

Jñānābha, Vol. 56(1P) (2026), 101-107

**A GRAPH-THEORETIC APPROACH TO CHATURANGA TURANGA BANDHĀM IN
ANCIENT INDIAN KĀVYĀ**

Rama Vijaykumar

Department of Mathematics, S.I.W.S. N.R. Swamy College of Commerce & Economics and
Smt. Thirumalai College of Science (Autonomous), Wadala, Mumbai, Maharashtra, India- 400031

Email: ramav@siwscollege.edu.in

(Received: August 01, 2025; Revised: June 08, 2026; Accepted: June 10, 2026)

DOI: <https://doi.org/10.58250/jnanabha.2026.561P-11>

Abstract

Ancient Indian scriptures are a vast ocean of knowledge, embedding profound insights within verses, shlokas, and sutras. These texts often connect deeply with disciplines such as mathematics, medicine, astronomy, astrology, cosmology, and philosophy. This paper aims to explore a specific verse that discloses the existence of graph-theoretic concepts in ancient Indian text, dating back to the 9th century A.D. Through an analytical study of a shloka arranged in the poetic structure of Kāvyaṃ, which stands out for its intellectual and visual appeal, we uncover a representation of the Knights Tour on a chessboard traversing each cell exactly once without repetition. This poetic construct mirrors the properties of a Hamiltonian path and introduces the foundational idea of what is now formalized as a Hamiltonian graph, thus showcasing a remarkable early instance of graph-theoretic thought in ancient Indian tradition.

2020 Mathematical Sciences Classification: 05C45 (Primary), 05C38, 68R10, 01A32 (Secondary)

Keywords and Phrases: : Kāvyaṃ, Chitrakāvyaṃ, Chaturanga, Turanga, Shlokas, Hamiltonian Path, Bipartite graph and Regular graph.

1 Introduction:

Chaturanga Turanga (Knight) Bandhām in Ancient India is a smart solution to a mathematical puzzle, which solves the knight tour problem. The knight tour problem states that the chess knight travels each and every cell exactly once, which was once deeply rooted in our ancient Indian work. Knight tour problem is beautifully explained in our ancient Indian text with the help of poem or Kāvyaṃ around 9th century, and as a mathematical problem it was first noticed by Leonhard Euler in 18th Century [7]. This paper provides the exploration of the knight tour problem in our Indian texts along with the graphical theoretical approach.

2 Meaning of Chaturanga Turanga Bandhām:

Chaturanga means chess board & Turanga means horse or knight which is a coin of a chess game. Chaturanga Turanga Bandhām, a technique which cleverly describe the movement of the Turanga in chaturanga to solve knight tour problem.

3 Origin of the Knight Tour Problem:

Early known mathematician to work on Knight problem was Leonhard Euler [7, 1759], he attempted to find a knight tour that will allow knight to visit each grid once and come back to the origin. Euler was not successful in getting these results but it became a base for the mathematicians who came after him like De Moivre [6], Vandermonde [13, 1774], H. C. Von Warnsdorff [1, 1823], Pratt [11, 1825], Legendre [8, 1830] and De Lavernade [5, 1839]. Warnsdorff's method [1] was to place the knight on the first cell of an empty 8x8 board and move as per knights L steps rule, ensuring that each cell is visited exactly once.

0	59	38	33	30	17	8	63
37	34	31	60	9	62	29	16
58	1	36	39	32	27	18	7
35	48	41	26	61	10	15	28
42	57	2	49	40	23	6	19
47	50	45	54	25	20	11	14
56	43	52	3	22	13	24	5
51	46	55	44	53	4	21	12

Figure 3.1: Warnsdorffs Knight tour Problem.

According to Warnsdorffs Rule, Knight follows the following steps:

- The Knight can start from any initial cell; in the above figure, the starting cell is marked as 0.
- The first L step of knight leads to the cell marked 1, the second step to 2, and so on.
- For each step, the Knight must move L steps to an unvisited cell/grid, selecting the cell with the minimal degree.
- The process will be followed until the Knight has visited all the cell exactly once.

In 20th Century the research work connected to Knight tour took a new direction, problem was extended to square and rectangle of different sizes and dimension to get different possible solution. The knight tour problem was treated as an instance for Hamiltonian path problem in graph theory.

4 Poetry meets Mathematics- Knight journey in ancient Indian texts:

Sanskrit is an old classical language of Hindus, and lot of our ancient work is written in Sanskrit script. Sanskrit poetry or Kāvyaṃ [9] is rich for its uniqueness and diversity which flows from Mahakāvyaṃ (epic Kavyas), devotional kāvyāṃ and chitra kāvyāṃ (figure or image poetry).

Before stating the knight tour in ancient India, it is worth going through the background of kāvyāṃ. Let us provide with the brief introduction about sabda alankara and chitrakāvyaṃ. Sabda alankara namely word-based ornamentation consists of well recognized alliteration and rhyme, which in Sanskrit resembles to anuprasa and yamaka. Anuprasa (alliteration) corresponds consonant which is repeatedly used in line of verse, and where as yamaka (rhyme) is associated with end of the line in a verse.

Chitrakāvyaṃ, is categorized into śabda-chitra (sound-based patterns), artha-citra (meaning-based patterns), and ubhaya-citra (mixture of both). It includes subtypes like bandha -geometric or object-based patterns, gati-movement-based patterns such as gomūtrika (cows zigzag path), rathapada (chariot movement), and turagapada (knights movement). This work sometimes represented as Adhama kāvyāṃ inferiority poetry, which targets the emotion and used to demonstrate the poetic and mathematical powers together.

5 Rudrata 4 x 8 knight tour:

In India the solution for the knight tour problem was known as early as 9th century AD which were a pair of verses in the form of kāvyām. The earliest example of this can be seen in Sanskrit treatise Kāvyaṭaṅkāra by Rudrata (9th Century). This type of kāvyā in sanskrit comes under chitrakāvyaṃ which means figure or image poetry. Rudrata Kāvyaṭaṅkāra, a sanskrit kāvyām works gives the pattern of knight steps on the chess board. It is a four-line verse read from left to right or by the path followed by the knight, where each syllable represents the single square of grid/cell [4]. Rudrata Kāvyaṃ verses are as follows:

अथ तुरगपदपाठः—

सेना लीलीलीना नाली लीनाना नानालीलीली ।

नालीनालीले नालीना लीलीली नानानानाली ॥१५॥

→ Sutra 1

The following translation is based on Choudharys Kāvyaṭaṅkāra [4], verse 15: I, who is a truthful well-read man, a group leader and helpful to my servants, praise the army whose leader a man who praises playful persons.

Rudratas verse is a kāvyām in which the syllables of the verses are first arranged in an 8x4 grid from left to right. Remarkably, the same syllables of the verse can be read again from the beginning by following the knights L moves, forming a Hamiltonian path. According to the syllabic arrangement of the verse, the knights follows the following path stated in the Figure 5.1:

१ से	३० ना	६ ली	२० ली	३ ली	२४ ना	११ ना	२६ ली
१६ ली	१९ ना	२ ना	२९ ना	१० ना	२७ ली	४ ली	२३ ली
३१ न	८ ली	१७ ना	१४ ली	२१ ले	६ ना	२५ ली	१२ ना
१८ ली	१५ ली	३२ ली	७ ना	२८ ना	१३ ना	२२ ना	५ ली

Figure 5.1: Representation of Turagapadabandha on 8x4 grids.

According to Figure 5.1, Firstly the syllable of the verse is arranged in 8x4 grid form (i.e. 8 columns and 4 rows), Now by taking the syllable of verses from left to right exactly the way it appears in an 8x4 grid, Turaga starts from से which is marked as 1, from here the first L step of knight moves to ना which is marked as 2. Following the syllable of the verses, Knight move from ना to ली which is marked as 3, then from ली to ली, marked as 4 and so on, the Turga follows the syllable of sutra 1 verses and moves on without repeating any of the cell or revisiting any of the cell, this uninterrupted movement of knight forms the bases for Hamiltonian path, which is a fundamental concept of graph theory. Thus Rudrata Kāvyaṭaṅkāra verses not only gives the literary classiness but also states the movement of Knight, landing to an example of Hamiltonian path.

6 Desikas 8x4 knight tour problem:

In 13th century Swami Vedanta Desika [2], who was an Indian scholar, expertise in poetic work of philosophical & religious framed a pair of verses for 8x4 knight tour problem -Chaturanga Turanga Bandhm, which is a poetic technic used in the Paduka Sahasram, meaning thousand verses on the sandals God Ranganatha, it is a Sanskrit 1008 verse kāvyām. Swami Vedanta Desika composed works in Sanskrit, Tamil and Manipravalam (a Sanskritised form of Tamil). He is regarded as a great Āchārya of his era and one of a true polymath, excelling in numerous skills including mathematics, civil, law, Medicine, kāvyām, philosophy, teaching, sports, acting, magic, linguistics, cryptography and many more.

Unlike Rudrata single verse solution, Desikas pair has two specific 32 syllable verses from Paduka sahasram i.e. 929 and 930, which were emphasized to the solution of knight problem about 500 years before Euler. The verses were meaningful, aligned with the approach of traditional chitrakāvya style and presented in 8x4 grid form. The movement or path of the knight is reflected in the way the verses are structured and read. The verse 929 talks about the filling up of half chess board i.e. 8x4 grids where the other one specifies the movement of the knight on the chess board. Following verses highlights, the beauty of literature, Mathematics, and the game of chess. The verses listed below carries a poetic or kvym feel, intertwined with the Mathematical classiness of Knights path.

स्थिरागसां सदाराध्या विहताकततामता ।
सत्पादुके सरासा मा रङ्गराजपदं नय ॥²¹

→ Sutra 2 (929 verse)

स्थिता समयराजत्पागतरा मादके गवि ।
दुरंहसां सन्नतादा साध्यातापकरासरा ॥

→ Sutra 3 (930 verse)

The following is a translation from Swami Vedanta Desika [2]: Sutra 2 (929 verse)- Oh sacred sandals of the Brahman, you are worshipped by those who have committed unforgiveable sins; you eliminate all that is sorrowful and unpleasant; you lead me to jingling/joyfully to the feet of Lord Rangaraja.

Sutra 3 (930 verse)- O sandals, you protect those who shine by their right attitude, whose place is the center of the sun blissful rays, you abolish the intoxication of the distressed; your radiance brings peace to those who take refuge in you, which move everywhere, may! those sandals which are golden and radiating lead me to the feet of Lord Rangaraja.

Figure stated below gives the 8x4 grid formation along with the knight path, formed with the help of two verses 929 and 930, which have to be read together as one set,

स्थि	रा	ग	सां	स	दा	रा	ध्या
1	30	9	20	3	24	11	26
वि	ह	ता	क	त	ता	म	ता
16	19	2	29	10	27	4	23
स	त्पा	दु	के	स	रा	सा	मा
31	8	17	14	21	6	25	12
रं	ग	रा	ज	प	द	न्न	य
18	15	32	7	28	13	22	5

Figure 6.1: Desikas Knight Tour Problem.

Working of Desikas verses: Desikas pair consists of two specific 32 syllable verses-929 and 930, Using verse 929, he arranges the syllables in an 8 x 8 grid from left to right. Then, with the assistance of verse 930, the path of Turanga (knight) is stated. The knight follows the sequence of syllables from verse 930, moving through the grid without repeating any of the cell and covering each and every cell once. This sequence forms the basis for a Hamiltonian path and a bipartite graph. Bhoja is another important poet of that century, who has given knights tour problem solution in his book on poetics, Sarasvatīkahābharaam. He described the chess puzzle as a mathematical problem and a literary challenge [9].

7 Ratnakara 4 x 8 Knight tour:

Rajanaka Ratnakara [9], a 9th-century Kashmiri poet, is best known for his epic Sanskrit kāvyām Hārāvijāyām meaning victory of Hara (God shiva) over the demon Andhaka. This kāvyām consists of 50 sargas (cantos or chapters) that gives in the details of the heroic deeds of Lord Shiva. His work is famous for its complexity, rich metaphors and the vivid description of Shivas celestial city and his powerful Tāṇḍava dance.

Rudrata, Bhoja and Vedanta Desikas approach was different from Ratnakaras approach, He employed a different knight tour path that was closed, meaning the knight returns to its initial point, forming a Hamiltonian cycle. He used 64 syllables for this structure. The verses of Ratnakara from his Hārāvijāyām kāvyām are as follows:

।तामभ्ययुङ्क्त यासङ्क्ष्यसङ्ख्यावद्विर्वृता भटैः ।

सूचिरोम्णः चमूनदैः सूचितप्रलयागमा ॥ १४४ ॥

भासेद्धाहाततनना सादिमायादसेविता ।

→ Sutra 4

सासहस्ततयाशान्तसाराहितसमिद्युता ॥ १४५ ॥

सहसादितसेनाशा तयासेसाहितद्युता ।

याता वित्रस्तरासाद्धा सभामातदनन्तसित् ॥ १४६ ॥] → Sutra 5

The following translation is based on Rajanaka Ratnakaras Sanskrit kāvyām Hārāvijāyām (verse 144,145 & 146), translated by G. S. S Murthy [9]:

Fearless roar of Suchiromanhs Army, which was served by horsemen and magicians, having swords in hand, and comprised of soldiers protected by the large fearless army, whose sounds indicated the arrival of total destruction and which attacked Kali, and the great force was defeated by her.

The three verses together form a solo sentence. The second verse (145) is the main one, which arranges the syllables in a 4 x 8 grid, whereas the third verse (146) provides the knight tour path.

	A	B	C	D	E	F	G	H
1	भा26	से11	द्वा24	हा5	त20	त9	न30	ना7
2	सा23	दि4	मा27	या10	द29	से6	वि19	ता16
3	सा12	स25	ह2	स्त21	त14	या17	शा8	न्त31
4	सा3	रा22	हि13	त28	स1	मि32	द्यु15	ता18

Figure 7.1: Ratnakara 4 x 8 Knight Tour Problem.

8 Graphical Framework for the Knights Tour Problem:

Graph Construction for the Knights Tour Problem is as follows:

- The knight placed on any cell of an empty board, is moved on the basis of knight rule.
- For building knight tour graph take each cell of chess board as node or vertices.
- Each L (legal) steps by the knight can be marked as edge in the graph.
- L steps moves knight one square along one axis and two steps along the other axis.
- For $n \times m$ knight tour graph the total number of vertices is $n \times m$ only.
- For $n \times n$ knight tour graph the total number of vertices is $n \times n = n^2$ and the total number of edges is $4(n-2)(n-1)$.
- Initial condition for knight tour path is $n \geq 5$ Knight follows L move and steps on each cell only once and cover each and every nodes presenting Hamiltonian path.
- Knight moves alternate nodes between white and black, example if the first knight step is in black cell the second step will be in white, third jump will be in black and so on, hence knight tour graph is bipartite graph along with Hamiltonian path. It is k -partite graph with value of $k = 2$.
- Knight graph follows two colorable graph and its chromatic number is 2.
- Degree of each vertex in knight graph is 2, therefore it is also known as regular bipartite graph, as degree is same for each vertex and in general every bipartite graph is perfect.
- It is a perfect graph as for every induced subgraph, chromatic number (Minimum number of colors needed) is equal to clique number (Maximum number of mutually connected vertices) i.e. $\chi(G) = \omega(G) = 2$
- Knight path can be represented as tree (Tree is a special type of graph which is undirected, connected and acyclic, and there is only one path between two nodes), with each node as position on chess board and the movement is represented by edges. The root of the tree is the starting position which follows the path and reaches the leaf node.

9 Conclusion

The Knights Tour problem, though widely associated with mathematics and chess, finds its origins in ancient intellectual traditions, including early Indian and Arabic mathematical texts. These historical verses reveal not just a fascination with the knight's steps movement, but also a deeper understanding of structure, logic, and strategy long before modern graph theory framed such problems. By expressing the Knights Tour as a graph, where each square of the chessboard is a vertex or node and each valid knight move is an edge. We bridge ancient curio with modern mathematical frameworks. This transformation helps us to apply powerful graph theory concepts such as Hamiltonian paths, Bipartite graph, tree, Perfect graph and algorithmic traversal strategies, presenting both analytical depth and computational possibilities.

Ultimately, the Knights Tour stands as an endless example of how ancient mathematical thinking in the form of shlokas and verses, which when interpreted through modern graphical lenses, continues to inspire exploration, innovation and bridging ancient Indian scholarship with modern analysis. This exploration not

only celebrates India's intellectual heritage but also underlines the cross-cultural appreciation of mathematical heritage.

Acknowledgment

On the successful completion of my dissertation, I would like to extend my sincere thanks to my guide, Dr. Prabha Rastogi, for encouraging me to delve into ancient mathematical works. I am also grateful to the editor and the reviewer for their appreciation and valuable suggestions, which have helped bring my paper to its present form. I would also like to thank the supporting staff for their assistance and coordination throughout this process.

References

- [1] <https://www.geeksforgeeks.org/warnsdorffs-algorithm-knights-tour-problem/>, 2025.
- [2] https://sridharshanam.org/wp-content/uploads/2019/01/Dextreous-Desika_1.pdf, 2019.
- [3] W. W. R. Ball and H. S. M. Coxeter, *Mathematical Recreations and Essays*, Dover Publications, 1987.
- [4] S. Choudhary, *Kāvyaṅkārā of Rudraṭa with Hindi Vyākhyā*, Delhi: Vasudev Prakashan, 1965. (In Sanskrit and Hindi).
- [5] J. E. De Lavernède, *Solution du problème du cavalier au jeu des échecs*, Mémoires de l'Académie de Nmes, (1839), 297–303.
- [6] A. De Moivre, *Notes on Knight's Tour Problems* (cited in W. W. R. Ball and H. S. M. Coxeter, *Mathematical Recreations and Essays*, Dover Publications), (1987).
- [7] L. Euler, *Solution d'une question curieuse qui ne paroît soumise aucune analyse*, Mémoires de l'Académie Royale des Sciences et Belles Lettres de Berlin, **15** (1759/1766), 310–337.
- [8] A.-M. Legendre, *Théorie des Nombres*, 3rd ed., Firmin Didot Frères, Paris, France, 1830.
- [9] G. S. S. Murthy, *The Knight's Tour Problem and Rudrata's Verse: A View of the Indian Facet of the Knight's Tour*, *Resonance*, **25**(8), (2020), 1095–1116.
- [10] R. R. Pāṭhaka, *Citrakāvya-kautukam*, Motilal Banarsidass, 1979. (In Sanskrit).
- [11] P. Pratt, *Studies of Chess*, 6th ed., London: Samuel Bagster, 1825.
- [12] Rudraṭa, *Kāvyaṅkārā*, Bombay: Nirnayasagar Press, 1886. (In Sanskrit).
- [13] A.-T. Vandermonde, *Remarques sur des problèmes de situation*, Mémoires de l'Académie Royale des Sciences, Paris, (1771/1774), 566–574.
- [14] Warnsdorff, H. C. von, *Des Rsselsprungs einfachste und allgemeinste Lsung*, Schmalkalden, 1823.