets contained in a soft (F_1, H) in a soft topological space $(U_1, \tau.\ ,S)$ is called the Soft δ-pre interior of (F_1, H). The intersection of all Soft δ-pre closed sets containing a soft set (F_1, H) in a soft topological space (U_1, τ, S) is called the Soft δ-pre closure of (F_1, H).

Definition 2.9 ([2]). *In a soft topological spaces (U_1, τ, H) a soft set (F_1, H) is called :*

(i) *Soft M-open set if $(F_1, H)\tilde{\subseteq}\,\mathrm{cl}(\mathrm{int}_\theta(F_1, H))\tilde{\cup}int(\mathrm{cl}_\delta(F_1, H))$*
(ii) Soft *M-closed set if $(F_1, H)\tilde{\supseteq}\,\mathrm{int}(cl_\theta(F_1, H))\tilde{\cap}cl(int_\delta(F_1, H))$*

3 Soft M^*-open sets

In this section we introduce soft M^*-open sets in soft topological spaces and study some of their properties.

Definition 3.1. *Let (U, E, τ) be a soft topological space. Then a soft subset (F, E) of a soft topological spaces (U, E, τ) is said to be,*

(i) *A soft M^*-open set (briefly $\tilde{M}^*O$) if $(F, E)\tilde{\subseteq}(\mathrm{int}(\mathrm{cl}(\mathrm{int}_\theta(F, E))))$.*
(ii) *A soft M^*-closed set (briefly $\tilde{M}^*C$) if $(F, E)\tilde{\supseteq}(\mathrm{cl}(\mathrm{int}(cl_\theta(F, E))))$.*

Lemma 3.1. *Let (F, E) be a soft subset of a soft topological space (U, E, τ),*

(I) *Every $\tilde{M}^*O$ is a soft θ-semi open set.*
(II) *Every $\tilde{M}^*O$ is an soft M-open set.*

Proof. (i) Obvious from the definition.
(ii) Let (F, E) be $\tilde{M}^*O$.
Then $(F, E)\tilde{\subseteq}\,\mathrm{int}(cl(\mathrm{int}_\theta(F, E)))$
$\tilde{\subseteq}cl(\mathrm{int}_\theta(F, E))$
$\tilde{\subseteq}\,\mathrm{cl}(\mathrm{int}_\theta(F, E))\tilde{\cup}\,\mathrm{int}(cl_\delta(F, E))$.
Hence (F, E) is a $\tilde{M}^*O$. ∎

Lemma 3.2. *In a soft topological space (U, E, τ), we have*

(i) *The arbitrary union of $\tilde{M}^*O$ is $\tilde{M}^*O$.*

(ii) The arbitrary intersection of $\tilde{M}^*C$ sets is $\tilde{M}^*C$.

Proof. (i) Let $\{(F,E)_\alpha : \alpha \in \wedge.$ an index set $\}$ is a collection of $\tilde{M}^*O$.

Hence for each $\alpha, (F,E)_\alpha \tilde{\subseteq} \operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)_\alpha))$. Taking the union of all such relations we get,

$$\tilde{\cup}\{(F,E)_\alpha\} \tilde{\subseteq} \tilde{\cup}(\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)_\alpha)))$$

$$\tilde{\subseteq} \operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(\tilde{\cup}(F,E)_\alpha)).$$

Hence $\tilde{\cup}(F,E)_\alpha$ is $\tilde{M}^*O$.

(ii) As (i) by taking the complements.

□

Theorem 3.1. *For a soft set (F,E) in the soft topological space (U,E,τ)*

(i) (F,E) is a soft M^*-open set iff $(F,E)^c$ is a soft M^*-closed set.

(ii) (F,E) is a soft M^*-closed set iff $(F,E)^c$ is a soft M^*-open set.

Proof. Obvious. □

4 Soft M^*-Closure and Soft M^*-Interior

Definition 4.1. *Let (F,E) be a soft subset of a soft topological space (U,E,τ). Then,*

(i) The intersection of all soft M^*-closed sets containing (F,E) is called the soft M^*-closure of (F,E) and is denoted by $\tilde{M}^*$-cl(F,E).

*(ii) The union of all soft M^*-open sets contained in (F,E) is called the soft M^*-interior of (F,E) and is denoted by $\tilde{M}^*$-*int*(F,E).*

Theorem 4.1. *The following hold for a soft subset of a soft topological space (U,E,τ).*

(i) (F,E) is $\tilde{M}^*O$ if and only if $(F,E) = (F,E)\tilde{\cap} \operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$.

(ii) (F,E) is $\tilde{M}^*C$ if and only if $(F,E) = (F,E)\tilde{\cup} \operatorname{cl}(\operatorname{int}(cl_\theta(F,E)))$.

Proof. (i) Let (F,E) be an $\tilde{M}^*O$.
Then $(F,E)\tilde{\subseteq}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$.
Hence $(F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E))) = (F,E)$.
Conversely let $(F,E) = (F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$. Then, $(F,E)\tilde{\subseteq}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$. Hence (F,E) is $\tilde{M}^*O$.
□

Theorem 4.2. *The following hold for a soft subset of a soft topological space (U,E,τ).*

(i) $\tilde{M}^*$-int$(F,E) = (F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$.

(ii) $\tilde{M}^*$-cl$(F,E) = (F,E)\tilde{\cup}\operatorname{cl}(\operatorname{int}(\operatorname{cl}_\theta(F,E)))$.

Proof. (i) since $\tilde{M}^*$-int(F,E) is $\tilde{M}^*O$,
$\tilde{M}^*$-int$(F,E)\tilde{\subseteq}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(\tilde{M}^*\text{-int}(F,E))))$
$\tilde{\subseteq}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$. Also,
$(F,E)\tilde{\cap}\tilde{M}^*$-int$(F,E)\tilde{\subseteq}(F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$.
Hence $\tilde{M}^*$-int$(F,E)\tilde{\subseteq}(F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$.
Conversely since, $\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta((F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))))$.
$\tilde{\supseteq}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta((F,E)\tilde{\cap}\operatorname{int}_\theta(\operatorname{cl}(\operatorname{int}_\theta(F,E))))))$
$\tilde{\supseteq}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)\tilde{\cap}\operatorname{int}_\theta(\operatorname{cl}(\operatorname{int}_\theta(F,E)))))$
$\tilde{\supseteq}\operatorname{int}(cl(\operatorname{int}_\theta(F,E)\tilde{\cap}\operatorname{int}_\theta(\operatorname{int}_\theta(F,E))))$
$= \operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta((F,E)\tilde{\cap}\operatorname{int}_\theta(F,E))))$
$= \operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$
$= (F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E)))$.
Hence, $\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta((F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E))))))\tilde{\supseteq}(F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(int_\theta(F,E)))$.
This implies that,
$(F,E)\tilde{\cap}\operatorname{int}(\operatorname{clc}\operatorname{int}_\theta(F,E))$ is an $\tilde{M}^*O$ contained in (F,E). Hence, $(F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E))\tilde{\subseteq}\tilde{M}^*$-int$(F,E)$.
Therefore $\tilde{M}^*$-int$(F,E) = (F,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(\operatorname{int}_\theta(F,E))$. □

5 Soft M^*-Limit Point, Soft M^*-Derived Set and Soft M^*-Neighborhood

Definition 5.1. *If (F, E) is a soft subset of soft topological space(U, E, τ). Then a point $x \in U$ is called soft M^*-limit point of a soft set $(F, E)\tilde{\subseteq}U$ if every soft M^*-open set $(T, E)\tilde{\subseteq}U$ containing x contains a point other than x.*

Definition 5.2. *The set of all soft M^*-limit points of (F, E) is called soft M^*-derived set of (F, E) and is denoted by $\tilde{M}^*$-$d(F, E)$.*

Definition 5.3. *A soft subset (F, E) of a soft topological spaces (U, E, τ) is said to be a soft M^*-neighborhood (Briefly $\tilde{M}^*$-nbd) of a point $x \in U$ if there exists an soft M^*-open set (G, E) such that $x \in (G, E)\tilde{\subseteq}(N, E)$. Then class of soft M^*-neighborhoods of $x \in U$ is called the soft M^*-neighborhood system of x and denoted by $\tilde{M}^*$-N_x.*

Theorem 5.1. *If (F, E) is a soft subset of (G, E). Then every soft M^*-limit point of (F, E) is also a soft M^*-limit point of (G, E) i.e.*
$(F, E)\tilde{\subseteq}(G, E)$ then $\tilde{M}^*$-$d(F, E)\tilde{\subseteq}\tilde{M}^*$-$d(G, E)$.

Proof. Let $p \in \tilde{M}^*$-$d(F, E)$. Then
(5.1) $((M, E)$-$\{p\})\tilde{\cap}(F, E) \neq \varnothing, \forall p \in \tau$ s.t. $p \in (M, E)$
$(F, E)\tilde{\subseteq}(G, E) \Rightarrow (F, E)\tilde{\cap}((M, E)$-$p)\tilde{\subseteq}(G, E)\tilde{\cap}((M, E)$-$\{p\}) \Rightarrow (G, E)\tilde{\cap}((M, E)$-$\{p\}) \neq \varnothing$ according to eq. (5.1) $\Rightarrow p \in \tilde{M}^*$-$d(G, E)$
Therefore $p \in \tilde{M}^*$-$d(F, E) \Rightarrow p \in \tilde{M}^*$-$d(G, E) \Rightarrow \tilde{M}^*$-$d(F, E)\tilde{\subseteq}\tilde{M}^*$-$d(G, E)$. □

Theorem 5.2. *A soft subset (G, E) of a soft topological space (U, E, τ) is soft M^*-open if and only if it is soft M^*-nbd for every point $p \in (G, E)$.*

Proof. Let (G, E) be a soft M^*-open set. Then (G, E) is a $\tilde{M}^*$-*nbd* for each $p \in (G, E)$.
Conversely let (G, E) be an $\tilde{M}^*$-*nbd* for each $p \in (G, E)$. Then there exists an soft M^*-open set $(W, E)_p$ Containing p such that $p \in (W, E)_p\tilde{\subseteq}(G, E)$. So
$(G, E) = \tilde{\cup}_{p\in(G,E)}(W, E)_p$. Therefore (G, E) is a soft M^*-open set. □

6 Soft M^*-Bases and Soft M^*-Subbases

Definition 6.1. *Let (U, E, τ), be a soft topological spaces. A class $\mathfrak{B}^*$ of soft M^*-open subsets of U, i.e. $\mathfrak{B}^*\tilde{\subseteq}\tau$, is a soft M^*-base for the soft topology τ iff*

(i) Every soft M^*-open set $(G, E) \in \tau$ is the union of members of $\mathfrak{B}^*$
(ii) For any point p belonging to an soft M^*-open set (G, E), there exists $(B, E) \in \mathfrak{B}^*$ with $p \in (B, E)\tilde{\subseteq}(G, E)$.

Definition 6.2. *Let (U, E, τ), be a soft topological spaces. A class ψ of soft M^*-open subsets of U, i.e. $\psi\tilde{\subseteq}\tau$, is a soft M^*-subbase for the soft topology τ on U iff finite intersection of members of ψ form a soft M^* base for τ.*

Theorem 6.1. *Let $\mathfrak{B}^*$ and $\mathfrak{B}^{**}$ be soft M^*-bases, respectively, for soft topologies τ^* and τ^{**} on a set U. If $(B, E) \in \mathfrak{B}^*$ is the union of members of $\mathfrak{B}^{**}$. Then $\tau^*\tilde{\subseteq}\tau^{**}$.*

Proof. Let (G, E) be a soft τ^*-open set. Then (G, E) is the soft union of members of $\mathfrak{B}^*$, i.e. $(G, E) = \cup_i(B_i, E)$ where $(B_i, E) \in \mathfrak{B}^*$. But by hypothesis, each $(B_i, E) \in \mathfrak{B}^*$ is the soft union of members of $\mathfrak{B}^{**}$ and so $(G, E) = \cup_i(B_i, E)$ is also the soft union of members of $\mathfrak{B}^*$ which are τ^{**}-open sets. Hence (G, E) is also a τ^{**}-open set and so $\tau^*\tilde{\subseteq}\tau^{**}$. □

Theorem 6.2. *If ψ is a soft M^*-subbases for soft topologies τ^* and τ^{**} on U, then $\tau^* = \tau^{**}$.*

Proof. Suppose $(G, E) \in \tau^*$. Since ψ is a soft M^*-subbases for $\tau^*, (G, E) = \cup_i(s_{i1} \cap \ldots \cap s_{ini})$ where $s_{ik} \in \psi$. But ψ is also a soft M^*-subbases for τ^{**} and so $\psi\tilde{\subseteq}\tau^{**}$; hence each $sik \in \tau^{**}$.since τ^{**} is a soft topology, $(s_{i1} \cap \ldots \cap s_{ini}) \in \tau^{**}$ and hence $(G, E) \in \tau^{**}$. Thus $\tau^*\tilde{\subseteq}\tau^{**}$. Similarly $\tau^{**}\tilde{\subseteq}\tau^*$ and so $\tau^* = \tau^{**}$. □

7 Conclusion

In the present work, we have introduced the concept of soft M^*-open set in soft topological space such as soft M^*-interior, soft M^*-closure, soft M^*-limit point, soft M^*-derived set and soft M^*-neighborhood. We also define and discuss the properties of soft M^*-bases and soft M^*-subbases.

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