

Jñānābha, Vol. 56(1P) (2026), 116-121

ANALYZING SOCIAL NETWORKS STRUCTURALLY USING PETRI NETS

Gajendra Pratap Singh¹, Nikita², Pooja³ and Riddhi Jangid^{4,*}

^{1,4}School of Computational and Integrative Sciences Jawaharlal Nehru University, New Delhi-110067

^{2,3}Department of Mathematics, Kamla Nehru College, University of Delhi, India-110067

*Corresponding author

Email: gajendra@mail.jnu.ac.in, nikita.mat2325@knc.du.ac.in, poojaa@knc.du.ac.in,
riddhijangid97@gmail.com

(Received: August 01, 2025, Revised: February 15, 2026; Accepted: June 18, 2026)

DOI: <https://doi.org/10.58250/jnanabha.2026.561P-13>

Abstract

Social networks are commonly expressed, modelled and analyzed using discrete mathematics as graph theory. This paper proposes a directed bipartite based model in the framework of Petri-net for behavioral and structural analysis of social works using places, transitions, directed arcs, and marking vectors. The approach establishes a formal relationship between graph-based social network structures and Petri-nets, enabling the analysis of connectivity, surfaces, P and T- invariants, marked graph, liveness, reachability, and structural properties. Illustrative examples demonstrate the applicability of Petri nets for representing and analyzing social network structures.

2020 Mathematical Sciences Classification: 05C82, 68R10, 68M11, 68Q85

Keywords and Phrases: Social networks, Graph theory, Petri nets, Reachability, Invariants, Firing Rule, Marking Vector.

1 Introduction

Social networks have been studied for a long-time using graph theory and it has been proven a powerful way to analyze such networks in different ways. Even we use the term 'graph' and 'network' interchangeably very oftenly. A graph is nothing but a set of vertices (nodes) linked with a set of arcs (edges). Nodes in a graph depicts the entities that we want to model and edges in the graph represent the connection between the nodes in our network. For instance, suppose we want to show the relation of 'friendship' among a group of people. The graph constructed is such that each node in the graph will depict the persons among which we want to show the relationship. And links are shown between only the persons who are friends with each other by the undirected arcs. Petri nets, A formalized modeling language which goes back around 1960s, has been used to describe and model discrete event dynamic systems (*DEDS*). These are topologically bipartite multi-digraphs, where we study and use some well-defined couples of rules with the digraph and in turn helps in analyzing *DEDS*. Bipartite directed graph means, we have two nonempty disjoint sets of nodes denoting using oval/circles (called places) and rectangles/bar (called transitions) and flow relation is denoted by directed arc in the graph. The arcs must be used to connect oval/circles and bar/rectangles but in such a way that no direct connection in between place to place or transition to transition. An edge which is directed from circle/place to rectangle/transition is the pre place of that bar/rectangle and vice-versa.

Marking/Token vector in a Petri Net: We assign dots/ token or write number to the places, called as marking vector. These tokens/markings are further used for change of state/execution of Petri nets. Theoretically, marking or n -vector = $(M_1, M_2, M_3, \dots, M_n)$, where n is the cardinality of number of circles/places in the Petri net structure and each $M_i \in \{0, 1, 2, 3, \dots, n\}$. In many cases, we assign some non-negative integer to the arcs in a Petri net, which we call as weight of the arc. and if weight is not assigned, we take it 1 by default.

Formally we define Petri net as a 5-tuple $PN = (\text{Place Set } P, \text{Transition Set } T, \text{Flow Relation } F, \text{Weight}$

Function W , Initial Marking M_0) (See the Fig 1.1), where:

$P = \{p_1, p_2, p_3, \dots, p_m\}$: A nonempty set of places,

$T = \{t_1, t_2, t_3, \dots, t_n\}$: A nonempty of transitions,

$F \subseteq (P \times T) \cup (T \times P)$: Flow-relation/arc set,

$W = F \rightarrow \{1, 2, 3, \dots\}$: Weight function,

$M_0 = P \rightarrow \{0, 1, 2, 3, \dots\}$: Marking Function called the initial marking vector.

Here $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$.

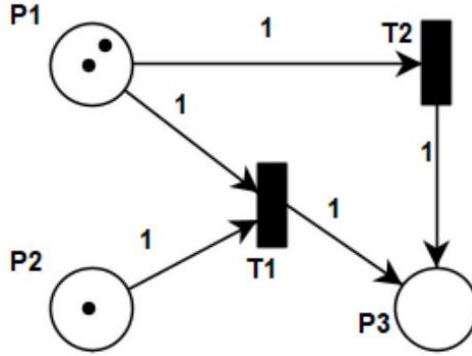


Figure 1.1: An illustration for a marked Petri net with initial marking vector $(2,1,0)$.

2 Petri net analysis using reachability

Firing-rule in Petri Net: Consuming of $w(p, t)$ tokens weight from pre place of a transition and producing $w(t, p)$ tokens weight into post place of the transition, responsible for the Petri Nets's change of state, called the transition firing rule. Here, $w(p, t)$ place to transition arc weight and similarly $w(t, p)$ means transition to place arc weight. In the following illustration shown in Fig 2.1 and Fig 2.2 (Reachability tree), we consider equal arc weight $w(p, t) = w(t, p) = 1$ for all the transitions.

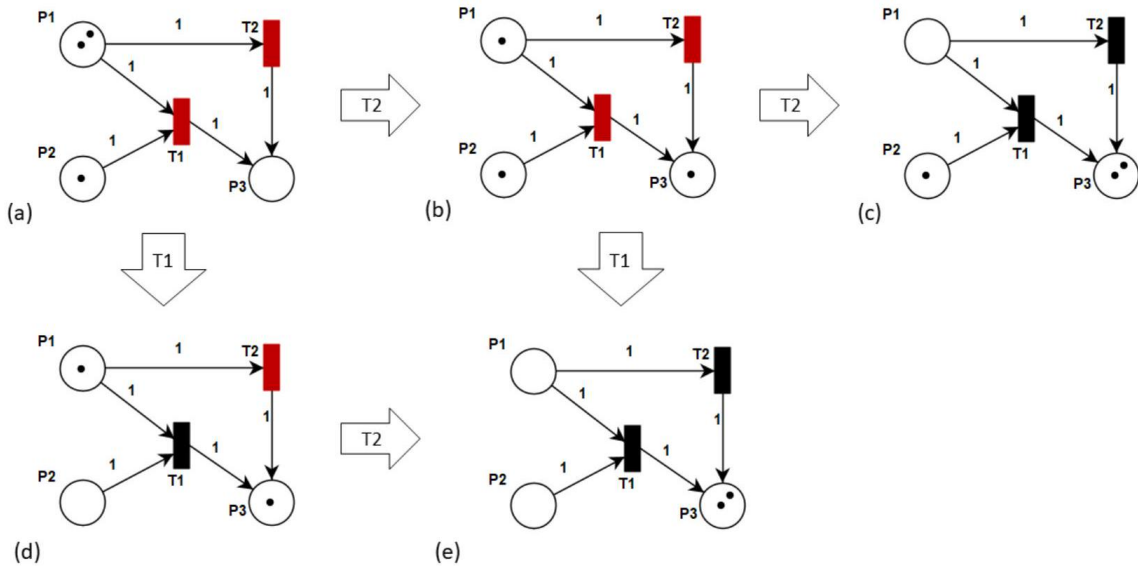


Figure 2.1: Illustration of firing rule in Petri net depicted in Fig.1.1.

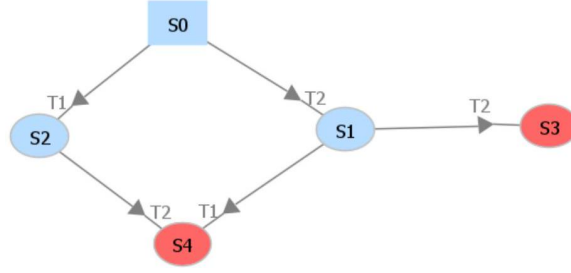


Figure 2.2: Reachability/Coverability graph of Petri net in Fig.1.1.

In the concept of social network analysis, we wish to study on how many different ways there are for a person to create connections with any other person in the community or an organization if they are unknown to each other directly using Petri nets.

3 Methodology and Construction

We have chosen a very small network, a group of seven people in an organization. We have shown the number of people in the network using a set of places in the Petri net. Now there can be multiple conditions one can think of to study the network-

- i. Initially when there is no interconnection among any person in the organization, there is one manager who is leading the organization and who recruits all the other employees into the organization. Such networks can be easily understood as similar to star networks. (Fig. 3.1.)
- ii. Other possibilities that can be studied are there can be multiple connections among employees before joining the organization. Creating and removing the links between the star networks can easily be understood for visualizing.

Studying the first case, we have a recruiter of the employees in an organization and we have depicted here the graphical model in the figure below. P0 place in the figure shows there is a recruiter of the organization.

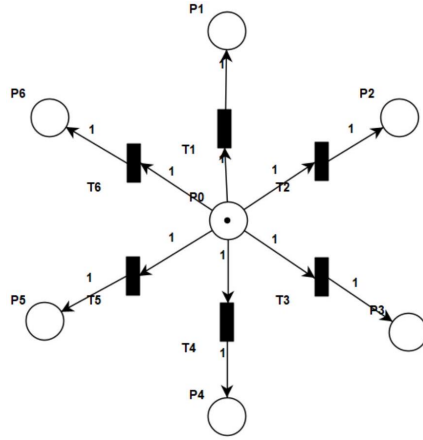


Figure 3.1: Petri net illustrating a kind of social network

This Petri net can be studied as a dual of the Safe Star Petri net as defined by Kansal *et al.* [3] in Boolean Petri nets. The social network as discussed can be studied using reachability-based analysis of this Petri net. For validation, we have shown here the generated reachability/coverability graph of the Petri net. The reachability graph shows us the connections that are there among the employees of the organization (see Fig. 3.2).

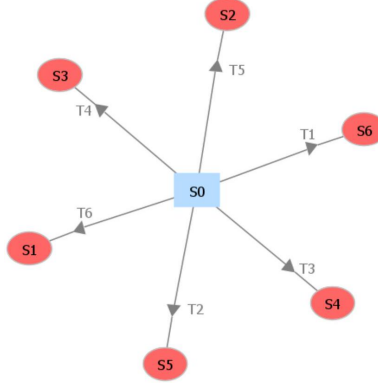


Figure 3.2: Reachability/Coverability graph of Petri net in Fig.3.1.

4 Reachability/Coverability Graph Results

Generating Reachability graph took 0.082s

Constructing it took 0.064s

Total time was 0.146s

We can clearly see from the results that the computation time is very fast. Besides computation time, there are many more results obtained with the Petri net analysis results that are shown below. Creating Incidence matrix for the state space analysis and looking for all possible firings and markings of the Petri net, we have used here the free software available for Petri nets, Platform Independent Petri Net Editor (*PIPE* v4.3.0). Besides this, we can also study the structural and behavioral properties of the Petri net. For e.g. Boundedness, Liveness, Safeness, Home State and Reversibility, etc. using the software.

Our aim was to look for possible shortest connection among the community, this is depicted using the shortest path to deadlock. The figure only depicts one path to deadlock3

5 Petri net state space analysis results

Combined incidence matrix /

	T1	T2	T3	T4	T5	T6
P0	-1	-1	-1	-1	-1	-1
P1	1	0	0	0	0	0
P2	0	1	0	0	0	0
P3	0	0	1	0	0	0
P4	0	0	0	1	0	0
P5	0	0	0	0	1	0
P6	0	0	0	0	0	1

(a)

Marking

	P0	P1	P2	P3	P4	P5	P6
Initial	0	1	0	0	0	0	0
Current	1	0	0	0	0	0	0

Enabled transitions

T1	T2	T3	T4	T5	T6
yes	yes	yes	yes	yes	yes

(b)

Petri net state space analysis results

Bounded	true
Safe	true
Deadlock	true

Shortest path to deadlock: T1

(c)

Figure 5.1: PIPE v4.3.0 analysis results of Petri net in Fig. 3.1.

There are another analysis techniques also available in the Petri net literature besides Reachability, that are Matrix and State Equations. One can study Invariant analysis results and Petri net classification results shown in the figure below (see Fig 5.1, Fig 5.2 and Fig 6.1).

Petri net classification results

State Machine	true
Marked Graph	false
Free Choice Net	true
Extended Free Choice Net	true
Simple Net	true
Extended Simple Net	true

Figure 5.2: Classification analysis results of Petri net in Fig. 3.1

6 Petri net invariant analysis results

Petri net invariant analysis results

T-Invariants

T1 T2 T3 T4 T5 T6

The net is not covered by positive T-Invariants, therefore we do not know if it is bounded and live.

P-Invariants

P0 P1 P2 P3 P4 P5 P6
1 1 1 1 1 1 1

The net is covered by positive P-Invariants, therefore it is bounded.

P-Invariant equations

$$M(P0) + M(P1) + M(P2) + M(P3) + M(P4) + M(P5) + M(P6) = 1$$

Analysis time: 0.0s

Figure 6.1: Invariant analysis results of Petri net in Fig. 3.1.

7 Conclusions and Scopes

In this article, we have shown graphical model for social relationships with the help of graph theory, Petri nets and readability. This idea may be applied to study in various biological pathways and process where there is a possibility of interconnection of nodes.

Acknowledgement. The second author, Dr. Gajendra Pratap Singh is thankful to the Department of Science and Technology(DST)-Science and Engineering Research Board (SERB) Projects (PID:MTR/2021/000378), DBT/BIC, and the authors are expressing their deep gratitude to anonymous reviewers and editors for their valuable suggestions and comments.

References

- [1] N. J. Dingle, W. J. Knottenbelt and T. Suto, PIPE2: a tool for the performance evaluation of generalised stochastic Petri Nets, *ACM SIGMETRICS Performance Evaluation Review*, **36**(4) (2009), 34–39.
- [2] T. Freytag and M. Sanger, WoPeD—An Educational Tool for Workflow Nets, *BPM (Demos)*, **31** (2014).
- [3] S. Kansal, M. Acharya and G. P. Singh, Boolean petri nets, in P. Pawlewski (Ed.), *Petri nets—Manufacturing and Computer Science*, (2012), 381–406.
- [4] S. Kansal, G. P. Singh and M. Acharya, A Disconnected 1-Safe Petri Net Whose Reachability Tree Is Homomorphic to a Complete Boolean Lattice, in *2011 International Conference on Process Automation, Control and Computing*, IEEE (2011), 1–5.
- [5] M. A. Marsan, Stochastic Petri nets: an elementary introduction, in *European Workshop on Applications and Theory in Petri Nets*, Springer, Berlin, Heidelberg (1988), 1–29.
- [6] T. Murata, Petri nets: Properties, analysis and applications, *Proceedings of the IEEE*, **77**(4) (1989), 541–580.
- [7] M. Notomi and T. Murata, Hierarchical reachability graph of bounded Petri nets for concurrent-software analysis, *IEEE Transactions on Software Engineering*, **20**(5) (1994), 325–336.
- [8] J. L. Peterson, *Petri Net Theory and the Modeling of Systems*, Prentice Hall PTR (1981).
- [9] M. Pezzè, R. N. Taylor and M. Young, Graph models for reachability analysis of concurrent programs, *ACM Transactions on Software Engineering and Methodology (TOSEM)*, **4**(2) (1995), 171–213.
- [10] Y. Saad and M. H. Schultz, Topological properties of hypercubes, *IEEE Transactions on Computers*, **37**(7) (1988), 867–872.
- [11] G. P. Singh, Applications of Petri nets in electrical, electronics and optimizations, in *2016 International Conference on Electrical, Electronics, and Optimization Techniques (ICEEOT)*, IEEE (2016), 2180–2184.
- [12] G. P. Singh and A. Gupta, A Petri net analysis to study the effects of diabetes on cardiovascular diseases, in *2019 6th International Conference on Computing for Sustainable Global Development (INDIACom)*, IEEE (2019), 481–488.
- [13] G. P. Singh and S. K. Singh, Petri net recommender system for generating of perfect binary tree, *International Journal of Knowledge and Systems Science (IJKSS)*, **10**(2) (2019), 1–12.
- [14] G. P. Singh, S. K. Singh and M. Jha, Existence of forbidden digraphs for crisp boolean Petri nets, *International Journal of Mathematical, Engineering and Management Sciences*, **5**(1) (2020), 83.
- [15] J. Wang, Petri Nets for Dynamic Event-Driven System Modeling, *Handbook of Dynamic System Modeling*, **1** (2007), 24.