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OPTIMIZATION INVENTORY MODEL FOR DETERIORATING ITEMS WITH STOCK DEPENDENT DEMAND AND CARBON TAX UNDER FUZZY ENVIRONMENTS

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Abstract

Inventory control models provide a structured framework for optimizing replenishment decisions under operational and economic constraints. To address the uncertainty commonly observed in real-world systems, this study proposes a fuzzy inventory control model that incorporates parameter imprecision within a fuzzy framework. The model considers deteriorating items with stock-dependent demand, allowable shortages, and complete backordering. Under the fuzzy environment, key parameters are represented as trapezoidal fuzzy numbers and defuzzified using the centroid method. A total cost function is developed to determine the optimal order quantity, the time at which inventory becomes zero, and the corresponding minimum cost. Numerical illustrations are presented for both fuzzy and crisp models, and the convexity of the cost function is verified graphically. Sensitivity analysis is conducted to examine the impact of variations in fuzzy parameters on optimal inventory decisions. The results indicate that the optimal total cost under the fuzzy environment is lower than that in the crisp case, demonstrating that incorporating fuzziness enables retailers to better capture uncertainty and improve inventory planning.

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Keywords and Phrases: Inventory model; deteriorating product; carbon tax policy; fuzzy set theory.

1 Introduction

Inventory control models form the foundation of effective supply chain management by determining optimal policies for ordering and replenishment. Their importance lies in ensuring product availability, minimizing operational costs, and enhancing overall system efficiency. In traditional inventory models, the majority of solutions are developed in deterministic environments. However, market volatility prevents them from accurately reflecting real-world problems. But this problem could be resolved with the help of fuzzy set theory introduced by Zadeh [39, 1965]. A significant research has been made in the field of fuzzy inventory control models. Vujovesvic *et al.* [31] developed an inventory control model in which the various costs like holding cost, ordering cost or more, involved in economic order quantity formula are assumed to be fuzzy. A fuzzy inventory model with or without backorder for fuzzy order quantity with trapezoid fuzzy number was developed by Yao *et al.* [34] while Lee and Yao [15] examined an economic order quantity model in fuzzy sense for inventory without backordering model. Kao and Hsu [12] discussed a single-period inventory model with fuzzy demand. Wee [32] gave a replenishment policy for items with a price-dependent demand and a varying rate of deterioration. Fuzzy inventory control model with and without backordering along with various fuzzy parameters were developed by Yao *et al.* [34, 35, 36, 37]. Syed [23] discussed a fuzzy inventory control model without shortages using signed distance method. A periodic review of inventory control model involving fuzzy expected demand and fuzzy backorder rate was given by Lin [16]. Singh and Singh [24] discussed fuzzy inventory model for finite rate of replenishment using signed distance method while Ritha [22] discussed fuzzy inventory control model for imperfect quality items with shortages. De [4]

gave a fuzzy order quantity model with fuzzy shortage quantity and fuzzy promotional index. Samal [25] optimized variable demand fuzzy economic order quantity inventory models with and without backordering. Priyam [19] gave continuous review of inventory model with controllable lead time, lost sales rate and order processing cost when the received quantity is uncertain. An *EOQ* model with backlogging under fuzzy environment was developed by De and Sana [5]. Kazemi *et al.* [13] incorporated human learning into a fuzzy *EOQ* inventory model with backorders. An *EOQ* model under fuzzy reasoning was solved by De [6].

Deteriorating items are characterized by a gradual decline in usability or value during storage, driven by factors such as spoilage, evaporation, or obsolescence. Effective management of such items necessitates specialized inventory models that incorporate deterioration rates to ensure timely replenishment and cost efficiency. Traditional inventory models can find it difficult to effectively control the uncertainties associated with deteriorating items, such as fluctuating demand, undetermined lead times, and variable degradation rates. Here fuzziness allows a more versatile and accurate approach. Various scholars have conducted research in the field of fuzzy inventory control model for deteriorating items. A fuzzy *EOQ* model for time-dependent deteriorating items and time-dependent demand with shortages was discussed by Saha [26]. Weibull deteriorating items with a time-dependent exponential demand rate are the subject of this paper's infinite time horizon fuzzy Economic Order Quantity (*EOQ*) models, where the deterioration parameter is a triangular fuzzy number. Halim *et al.* [10] developed fuzzy economic order quantity model for deteriorating items with stochastic demand, partial backlogging and fuzzy deterioration rate. Additionally, the suggested model is expanded to include the scenario in which a fuzzy value is assumed for the partial backlog factor. By using the signed distance approach to defuzzify the total cost, the minimum total cost and ideal order quantity are determined. Kumar and Rajput [14] gave fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. It is expected that the backlog rate of unmet demand decreases exponentially with waiting time. It is assumed that the demand, degradation, and backlog rates are triangular fuzzy values and goal is to defuzzify the total profit function using the centroid and signed distance methods. A fuzzy inventory model for deteriorating items with exponentially decreasing demand under fuzzified cost and partial backlogging was given by Mohanty and Tripathy [17]. This model additionally takes degradation and fuzzy-based cost components (such as holding costs and shortage costs) into account. It is assumed that all associated expenses are trapezoidal fuzzy numbers and defuzzification is accomplished using the gradient mean integration approach and signed distance. Shekarian *et al.* [27] explored an economic order quantity model considering different holding costs for deteriorating items subject to fuzziness. This study develops a fuzzy model that uses triangular fuzzy numbers (*TFNs*) to describe the fuzzified parameters and decision variables. Fuzzy arithmetic operations are used to determine the total profit per unit of time, and the graded mean integration value (*GMIV*) method is used to defuzzify the results. Indrajit *et al.* [11] discussed a fuzzy inventory model for deteriorating items with stock dependent demand rate in which the triangular fuzzy numbers represent the demand, holding cost, deterioration cost, and deterioration rate and the total cost function is defuzzified using the centroid method, signed distance method, and graded mean representation. A fuzzy inventory model for a deteriorating item with variable demand, permissible delay in payments and partial backlogging with shortage that follows inventory (*SFI*) policy was given by Shaikh *et al.* [28]. Soni *et al.* [29] *EOQ* examined an *EOQ* model of deteriorating items for fuzzy demand and learning in fuzziness with finite horizon in which an algorithm is created to calculate the number of replenishment once the model is expanded by including learning in fuzziness.

With increasing awareness of environmental impacts, incorporating carbon emission policies into inventory control models has become essential. It allows firms to align inventory planning with modern environmental standards. Arora *et al.* [1] presented a re-manufacturing inventory model to control the carbon emission using cap-and-trade regulation with the hexagonal fuzzy number. This study explored the fuzzy model of economic production for both remanufacturing and recycling with uncertain cost parameters under the cap-and-trade rule to control the carbon emission due to different modes of transportation. Due to imprecise cost parameters, the hexagonal fuzzy numbers are set to fuzzify the overall cost, which leads to correct decisions in a more confident way. The result is defuzzified by using graded mean integration. De *et al.* [7] explored carbon emission issues over production manufacturing system with the context of joint inventory control and sustainable trade credit financing for deteriorating items in a supplier-retailer-customer model under volumetric fuzzy system. Mahato *et al.* [18] devised inventory models for deteriorating items with fixed lifetime, partial backordering and dynamic demand under two different scenarios (a) carbon cap and trade policy (b) carbon tax policy. Bhardwaj *et al.* [2] developed an innovative sustainable fuzzy

inventory approach that integrates green considerations, carbon emissions, and a bi-phase credit policy.

In this article, a fuzzy inventory control model for deteriorating items with stock-dependent demand and complete backlogging is presented, both fuzzy and crisp scenarios are examined through numerical analysis, and managerial insights are derived through sensitivity analysis.

1.1 Motivation and Contribution

Inventory control models generally involve uncertain or imprecise data for demand, deterioration, and costs. This study addresses such uncertainty by modeling all functions and parameters using fuzzy mathematics, offering a robust and realistic framework for decision making in inventory. Table 1.1 compares selected inventory control studies, outlining their features, assumptions, and methodologies, and highlighting existing gaps. Building on this, the present study introduces a model that simultaneously integrates several distinctive features not previously addressed together. The main contributions are as follows:

- A key distinction of the proposed model is its treatment of demand during stock-out. Unlike conventional models, which assume demand drops to zero or equals the linear demand intercept, the present model allows demand to transition to an independent constant level. This flexibility captures realistic market behavior where demand persists regardless of inventory, extending the applicability of stock-dependent frameworks.
- All model parameters are assumed to be fuzzy parameters, distinguishing the proposed framework from most existing studies that consider fuzziness only in selected parameters.
- Trapezoidal fuzzy numbers are employed to represent parameter uncertainty, ensuring analytical tractability while realistically capturing imprecision.
- The model integrates a carbon emission policy in the form of a carbon tax, enabling the simultaneous assessment of economic and environmental impacts of inventory decisions.

Table 1.1: Overview of some of the related study

Author	Year	Demand	Carbon emission policies	Fuzzy Logic
Xu <i>et al.</i> [33]	2019	Time dependent	Yes	No
Yadav <i>et al.</i> [38]	2021	Carbon emission dependent	Yes	No
Sahoo and Acharya [30]	2022	Time dependent	Yes	Yes
Parida <i>et al.</i> [20]	2023	Time dependent	Yes	Yes
Pakhira <i>et al.</i> [21]	2024	Constant	Yes	Yes
Dave <i>et al.</i> [8]	2025	Time dependent	No	Yes
Das <i>et al.</i> [9]	2025	Price dependent	No	Yes
This paper	2026	Stock dependent	Yes	Yes

1.2 Structural outline of paper

The paper is organized as follows. All the notations used in the paper are given in Section 2 followed by the assumptions made for the mathematical model given in Section 3. Section 4 provides the formulation of mathematical model. The solution of the mathematical model is presented in Section 5. Section 6 of this paper deals with the numerical computation part of the paper. Sensitivity analysis is done in Section 7. Finally the conclusion is discussed in Section 8.

2 Preliminaries:

Definition 2.1 ([3]). *Fuzzy Set:* Fuzzy set A on universe X is defined by function $\mu_A(x)$ called the membership function of set A . Membership function basically assigns a membership value to the elements from the universal set. It determines the degree of membership of each element in fuzzy set. The membership function is a real valued function which is defined as follows:- $\mu_A(x) : x \rightarrow [0, 1]$ Where,

$$\mu_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \\ (0, 1) & x \text{ is partly in } A. \end{cases}$$

Definition 2.2 ([3]). *Trapezoidal fuzzy numbers:* Let $\tilde{A}$ be a trapezoidal fuzzy number which is represented as (a, b, c, d) where a, b, c, d are real numbers and $a \leq b \leq c \leq d$. Then the membership function $\mu_A(x)$ will

be given as

$$\mu_A(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & b < x \leq c \\ \frac{d-x}{d-c}, & c < x \leq d \\ 0, & x > d. \end{cases}$$

Definition 2.3 ([40]). *Centroid Method: The centroid (center-of-area) defuzzification of the trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ yields the crisp or defuzzified value*

$$A^* = \frac{a + 2b + 2c + d}{6}.$$

Defuzzification is the systematic procedure used to transform a fuzzy set, typically obtained from a fuzzy inference process, into a corresponding crisp numerical value. This step enables the interpretation of fuzzy conclusions in precise, actionable terms and is essential for implementing fuzzy models in real-world decision-making environments.

3 Notations and Assumptions:

3.1 Notations

The notations which are used for different variables are listed in Table 3.1.

Table 3.1: List of parameters with their description and symbols

Description of variables	Symbol of the variables
Inventory level at time t	$I(t)$
Fuzzy inventory level at time t	$\tilde{I}(t)$
Ordering quantity per order	q_d
The quantity of products that deteriorate during the interval $[0, t_0]$	P_d
Fuzzy deterioration rate	θ
The fuzzy integrated market demand	$\tilde{D}[t, \tilde{I}(t)]$
Fuzzy base demand rate when inventory is available	$\tilde{m}$
Fuzzy inventory dependent demand coefficient	$\tilde{n}$
Fuzzy demand rate	$\tilde{D}_0$
The point of time where the inventory becomes zero	t_0
Cycle length of inventory	T
Fuzzy total cost function	$\tilde{TC}$
Fuzzy unit buying cost	$\tilde{a}$
Fuzzy per unit inventory holding cost per unit time	$\tilde{b}$
Fuzzy per unit shortage cost	$\tilde{c}$
Fuzzy per unit handling cost of the deteriorated product	$\tilde{d}$
Fuzzy fixed cost per order	$\tilde{K}$
Fixed carbon emissions per order	A_e
Per unit carbon emission of orders	c_1^e
Per unit carbon emission for inventory per unit time	h_1^e
Fuzzy carbon tax under the carbon tax policy(rupees/per unit carbon emission)	$\tilde{\lambda}$

3.2 Assumptions

The assumptions for the proposed mathematical model are stated as follows:

- The lead time is assumed to be zero; that is, the time elapsed between placing an order and receiving the ordered items is negligible.
- The total length of the inventory cycle (or planning horizon) is denoted by T .
- The demand rate is assumed to be stock-dependent during the positive-inventory period and is given by $m + nI(t)$, where m represents the base demand rate when inventory is available, reflecting the fundamental level of customer demand independent of stock visibility, and n denotes the inventory-dependent demand coefficient, which measures the rate of change in demand with respect to inventory

level $I(t)$. Once the inventory level reaches zero, the demand becomes constant at the base rate D_0 . This assumption captures both the influence of available inventory on customer demand and the persistence of baseline demand during stock-out periods.

- Shortages are permitted and are completely backlogged.
- A single type of deteriorating item is considered in the inventory control model. The item is assumed to be in its optimal condition initially and begins to deteriorate over time until t_0 . The deterioration rate is assumed to be constant and is denoted by θ .
- Carbon emissions are assumed to arise from both inventory holding and transportation activities. To mitigate environmental impact, a carbon tax policy is imposed, whereby a cost proportional to the amount of carbon emitted is charged. This policy incentivizes firms to optimize inventory and transportation decisions so as to reduce both operational costs and carbon emissions.
- The uncertain parameters in the model are represented using fuzzy set theory. In particular, trapezoidal fuzzy numbers are employed to effectively capture the vagueness and imprecision inherent in real-world inventory systems.

4 Formulation of Mathematical Model:

In this model, during the initial interval $[0, t_0)$, the demand is assumed to be dependent on the inventory level and is expressed as $m + nI(t)$, where m denotes the base demand rate when inventory is available, n is the inventory-dependent demand coefficient, and $I(t)$ represents the inventory level at time t . Once the inventory is depleted, the demand becomes constant at the rate D_0 and is completely backlogged until the end of the cycle at time T . The integrated demand function is denoted by $D[t, I(t)]$ and is defined as follows:

$$D[t, I(t)] = \begin{cases} m + nI(t), & 0 \leq t < t_0 \\ D_0, & t_0 \leq t < T. \end{cases}$$

The items considered in this model are of a deteriorating nature and are assumed to deteriorate at a constant rate θ . The maximum inventory level at time $t = 0$ is denoted by S_0 , which gradually decreases and becomes zero at time t_0 due to the combined effects of demand and deterioration. Shortages are permitted and are completely backlogged during the interval $[t_0, T]$. At time T , the next replenishment occurs, restoring the inventory level to S_0 .

In real-world situations, it is often difficult to describe system parameters precisely using exact numerical values alone. Therefore, the above model is extended using fuzzy set theory, wherein the parameters m , n , D_0 , and θ are treated as imprecise. For instance, a decision-maker may be unable to specify an exact value for the deterioration rate; however, based on market studies, an approximate range within which the deterioration rate typically lies may be available. To capture such vagueness and uncertainty, the model represents these parameters using trapezoidal fuzzy numbers, thereby reflecting the imprecision inherent in their estimation from market surveys and empirical data.

The new demand function in fuzzy environment $\tilde{D}[t, \tilde{I}(t)]$ is taken as follows:

$$\tilde{D}[t, \tilde{I}(t)] = \begin{cases} \tilde{m} + \tilde{n}\tilde{I}(t), & 0 \leq t < t_0 \\ \tilde{D}_0, & t_0 \leq t < T. \end{cases} \quad (4.1)$$

Since the values of these parameters are specified in an imprecise manner, their corresponding defuzzified values are obtained using the centroid method. By substituting these defuzzified parameters into equation (4.1), the resulting crisp demand function is derived as follows:

$$D_d[t, I_d(t)] = \begin{cases} m_d + n_d I_d(t), & 0 \leq t < t_0 \\ D_{0d}, & t_0 \leq t < T. \end{cases}$$

Here x_d denotes the defuzzified value of parameter x .

In this model, the change in level of inventory at any point of time for an item following the demand is given in equation 4.2

$$\frac{dI_d(t)}{dt} = \begin{cases} -m_d - (n_d + \theta_d)I_d(t) & 0 \leq t < t_0 \\ -D_{0d} & t_0 \leq t < T. \end{cases} \quad (4.2)$$

Solving equation (4.2) with the condition $I_d(t_0) = 0$ gives $I_d(t)$ as follows

$$I_d(t) = \begin{cases} \frac{m_d}{n_d + \theta_d} [e^{(n_d + \theta_d)(t_0 - t)} - 1], & 0 \leq t < t_0 \\ D_{0d}(t_0 - t), & t_0 \leq t < T. \end{cases}$$

The initial inventory level can be computed by $I_d(0)$ in equation which gives

$$S_0 = I_d(0) = m_d t_0 \left\{ 1 + \frac{(n_d + \theta_d)t_0}{2} \right\}.$$

The total number of items deteriorated in the time interval 0 to t_0 can be computed as

$$P_d = S_0 - \int_0^{t_0} D_d(t, I_d(t))dt = \frac{m_d t_0^2 \theta_d}{2} - \frac{m_d n_d (n_d + \theta_d) t_0^3}{6}.$$

The order quantity in this cycle is calculated as:-

$$q = S_0 + \int_{t_0}^T D_d(t, I_d(t))dt = m_d t_0 + \frac{m_d t_0^2 (n_d + \theta_d)}{2} + D_{0d} (T - t_0).$$

To compute the components of the total cost function, the purchasing cost, holding cost, storage cost, handling cost of deteriorated items, carbon emission cost, and deterioration rate are treated as fuzzy parameters. All parameters are represented using trapezoidal fuzzy numbers, which are defuzzified to obtain crisp values for determining the optimal inventory decisions.

Let $\tilde{m} = (m_1, m_2, m_3, m_4)$, $\tilde{n} = (n_1, n_2, n_3, n_4)$, $\tilde{D}_0 = (D_{01}, D_{02}, D_{03}, D_{04})$, $\tilde{a} = (a_1, a_2, a_3, a_4)$, $\tilde{b} = (b_1, b_2, b_3, b_4)$, $\tilde{c} = (c_1, c_2, c_3, c_4)$, $\tilde{d} = (d_1, d_2, d_3, d_4)$, $\tilde{K} = (K_1, K_2, K_3, K_4)$, $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$ are trapezoidal fuzzy numbers.

Here, centroid method is employed here to defuzzify the fuzzy parameters. If $\tilde{x} = (x_1, x_2, x_3, x_4)$ is the fuzzy trapezoidal number then the defuzzified value of $\tilde{x}$ using centroid method is $x_d = \frac{x_1 + 2x_2 + 2x_3 + x_4}{6}$.

Now the components of total cost function under crisp environment are computed as follows:-

Purchase cost:

$$c_1 = a_d q = a_d \left\{ m_d t_0 + \frac{m_d t_0^2 (n_d + \theta_d)}{2} + D_{0d} (T - t_0) \right\}.$$

Holding cost:

$$c_2 = b_d \int_0^{t_0} I(t)dt = \frac{b_d m_d t_0^2}{2} \left[1 + \frac{(n_d + \theta_d)t_0}{3} \right].$$

Shortage cost:

$$c_3 = c_d \int_{t_0}^T (-I(t))dt = c_d D_{0d} \left[\frac{t_0^2}{2} + \frac{T^2}{2} - T t_0 \right].$$

Handling cost of deteriorated products:

$$c_4 = d_d P_d = d_d \left\{ \frac{m_d t_0^2 \theta_d}{2} - \frac{m_d n_d (n_d + \theta_d) t_0^3}{6} \right\}.$$

Fixed ordering cost:

$$c_5 = K_d.$$

Carbon tax policy cost:

$$c_6 = \lambda_d \left\{ A_e + c_1^e \left\{ m_d t_0 + \frac{m_d t_0^2 (n_d + \theta_d)}{2} + D_{0d} (T - t_0) \right\} + h_1^e \left\{ \frac{m_d t_0^2}{2} \left[1 + \frac{(n_d + \theta_d)t_0}{3} \right] \right\} \right\}.$$

The retailers total cost function is as follows:

$$TC_d = a_d \left\{ m_d t_0 + \frac{m_d t_0^2 (n_d + \theta_d)}{2} + D_{0d} (T - t_0) \right\} + \frac{b_d m_d t_0^2}{2} \left[1 + \frac{(n_d + \theta_d)t_0}{3} \right] + c_d D_{0d} \left[\frac{t_0^2}{2} + \frac{T^2}{2} - T t_0 \right] \quad (4.3)$$

$$+ d_d \left\{ \frac{m_d t_0^2 \theta_d}{2} - \frac{m_d n_d (n_d + \theta_d) t_0^3}{6} \right\} + K_d + \lambda_d \left\{ A_e + c_1^e \left\{ m_d t_0 + \frac{m_d t_0^2 (n_d + \theta_d)}{2} + D_{0d} (T - t_0) \right\} \right. \\ \left. + h_1^e \left\{ \frac{m_d t_0^2}{2} \left[1 + \frac{(n_d + \theta_d)t_0}{3} \right] \right\} \right\}. \quad (4.1)$$

Then, this optimization problem is described as follows:

$$\text{Min } TC_d(t_0),$$

$$\text{Subject to } t_0 > 0.$$

This model can also be applied directly to crisp inputs by substituting the corresponding crisp values for the fuzzy parameters. In other words, instead of using defuzzified values from fuzzy inputs, the model can directly use crisp parameter values to determine the optimal inventory decisions.

5 Numerical Illustration and Convexity Validation:

To illustrate the model discussed above, two randomly generated data sets are presented and solved using *LINGO 18.0* under both fuzzy and crisp environments. In the first subsection 5.1, the inventory model is analyzed in a fuzzy environment that illustrates the optimal behavior of the total cost function under a fuzzy environment. In this case, the uncertain parameters are represented by trapezoidal fuzzy numbers to capture real-world imprecision and to assess the robustness of the proposed model in the presence of uncertainty. In the second subsection 5.2, the inventory model is analyzed in a crisp environment while maintaining the same demand structure and model assumptions. All parameters are treated as deterministic and are assigned precise or crisp numerical values, which are substituted into Equation (4.3). The model is then solved numerically to obtain the optimal time when inventory becomes zero, optimal order quantity, and the minimum total system cost. This crisp analysis serves as a computational benchmark for evaluating the performance of the proposed model.

5.1 Fuzzy Model

Let us consider $\tilde{a} = (6, 8, 12, 14)$, $\tilde{b} = (0.012, 0.016, 0.024, 0.028)$, $\tilde{c} = (0.6, 0.8, 1.2, 1.4)$, $\tilde{d} = (3, 4, 6, 7)$, $\tilde{K} = (60, 80, 120, 140)$, $\tilde{D}_0 = (16, 18, 22, 24)$, $\tilde{\theta} = (0.016, 0.018, 0.022, 0.024)$, $\tilde{n} = (0.006, 0.008, 0.012, 0.014)$, $\tilde{\lambda} = (0.06, 0.08, 0.12, 0.14)$ are given trapezoidal fuzzy inputs. Optimizing the above problem (Equation (4.3)) under fuzzy environment, we get the optimal value of TC_d i.e. $TC_d^* = 25890.32$, optimal value for t_0 i.e. $t_0^* = 30.84652$ and $q_d^* = 1495.6905$.

The convexity of the cost function TC_d with respect to t_0 for fuzzy case is shown graphically in the Figure 5.1.

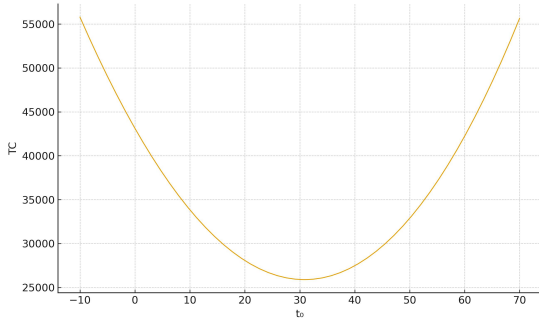


Figure 5.1: Total cost as function of t_0 (fuzzy model)

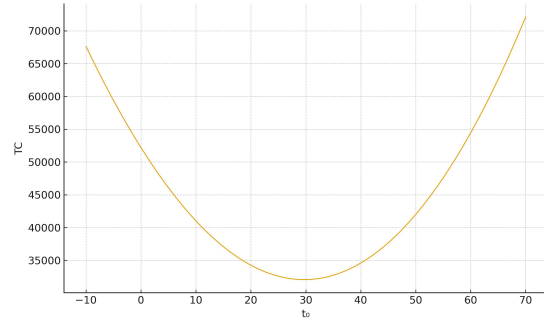


Figure 5.2: Total cost as function of t_0 (crisp model)

5.2 Crisp model

Instead of fuzzy inputs, if the crisp values are given as $T = 56$, $a = 11$, $b = 0.022$, $c = 1.1$, $d = 5.2$, $M = 105$, $D_0 = 22$, $\theta = 0.021$, $n = 0.013$, $m = 24$, $\lambda = 0.12$ then after optimizing Equation 4.3, we get the optimal total cost i.e. $TC_d^* = 32065.10$, optimal value for time when inventory becomes zero i.e. $t_0^* = 29.71537$ and optimal amount of quantity to be ordered i.e. $q_d^* = 1651.6951$.

The convexity of the cost function TC_d with respect to t_0 for crisp model is shown graphically in the Figure 5.2.

Upon comparing the optimal total costs obtained from the fuzzy and crisp models, it is observed that the total cost incurred under the fuzzy environment is lower than that of the crisp environment. Additionally, inventory depletion occurs later in the fuzzy environment.

6 Sensitivity Analysis

In this section, a sensitivity analysis is conducted to examine the impact of variations in key parameters on the optimal total cost of the system. Each parameter is independently decreased and increased by 50% to investigate its influence on the optimal depletion time (t_0^*), the economic order quantity (q^*), and the optimal total cost (TC_d^*) under both crisp and fuzzy environments. Tables 6.1 and 6.2 provide a systematic summary of the simulated results. Several important observations and managerial insights emerge from this analysis, which are discussed in detail in the subsequent section.

6.1 Observations

- The numerical results, along with the sensitivity analysis presented in Tables 6.1 and 6.2, indicate that the total cost under the fuzzy environment (25,890.32) is consistently lower than that under the crisp environment (32,065.10). This difference of 19.25% arises because the fuzzy approach incorporates parameter uncertainty through membership functions, allowing the model to distribute variability more flexibly across demand, deterioration rate, and other cost components. Consequently, the fuzzy total cost provides a more realistic and conservative estimate compared with the rigid, single-point assumptions of the crisp model.
- The results also indicate that incorporating a carbon tax policy increases the total cost of the inventory system in both crisp and fuzzy models. Specifically, increasing (decreasing) the carbon emission cost under the carbon tax policy (λ) by 50% leads to an increase (decrease) in total cost of 6.01% (7.24%) and 5.96% (7.09%) in the crisp and fuzzy environments, respectively. This increase occurs because the carbon tax directly penalizes emissions generated during ordering, transportation, storage, and deterioration processes, thereby adding an explicit cost component to the total system cost.
- Sensitivity analysis further reveals that the total cost is highly responsive to changes in both demand and purchasing cost in both environments. Even moderate fluctuations in these parameters result in significant variations in overall cost. For instance, a 50% increase (decrease) in demand (D_0) results in a total cost change of +18.2289% (−32.3350%) in the crisp environment and +17.1990% (−31.4086%) in the fuzzy environment. Similarly, a 50% increase (decrease) in purchasing cost (a) leads to a total cost change of +27.7002% (−29.0887%) in the crisp environment and +28.2901% (−29.5910%) in the fuzzy environment.
- The sensitivity analysis also shows that the total cost is relatively less sensitive to the inventory-dependent demand coefficient (n), shortage cost (c), carbon emission cost (λ), and deterioration rate (θ). The total cost increases moderately with changes in these parameters and is largely insensitive to holding cost and the handling cost of deteriorated products.

6.2 Insights for practitioners

- The numerical results emphasize that incorporating fuzziness into inventory parameters can reduce overall cost, particularly in systems with uncertain demand, deterioration, or other cost factors. By effectively capturing parameter vagueness, the fuzzy model provides a more flexible and economical framework for decision-making, enabling managers to establish reliable ordering policies under uncertainty. This suggests that companies dealing with imprecise data common in cases of deteriorating goods, fluctuating demand, or real-time supply chain disruptions can significantly lower total costs by adopting fuzzy-based inventory planning instead of traditional deterministic approaches.
- The incorporation of a carbon tax policy in the model indicates that firms operating under carbon taxation regimes must adjust their inventory strategies to account for the additional costs associated with environmental compliance. Managers are encouraged to adopt greener transportation modes, invest in low-emission preservation technologies, optimize order quantities and frequencies to minimize emission-related expenses, and redesign processes to reduce carbon footprints. Although carbon taxes may increase short-term operational costs, they incentivize firms to adopt more energy-efficient and sustainable practices, which can ultimately lower emissions and reduce long-term costs.
- Since the total cost in the model is highly sensitive to demand patterns and purchasing costs, managers must focus on accurate estimation and continuous monitoring of these parameters. Firms should enhance demand forecasting using advanced predictive tools or real-time data, negotiate stable supplier contracts to reduce purchasing cost volatility, consider bulk purchasing agreements where feasible, and review ordering policies more frequently under high demand uncertainty. Recognizing the significant impact of demand and purchasing costs on overall expenses helps managers implement strategies to stabilize costs, mitigate financial risk, and enhance the reliability of the inventory system.

Table 6.1: Sensitivity analysis of parameters for crisp model

Parameter	% change	t_0^*	TC_d^*	Change in TC_d^*	q^*
a	-50	33.0750	22737.76	-29.0887	1651.6958
a	+50	26.9098	40947.21	+27.7002	1651.6958
b	-50	29.9683	31907.79	-6.4905	1651.6958
b	+50	29.46783	32219.45	+0.4813	1651.6958
c	-50	20.45807	26423.64	-17.5937	1651.6958
c	+50	35.0416	35395.95	+10.3877	1651.6958
d	-50	30.3077	31583.11	-1.5031	1651.6958
d	+50	29.1729	32500.17	+1.3568	1651.6958
λ	-50	32.9865	29741.79	-7.2456	1651.6958
λ	+50	27.1587	33994.89	+6.0183	1651.6958
m	-50	41.9955	20849.67	-34.9770	1171.8238
m	+50	22.02416	38.840 .42	+21.1298	1837.1971
n	-50	30.96951	31203.74	-2.6862	1610.4425
n	+50	28.64823	32816.94	+2.3447	1688.1669
D_0	-50	16.6682	21696.84	-32.3350	946.0407
D_0	+50	37.13576	37910.23	+18.2289	2076.4365
θ	-50	33.2273	29737.37	-7.2593	1731.382
θ	+50	26.9274	33931.24	+5.8198	2009.5396

Table 6.2: Sensitivity analysis of parameters for fuzzy model

Parameter	% change	t_0^*	TC_d^*	Change in TC_d^*	q^*
a	-50	34.1107	18229.09	-29.5910	1495.6905
a	+50	28.0868	33214.72	+28.2901	1495.6905
b	-50	31.1151	25752.04	-0.5340	1495.6905
b	+50	30.5837	26025.97	+0.5239	1495.6905
c	-50	21.4746	21555.85	-16.7416	1495.6905
c	+50	36.12410	28388.69	+9.6498	1495.6905
d	-50	31.5548	25438.47	-1.7452	1495.6905
d	+50	30.1587	26323.97	+1.6749	1495.6905
λ	-50	33.9833	24053.85	-7.0932	1495.6905
λ	+50	28.3475	27434.59	+5.9646	11995.6905
m	-50	43.0953	16593.58	-35.9081	1038.5811
m	+50	23.0133	31615.29	+22.1123	1681.3308
n	-50	29.9353	26415.95	+2.03021	1426.3102
n	+50	31.8885	25302.22	-2.27150	1575.1814
D_0	-50	17.4883	17758.52	-31.4086	870.7870
D_0	+50	38.2872	30343.21	+17.1990	1857.4526
θ	-50	34.5568	23868.31	-7.8099	1451.83153
θ	+50	27.9127	27506.04	+6.2406	1512.6376

7 Conclusions

This study developed a fuzzy inventory control model for deteriorating items with dynamic, stock-dependent demand, allowing shortages and complete backlogging. Key parameters were modeled as trapezoidal fuzzy numbers and defuzzified using the centroid method. The numerical illustrations were provided to optimize the total cost functions in both (fuzzy and crisp) environments using *LINGO 18.0*, and the convexity of the cost function was validated graphically using *MATLAB*. The results demonstrate that incorporating fuzziness lowers total cost by improving the handling of parameter uncertainty and enabling more robust and flexible inventory planning, making it particularly valuable for retailers managing deteriorating items

with stock-dependent demand. Sensitivity analysis was performed on the parameters, $m, n, D_0, a, b, c, d, \lambda$ and θ for both crisp and fuzzy models in Table 6.1 and Table 6.2 respectively which reveals that total cost is highly responsive to changes in demand and purchasing cost, emphasizing the importance of monitoring these factors for effective inventory management. Additionally, the analysis of carbon tax policy highlights that regulatory constraints increase total costs, suggesting that firms should adopt energy efficient technologies or preservation strategies to maintain sustainability and cost efficiency. Overall, the proposed fuzzy framework offers practical advantages over traditional models, supporting firms in making informed, sustainable, and cost effective inventory decisions under uncertain and imprecise market conditions.

This study provides a flexible framework for understanding inventory systems under uncertainty, while leaving room for further refinement. The model could naturally accommodate more complex demand patterns, different deterioration behaviors, and alternative shortage scenarios, and the fuzzy parameter approach allows for richer representations of uncertainty. At the same time, the sustainability perspective considered here could be explored in broader inventory contexts, helping to make the model even more practical and relevant for decision making in the real world.

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