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INTEGRATED MODELING OF COLD STANDBY, MAINTENANCE STRATEGY, AND REPAIR DELAY IN MACHINING SYSTEMS

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Abstract

This paper presents a comprehensive mathematical framework for analyzing the performance of machining systems incorporating cold standby units, preventive maintenance (*PM*), and repair delays. The model captures the real-world complexity of manufacturing environments, where machines may fail unpredictably, repairs are subject to resource limitations and delays, and preventive interventions are scheduled to minimize unexpected downtimes. Using a Markovian approach with state-space representation, the system is modeled through balance equations that account for various operational states, including delay queues and active maintenance. Key performance indicators such as system availability, expected number of operational machines, and server utilization are derived to quantify system efficiency under different failure, repair, and maintenance regimes. A cost function integrating repair, delay, maintenance, and unavailability penalties is formulated and optimized under system constraints. Numerical illustrations and *MATLAB*-based optimization reveal that strategic tuning of preventive maintenance frequency and repair responsiveness can significantly reduce operational costs and improve system reliability. Graphical results further demonstrate the impact of failure rate and maintenance parameters on performance metrics. This study provides valuable insights for decision-makers aiming to enhance productivity, reduce downtime, and optimize maintenance policies in complex industrial systems.

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1 Introduction

Mathematical modeling of machining systems with cold standby, preventive maintenance, and repair delay plays a crucial role in enhancing the reliability, availability, and overall efficiency of industrial production environments. In such systems, cold standby units are inactive yet available to replace failed primary machines, thus improving fault tolerance without incurring wear during standby. Preventive maintenance (*PM*) is a proactive strategy designed to service machines before failure occurs, minimizing unexpected downtime and extending equipment life. Meanwhile, repair delay accounts for real-world constraints like technician unavailability, spare part procurement, or administrative lags, which can significantly affect recovery times and system performance. By integrating these elements into a comprehensive mathematical framework often using tools like Markov processes, queueing theory, and matrix-analytic methods researchers can evaluate key performance metrics such as system availability, mean time to failure (*MTTF*), and expected repair load. These models support optimal decision-making for maintenance scheduling, resource allocation, and system design under uncertainty, contributing to cost-effective and resilient manufacturing operations.

Yang *et al.* [14] proposed a preventive maintenance policy that accounts for two-stage dependent deterioration and external shocks. Their model highlights the importance of coupling degradation processes with stochastic shock environments to develop more robust maintenance schedules. The study successfully integrates condition-based and time-based maintenance principles, setting the groundwork for

more sophisticated models in multi-state systems. Ruiz-Castro [6] addressed the reliability behavior of complex multi-state systems under the influence of repair vacations. The study introduced a flexible framework that integrates vacation policies into the repair process, providing key insights into the behavior of degraded systems that experience service interruptions. This work laid a foundational approach for later developments in cold standby modeling and repair delays. Ruiz-Castro and Dawabsha [9] advanced the modeling of multi-state warm standby systems by incorporating preventive maintenance, unit loss, and multiple repairpersons. The model acknowledged real-world challenges like the variability in repair capacity and failures during standby, offering a more comprehensive picture of availability and reliability under constrained service resources. Shekhar *et al.* [11] explored the significance of integrating vacation policies within reliability models. Their work emphasized the critical impact of repairman availability and idle time on system performance, particularly under realistic operational conditions. By reviewing various vacation models, the study contributed to broader applications of queuing theory in reliability analysis. Cha *et al.* [1] introduced the concept of delayed worse-than-minimal repair models, offering a novel perspective on preventive replacement policies. Their study provided analytical tools for comparing different repair strategies based on their long-term effects on system failure probability, proving especially useful for aging systems with repair inefficiencies. Ruiz-Castro [7] focused on optimizing cold standby systems under multiple repair vacations and unit losses. His model evaluated the interplay between standby unit availability and service interruptions, shedding light on how repair delays and multiple vacations degrade system reliability. The results emphasized the importance of synchronizing repair schedules and standby allocation. Yang *et al.* [13] presented a joint optimization framework that integrated preventive maintenance with production scheduling in multi-state systems. Leveraging reinforcement learning, the study offered a dynamic approach for adjusting maintenance and production decisions in real-time, based on system performance and resource availability. It demonstrated the feasibility of *AI* techniques in maintenance planning. Ruiz-Castro and Acal [8] employed Marked Markovian Arrival Processes (*MMAPs*) to model complex multi-state systems with embedded vacation policies. Their analytical approach captured the bursty and correlated nature of failures and repair demands, improving the realism and precision of modeling repair delays and system transitions. Gao [3] investigated a redundant system with dependent failures and variable repair rates, using advanced stochastic methods to explore system dynamics. His work underlined the impact of repair rate heterogeneity on system performance and provided optimization strategies for repair resource allocation in redundant architectures. Gao *et al.* [4] extended the analysis to series systems experiencing common-cause failures and delayed vacations. Their approach offered reliability bounds and cost-effective repair strategies in the presence of correlated failures and structured repair interruptions. This study highlighted how common-cause effects compound the risks introduced by repair delays. Dong and Bai [2] developed a preventive maintenance policy for consecutive k -out-of- n : F balanced systems subjected to shock environments. Their methodology focused on reliability optimization under high-failure-stress conditions, providing analytical tools for balancing system redundancy with scheduled maintenance to minimize downtime. Wu *et al.* [12] explored preventive maintenance in performance-sharing multi-state systems. By modeling how performance levels influence the maintenance needs of interconnected subsystems, their study provided a nuanced understanding of how degradation propagates through shared-resource systems, helping to prevent cascading failures. Ning *et al.* [5] proposed a joint optimization framework combining preventive maintenance scheduling and protective triggering mechanisms in k -out-of- n : F systems. Their approach emphasized periodic inspections and dynamic adaptation to environmental and operational conditions, offering efficient risk-reduction strategies under uncertain failure behavior. Ruiz-Castro and Zapata-Ceballos [10] presented an algorithmic modeling approach for redundant multi-state systems subject to multiple stochastic events, including preventive maintenance, unit losses, and vacation policies. Their *MMAP*-based model allowed for detailed state transitions and service dynamics, capturing the complex interdependencies among system components. This study represents one of the most comprehensive efforts in modeling modern manufacturing systems with cold standby, delayed repair, and maintenance strategies.

2 Assumption and Notations

Assuming the system operates with:

- (i) N active machines (can fail),
- (ii) S cold standby machines (not operating but ready to be activated),
- (iii) A single repair server with repair delay δ ,

- (iv) A preventive maintenance rate θ ,
- (v) A failure rate λ per machine,
- (vi) A repair rate μ ,

and states defined as a tuple (i, j, k) , where:

- i : number of operating machines,
- j : number of failed machines under repair delay,
- k : number of machines under active repair or PM.

3 Balance Equations (Steady Equations)

Let P_{ijk} denote the steady-state probability of the system being in state (i, j, k) .

State (N, 0, 0): All machines are operational; no failure, no repair yet

$$-N\lambda P_{N,0,0} + \mu P_{N-1,0,1} = 0. \quad (3.1)$$

Machines can fail at rate $N\lambda$, or a repair from state $(N-1, 0, 1)$ brings the system back to full operation.

State (N-1, 1, 0): One failure under delay; no active repair yet

$$N\lambda P_{N,0,0} - \delta P_{N-1,1,0} = 0. \quad (3.2)$$

From full operation, one machine fails, enters delay queue; it leaves delay at rate δ to go into repair.

State (N-1, 0, 1): One machine under repair, none under delay

$$\delta P_{N-1,1,0} - \mu P_{N-1,0,1} = 0. \quad (3.3)$$

Machine moves from delay to active repair (rate δ), then completes repair (rate μ).

State (N-2, 2, 0): Two failed machines under delay

$$(N-1)\lambda P_{N-1,1,0} - 2\delta P_{N-2,2,0} = 0. \quad (3.4)$$

A second failure occurs when one is already delayed. Both leave delay at rate 2δ .

State (N-1, 0, 1), triggered by PM:

$$\theta P_{N,0,0} - \mu P_{N-1,0,1} = 0. \quad (3.5)$$

Preventive maintenance starts at rate θ , treated similarly to repair; completes at μ .

State (N-2, 1, 1): One failed machine in delay, one under PM

$$\lambda P_{N-1,0,1} + \theta P_{N-1,1,0} - (\delta + \mu) P_{N-2,1,1} = 0. \quad (3.6)$$

Failure or PM leads here; exit due to delay completion or PM ending.

4 Performance Measures

- (i) **System Availability A:** Probability that at least one machine is operational.

$$A = \sum_{i>0, j, k} P_{ijk}. \quad (4.1)$$

This excludes all states with $i = 0$, i.e., when no machine is working.

- (ii) **Expected Number of Operational Machines E_{op} :**

$$E_{op} = \sum_{i, j, k} i \cdot P_{ijk}. \quad (4.2)$$

Represents the mean number of machines currently working.

- (iii) **Expected Number of Failed Machines in Delay Queue E_{delay} :**

$$E_{delay} = \sum_{i, j, k} j \cdot P_{ijk}. \quad (4.3)$$

Captures the average number of machines in the repair delay buffer.

- (iv) **Expected Number of Machines under Repair or PM E_{repair} :**

$$E_{repair} = \sum_{i, j, k} k \cdot P_{ijk}. \quad (4.4)$$

Indicates how many machines are, on average, being actively repaired or maintained.

- (v) **Probability of Complete System Failure P_{fail} :**

$$P_{fail} = \sum_{j, k} P_{0jk}. \quad (4.5)$$

This is the probability that all machines are down, either failed or under PM.

- (vi) **Server Utilization U :** Assuming a single repair/PM server, the utilization is the probability that at least one machine is under repair or PM:

$$U = \sum_{i,j,k>0} P_{ijk}. \quad (4.6)$$

- (vii) **Preventive Maintenance Rate (PM frequency):**

$$\text{PM Rate} = \theta \cdot \sum_{i,j,k} P_{ijk}. \quad (4.7)$$

- (viii) **Failure Frequency:**

$$\text{Failure Rate} = \sum_{i,j,k} i \cdot \lambda \cdot P_{ijk}. \quad (4.8)$$

5 Cost Analysis

We introduce cost components, formulate the total cost function, and then pose a constrained optimization problem.

$$\text{Total Cost (TC)} = C_r \cdot \lambda E_{op} + C_d \cdot E_{delay} + C_p \cdot \theta + C_f \cdot E_{down} + C_s \cdot U. \quad (5.1)$$

where $E_{down} = N - E_{op}$ (Expected downtime).

C_r : cost due to failures and repairs,

C_d : cost due to failed machines waiting in delay,

C_p : cost of performing preventive maintenance,

C_f : penalty due to unavailability,

C_s : operational cost of repair server.

Objective:

$$\min_{\theta, \delta, \mu} TC(\theta, \delta, \mu). \quad (5.2)$$

Subject to Constraints:

$$\text{Repairservercapacity : } U \leq 1, \quad (5.3)$$

$$\text{Availabilityrequirement : } A \geq A_{min}, \quad (5.4)$$

$$\text{Preventivemaintenancefrequencylimits : } \theta_{min} \leq \theta \leq \theta_{max}, \quad (5.5)$$

$$\text{Delayandrepairservicelimits : } \delta_{min} \leq \delta \leq \delta_{max}, \quad \mu_{min} \leq \mu \leq \mu_{max}. \quad (5.6)$$

6 Numerical Illustration

Sample Size: 2 Machines ($N = 2$), 1 cold standby ($S = 1$), 1 repair server.

State (i, j, k) : i = working machines, j = in delay, k = in repair or preventive maintenance.

Table 6.1: Numerical values of system parameters

Parameter	Symbol	Value
Failure rate per active machine	λ	0.1
Repair rate	μ	0.5
Delay rate	δ	0.2
Preventive Maintenance rate	θ	0.05

$$-2\lambda P_{2,0,0} - \theta P_{2,0,0} + \mu P_{1,0,1} = 0, \quad (6.1)$$

$$2\lambda P_{2,0,0} - \delta P_{1,1,0} = 0, \quad (6.2)$$

$$\delta P_{1,1,0} + \theta P_{2,0,0} - \mu P_{1,0,1} = 0, \quad (6.3)$$

$$\lambda P_{1,1,0} - 2\delta P_{0,2,0} = 0, \quad (6.4)$$

$$2\delta P_{0,2,0} - (\delta + \mu) P_{0,1,1} = 0, \quad (6.5)$$

$$\delta P_{0,1,1} - 2\mu P_{0,0,2} = 0. \quad (6.6)$$

Table 6.2: Possible States

Label	State (i,j,k)	Description
S_0	(2,0,0)	Both machines working
S_1	(1,1,0)	One fails, in delay
S_2	(1,0,1)	One under repair/PM
S_3	(0,2,0)	Two in delay
S_4	(0,1,1)	One in delay, one in repair
S_5	(0,0,2)	Both in repair or under PM (system failed)

Normalization Condition:

$$P_{2,0,0} + P_{1,1,0} + P_{1,0,1} + P_{0,2,0} + P_{0,1,1} + P_{0,0,2} = 1. \quad (6.7)$$

$$\text{From (6.2): } P_{1,1,0} = \frac{2\lambda}{\delta} P_{2,0,0} = \frac{2 \times 0.1}{0.2} P_{2,0,0} = P_{2,0,0}.$$

$$\text{From (6.3): } P_{1,0,1} = \frac{\delta P_{1,1,0} + \theta P_{2,0,0}}{\mu} = \frac{0.2 P_{1,1,0} + 0.05 P_{2,0,0}}{0.5} = 0.5 P_{2,0,0}.$$

$$\text{From (6.4): } P_{0,2,0} = \frac{\lambda}{2\delta} P_{1,1,0} = \frac{0.1}{2 \times 0.2} P_{2,0,0} = 0.25 P_{2,0,0}.$$

$$\text{From (6.5): } P_{0,1,1} = \frac{2\delta}{\delta + \mu} P_{0,2,0} = \frac{2 \times 0.2}{0.2 + 0.5} (0.25 P_{2,0,0}) \approx 0.142857 P_{2,0,0}.$$

$$\text{From (6.6): } P_{0,0,2} = \frac{\delta}{2\mu} P_{0,1,1} = \frac{0.2}{2 \times 0.5} (0.142857 P_{2,0,0}) \approx 0.028571 P_{2,0,0}.$$

$$\text{Using (6.7): } P_{2,0,0} + P_{2,0,0} + 0.5 P_{2,0,0} + 0.25 P_{2,0,0} + 0.142857 P_{2,0,0} + 0.028571 P_{2,0,0} = 1. \\ 2.921428 P_{2,0,0} = 1 \implies P_{2,0,0} \approx 0.3423.$$

Results:

$$P_{2,0,0} \approx 0.3423, P_{1,1,0} \approx 0.3423, P_{1,0,1} \approx 0.1712, P_{0,2,0} \approx 0.0856, P_{0,1,1} \approx 0.0489, P_{0,0,2} \approx 0.0097.$$

$$\text{Availability} \approx 1 - P_{0,0,2} \approx 0.9903.$$

$$E_{op} = 2P_{2,0,0} + 1P_{1,1,0} + 1P_{1,0,1} \approx 1.1981.$$

$$E_{delay} = 1P_{1,1,0} + 2P_{0,2,0} + 1P_{0,1,1} \approx 0.5624.$$

$$E_{repair} = 1P_{1,0,1} + 1P_{0,1,1} + 2P_{0,0,2} \approx 0.2395.$$

$$U = E_{repair} \approx 0.2395.$$

Table 6.3: Numerical values for cost analysis

Symbol	Value	Units
C_r	100	\$/repair
C_d	50	\$/unit delay
C_p	80	\$/PM
C_f	200	\$/unit downtime
C_s	30	\$/time
$\theta_{min}, \theta_{max}$	0.01, 0.1	PM rate bounds
μ_{min}, μ_{max}	0.2, 1.0	Repair rate bounds
$\delta_{min}, \delta_{max}$	0.1, 1.0	Delay rate bounds
A_{min}	0.95	availability

MATLAB Optimization Results:

Optimal PM rate (theta): 0.0100,

Optimal Delay rate (delta): 1.0000,

Optimal Repair rate (mu): 1.0000,

Minimum Total Cost: 94.53.

7 Results and Discussion

The Figure 7.1 illustrates how the performance of a machining system degrades as the failure rate (λ) increases, under various levels of repair delay (δ). The vertical axis represents the expected number of operational machines, while the horizontal axis shows the failure rate λ ranging from 0 to 0.5. Each curve corresponds to a different value of repair delay δ (from 0.1 to 1.0). As observed, for all δ values, the expected

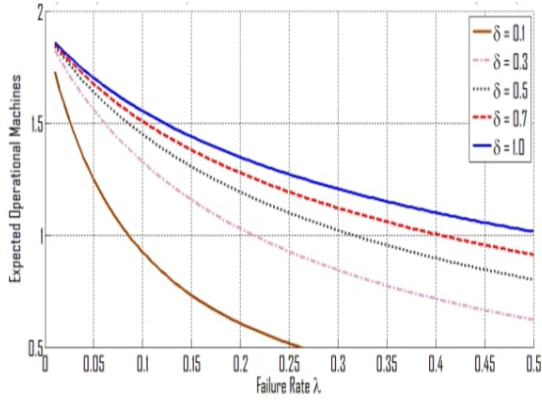


Figure 7.1: Expected number of operational machines vs. failure rate λ for different values of repair delay (δ).

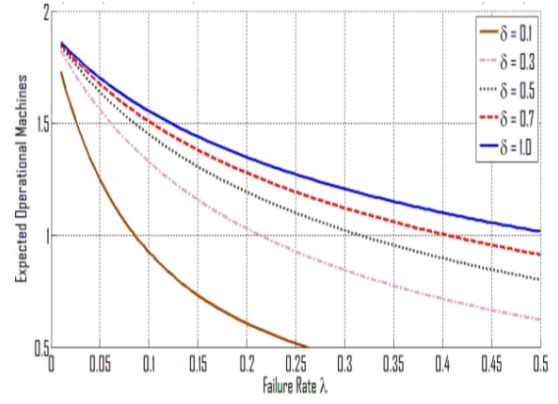


Figure 7.2: Expected number of operational machines vs. failure rate λ for different values of repair delay (δ).

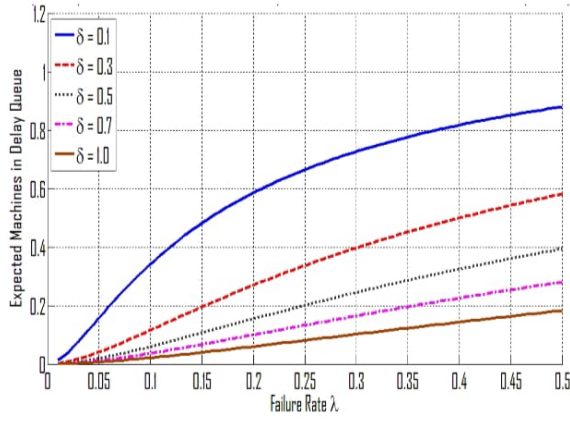


Figure 7.3: Expected number of failed machines in delay queue vs. failure rate λ for different values of repair delay (δ).

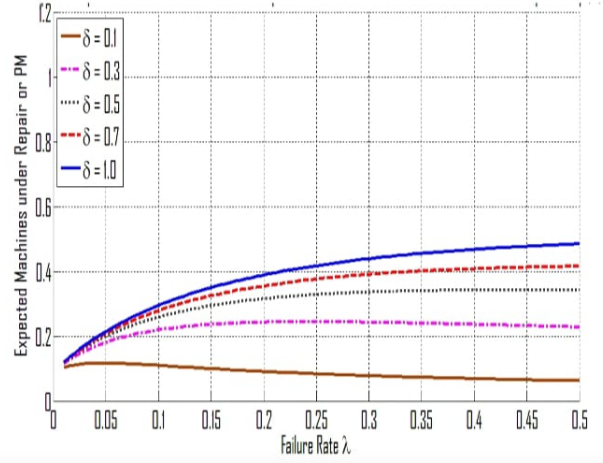


Figure 7.4: Expected number of machines under repair or PM vs. failure rate λ for different values of repair delay (δ).

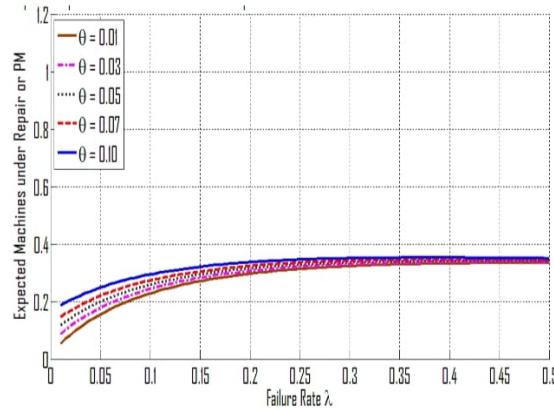


Figure 7.5: Expected number of machines under repair or PM vs. failure rate λ for different preventive maintenance rates (θ).

number of operational machines decreases with increasing λ , indicating that more frequent failures lead to fewer machines being available for operation. Furthermore, curves with higher δ values (i.e., longer repair delays) consistently lie above those with lower δ , showing that systems with faster repair (lower δ) experience a sharper drop in operational availability when failure rates increase. This suggests that repair delay plays a crucial role in sustaining machine operability, particularly under conditions of high failure risk.

The Figure 7.2 depicts the influence of varying preventive maintenance rates (θ) on the operational performance of machines as failure rate (λ) increases. The vertical axis shows the expected number of operational machines, while the horizontal axis represents the failure rate λ (ranging from 0 to 0.5). Each curve corresponds to a different value of θ (from 0.01 to 0.10). As λ increases, all curves show a decline in the expected number of operational machines, indicating that higher failure rates reduce system availability. However, the curves for different θ values are closely clustered, especially beyond $\lambda > 0.1$, suggesting that preventive maintenance rate has only a marginal impact on improving system availability under increasing failure rates. Minor differences appear in the low- λ region, where higher θ values (more frequent maintenance) slightly improve availability. This implies that while preventive maintenance does enhance reliability, its effectiveness diminishes as failure rates grow more severe.

The Figure 7.3 illustrates how the accumulation of failed machines waiting for repair (i.e., in the delay queue) varies with increasing failure rate λ for various repair delay parameters δ . The x-axis represents the failure rate (λ), while the y-axis shows the expected number of failed machines in the delay queue. Each curve corresponds to a different value of δ (from 0.1 to 1.0). The graph clearly shows that as the failure rate λ increases, the number of failed machines in the queue also increases. However, lower values of δ (i.e., higher repair delay) lead to a steeper and higher curve, meaning more machines accumulate in the queue. Conversely, higher δ values (faster repair initiation) result in flatter curves, indicating better system responsiveness and fewer queued failures. This emphasizes the critical role of minimizing repair delays to reduce system congestion and improve repair efficiency in machining systems.

The Figure 7.4 shows how the number of machines undergoing either repair or preventive maintenance (PM) changes with increasing failure rate λ , across various levels of the repair delay parameter δ . The x-axis represents the failure rate λ , while the y-axis shows the expected number of machines currently in the repair or PM phase. Each curve corresponds to a specific value of δ , ranging from 0.1 to 1.0. The trend demonstrates that as λ increases, more machines enter the repair or PM state. Importantly, higher values of δ (i.e., faster response to repair initiation) lead to higher values of the expected number under repair or PM, while lower δ values (i.e., longer delays before initiating repair) result in fewer machines actively being repaired or maintained. This is because at low δ , failures accumulate in the delay queue instead of transitioning to active repair or PM promptly. Thus, the graph emphasizes how improving the responsiveness of repair initiation (increasing δ) allows more machines to be processed for recovery, enhancing system efficiency.

Figure 7.5 illustrates the relationship between the Expected Number of Machines under Repair or Preventive Maintenance (PM) and the Failure Rate (λ) for different values of the Preventive Maintenance Rate (θ). As shown on the graph, the x-axis represents the failure rate (λ), while the y-axis shows the number of machines engaged in repair or PM activities. Each curve corresponds to a different value of θ , ranging from 0.01 to 0.10. The graph reveals that, in general, as λ increases, more machines are expected to be under repair or PM, though the increase becomes gradual at higher values of λ . It also shows that higher preventive maintenance rates (e.g., $\theta = 0.10$, blue line) result in a slightly greater number of machines in the repair or PM state. This is intuitive, as more frequent preventive maintenance increases the likelihood that machines are temporarily unavailable for operation due to ongoing maintenance. However, the differences between the curves are relatively small, indicating that θ has a subtler influence on this metric compared to other system parameters like repair delay. The graph highlights how preventive strategies impact the load on repair and maintenance capacity, especially as system stress increases due to rising failure rates.

8 Concluding Remarks

The study presents a robust mathematical framework that effectively integrates cold standby configuration, preventive maintenance strategies, and repair delay dynamics into the reliability modeling of machining systems. Through a Markovian state-space approach, it evaluates critical performance metrics such as system availability, server utilization, and the expected number of machines in repair or delay. The numerical analysis, supported by MATLAB-based optimization, reveals that minimizing preventive maintenance rate while maximizing repair and delay response rates significantly reduces operational costs and improves system efficiency. Graphical illustrations reinforce the finding that while preventive maintenance has a marginal

effect, reducing repair delays is paramount for sustaining operational readiness, especially under rising failure rates. Overall, the model provides valuable insights for industrial decision-makers to optimize maintenance schedules, manage repair resources effectively, and enhance the overall reliability of complex machining environments.

8.1 Declaration of Competing Interest

The authors declares that they have no known competing financial interest or personal relationships that could have appeared to influence the work reported in this paper.

8.2 Funding

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8.3 Data availability

No data was used for the research described in the article.

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References

- [1] J.H. Cha, M. Finkelstein and G. Levitin. On the delayed worse than minimal repair model and its application to preventive replacement, *IMA Journal of Management Mathematics*, **34**(1)(2021), 101-122.
- [2] Q. Dong and M. Bai. Reliability analysis and preventive maintenance policy for consecutive k-out-of-n-F balanced system under failure criterion operating in shock environment. *Quality and Reliability Engineering International*, **34**(2024), 108-114.
- [3] S. Gao. Reliability analysis and optimization for a redundant system with dependent failures and variables repair rates. *Mathematical and Computers in Simulation*, **208** (2023), 637-659.
- [4] S. Gao, J. Wang and J. Zhang. Reliability analysis of redundant series system with common cause failures delayed vacation, *Reliability Engineering and System Safety*, **239** (2023), 109467.
- [5] R. Ming, K. Wang, X. Zhao and Z. Li. Joint optimization of prevention maintenance and triggering mechanism for k-out-of-n:F system with protective devices based on periodic inspection, *Reliability Engineering & System Safety*, **25** (2024), 224-230.
- [6] J.E. Ruiz-Castro. A complex multistate system with vacations in the repair, *Journal of Mathematical and Statistics*, **15**(1)(2019), 225-232.
- [7] J.E. Ruiz-Castro. Optimizing a multi-state cold standby system with multiple vacations in the repair and loss of units, *Mathematics*, **9**(8)(2021), 9-13.
- [8] J.E. Ruiz-Castro and C. Acal. MMAPs to model complex multi-state systems with vacation policies in the repair facility, *Journal of Reliability and Statistical Studies*, **15**(2) (2022), 473-504.
- [9] J.E. Ruiz-Castro and M. Dawabsha. A multi-state warm standby system with preventive maintenance, loss of units and an indeterminate multiple number of repair persons, *Computers & Industrial Engineering*, **142** (2020), 178-187.
- [10] J.E. Ruiz-Castro and H.A. Zapata-Ceballos. Algorithmic modelling of a complex redundant multi-state system subject to multiple events, preventive maintenance, loss of units and a multiple vacation policy through a MMAP, *Mathematics and Computers in Simulation*, **230** (2015), 165-192.
- [11] C.H. Shekhar, S. Varshney and A. Kumar. Reliability and vacation: The critical issue in: *Advances in Reliability and its applications*, Eds. Mangey Ram and Hoang Pham, Springer Series in Reliability Engineering (2020), 251-292.
- [12] C. Wu, R. Pan, X. Zhao and X. Wang. Designing preventive maintenance for multi-state system with performance sharing, *Reliability Engineering & System Safety*, **241** (2024), 203-209.
- [13] H. Yang, W. Li and B. Wang. Joint optimization of preventive maintenance and production scheduling for multi-state production systems based on reinforcement learning. *Reliability Engineering & System Safety*, **214** (2021), 153-159.
- [14] L. Yang, X. Ma, R. Peng, Q. Zhai and Y. Zhao. A preventive maintenance policy based on dependent two-stage deterioration and external shocks, *Reliability Engineering & System Safety*, **160** (2017), 201-221.