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MAGNETO-CONVECTION IN A VISCOELASTIC FLUID SATURATED DEVELOPING SOLUBLE ANISOTROPIC POROUS MEDIUM

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Abstract

We consider the thermo-solutal magneto-convection in a horizontal anisotropic porous medium, saturated with viscoelastic fluid, which is heated and salted from below. Here, the chemical equilibrium is on the bounding surfaces, and the solubility of the dissolved components depends on temperature. The aim is to determine the criteria for the onset of magneto-convection by finding the critical Rayleigh number and wave number. Also, heat and mass transfer phenomena are captured by studying Nusselt and Sherwood numbers. The extended Darcy model is used to express the momentum equation for Oldroyd-type viscoelastic fluid with an externally imposed vertical magnetic field. Due to viscoelastic behaviour of the fluid, the convection is set in through oscillatory rather than stationary. An entire investigation has been done in two parts: (i) linear stability analysis and (ii) weakly non-linear stability analysis. The effect of main controlling parameters, such as the magnetic field parameter (Chandrashekhar number Q), viscoelastic parameters (relaxation (λ_1) and retardation (λ_2) parameters), and effective chemical reaction (i.e. Damköhler number, (χ)), on the stability of the system are investigated. Q, λ_2 are found to delay the oscillatory convection whereas λ_1 and χ speed up the onset of oscillatory convection. The non-linear theory based on the truncated representation of the Fourier series method predicts the occurrence of sub-critical instability in the form of finite amplitude motion. The effect of the above-mentioned parameters on heat and mass transfer is also discussed.
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1 Introduction

Viscoelastic fluid, a non-Newtonian fluid, has the interesting ability to develop both viscous and elastic properties under the same conditions. Due to these properties, it can be visualized in different types of liquids, colloids, polymers, organic and polymer alloys, and a number of biological materials. These non-Newtonian fluids retain stress even in the absence of a gradient of velocity and the ensuing ability to produce highly non-linear behaviour, regardless of the considered the specific chemical composition. Its molecules are also showing deformation (evolving with a characteristic time that does not match that of the main flow), while its initial flow can produce long-chain stretched molecules and can cause secondary flows, which further stretch them, thereby allowing the amplification of an initial small disturbance through an iterative cause-and-effect coupling mechanism. These fluids have a large number of technological applications in the chemical, cosmetic, pharmaceutical, materials, energy, and food industries. These applications are well mentioned in the research work of [1, 2, 5, 7, 21, 26, 27, 37]. Thermal convection in saturated porous media is significant in a wide range of technological applications, and it is becoming more interesting in geothermal energy [28] and astrophysical problems [3]. Theoretically, different mathematical models have been used to study the behavior of thermal convection in viscoelastic fluid saturated porous medium. Recently, many studies have examined the rheological characteristics associated with viscoelastic flow in porous media. Herbert [9] and Green [8] were the first, who investigated oscillatory convection problem in a viscoelastic fluid of the Oldroyd type under the condition of infinitesimal perturbations. Using linear stability theory, Yoon *et al.* [41] examined the onset of oscillatory convection in a horizontal porous layer saturated with viscoelastic fluid. Laroze *et al.* [14] found that the nature of convection solution depends on viscoelastic parameter after studying the effect of viscoelastic fluids on bifurcations of convective instabilities. The stability of Maxwell fluid in a porous medium using Darcy-Brinkman-Maxwell model is examined by Tan and Masuoka [38] and they found the criteria for the onset of oscillatory convection. Stability criteria for both stationary and oscillatory convection in viscoelastic liquid saturated anisotropic porous layer is analysed by [17]. Sheu *et al.* [15] studied the chaotic convection of viscoelastic fluids in porous media and found that the flow behavior may be stationary, periodic, or chaotic.

All above mentioned research was carried out for one-component, i.e. a pure fluid. However, several applications are known in which the fluid has to be treated as a binary mixture consisting of fluid and solute (see, for example [22]). Extension of the Hortan-Rogers-Lapwood problem for double diffusive convection in a porous layer with Newtonian fluid was initialized by [19]. In his work, he has used the linear theory of stability analysis and considered the iso-advective transport. For the classical Rayleigh-Benard system with and without porous medium, there has been a large amount of work dealing with binary mixtures and it is well documented in the book written by Nield and Bejan [20].

Initially, the effect of the chemical reaction on the double-diffusive convection in porous medium was introduced by Steinberg and Brand [30, 31] while studying the stability of the flow in a horizontal layer under free-free boundary conditions. In this paper, flow was induced by chemical reaction as well as thermal and concentration gradients applied to the boundaries of the system. They considered the regime where the reaction rate was so fast that solutal diffusion could be neglected. Pritchard and Richardson [23] have investigated the stability of flow under fast as well as slow reaction rates. They reported the effect of solutal diffusion on the stability of flow. According to their study, the reactive term stabilizes oscillatory convection when the solutal Darcy-Rayleigh number is positive. Wang and Tan [39] studied the onset of double-diffusive (thermo-solutal) convection with a reaction term in a horizontal sparsely packed porous medium, based on Brinkman model. They analysed that the Darcy number destabilizes the flow in stationary as well as oscillatory modes; however, the effects of Lewis number and reaction term depend on the values of the solutal Rayleigh number. All these studies were restricted to isotropic porous media. However, the influence of the chemical reaction on double-diffusive convection in an anisotropic porous layer is reported in the work of [18]. They have reported that the effect of the chemical reaction, as well as the anisotropy of the medium, may be stabilizing or destabilizing. Most of the work related to convection reaction is related to Newtonian fluid. In 2013, Srivastava and Bera [32], extended the work of [18], for non-Newtonian fluid, especially couple-stress fluid, for different boundary conditions (such as free-free, rigid-rigid, and rigid-free). They have reported that increasing the couple stress parameter increases flow stability in all three cases, but among all cases, its stabilization effect for rigid-rigid is maximum. Also, the chemical reaction stabilizes the flow for all three cases.

It should be noted that most of the above-mentioned research work has been done without external forces or with external forces such as coriolis force. But less attention is paid towards the onset of double-diffusive convection in viscoelastic fluid saturated porous medium under the influence of a magnetic field. Magneto-hydrodynamics is a branch of geophysics that studies and measures the velocities and positions of frames of reference on the earth's surface as they rotate towards the frame of inertia in the presence of a magnetic field. It is useful in the commercial production of the magnetic fluids, in chemical engineering, and in the performance of petroleum reservoirs, where the geothermal areas are influenced by the magnetic field of the earth [40]. Using linear and nonlinear stability analysis, [24] investigated the problem of establishing convection currents in a layer of viscous, electrically conducting fluid in the presence of a magnetic field. The stability of finite-amplitude and overstable convection of a conducting fluid through a fixed porous bed was examined by [29, 33]. Desaive *et al.* [6] investigated the thermo convection in a ferro-fluid saturating a rotating porous layer. Magneto-double diffusive convection in an electrically conducting fluid-saturated porous medium with temperature modulation of the boundaries is investigated by [4]. They reported the effect of frequency of temperature modulation, solute Rayleigh number (solutal effect), and Darcy Chandrasekhar number (magnetic field effect), mainly. In 2012, Srivastava *et al.* [34] examined the magneto-double-diffusive convection under the Soret effect. It is found that the magnetic field parameter, solutal Rayleigh number, delays the onset of magneto-double diffusive convection. Siddique *et al.* [35] investigated the unsteady free convection flow, with heat and mass transfer, of an electrically conducting viscoelastic fluid through a porous medium of variable permeability. The flow domain is a half-space bounded by a vertical porous plate, with a constant heat flux, constant concentration, and a rectilinear translation in its plane with constant velocity. The unsteady *MHD* flow in a porous medium past an inclined plate in a thermally stratified fluid flow under the impact of the Dufour and Soret effects was recently studied by Nidhi and Priyanka [25]. To put numerical solutions into practice, they employed the finite difference technique. They found that the fluid flow's velocity drops as the magnetic field is increased. Thermal convection of a magneto-hydrodynamic (*MHD*) micropolar fluid flow in a porous nonlinear media stretching sheet with thermal radiation was studied by Amrita and Bhupander [11]. They also came to the conclusion in their investigation that when the magnetic field gets stronger, the flow velocity of the fluid drops.

Here we focus on magneto-double diffusive convection in a viscoelastic fluid-saturated anisotropic porous layer with the influence of chemical reaction on the boundaries. The multi-fold intention of this work is: (i) to enrich the existing literature on the oscillatory states for viscoelastic fluids; (ii) the magnetic field effect on double diffusive convection reaction; and (iii) weakly non-linear stability analysis of magneto-convection in viscoelastic fluid saturated by an anisotropic layer in the presence of chemical reaction. This will give new fundamental knowledge about the possible convective phenomena in some technological processes, free-free surfaces, and binary viscoelastic liquids with positive reactions on boundaries.

The structure of the paper is organized as follows: Section 2 introduces the mathematical formulation of the physical problem. Section 3 focuses on the linear stability analysis, considering both stationary and oscillatory modes of convection. In Section 4, the weakly nonlinear theory is developed to explore the effects of heat and mass transfer. Section 5 presents and discusses the key results, while Section 6 concludes the paper by summarizing the main findings and their significance.

2 Mathematical model

2.1 The physical domain

We consider viscoelastic binary fluid saturated anisotropic porous layer of depth d, and confined between two parallel horizontal planes at $z = 0$ and $z = d$. A constant magnetic field $\mathbf{H}_b = H_b\hat{\mathbf{k}}$ is maintained externally in the vertical upward direction, and a Cartesian frame of reference is chosen with the origin in the lower boundary; the horizontal component x and vertical component z increases upwards as shown in Fig. 5.1. The surfaces are stretched indefinitely in both x and y directions while maintaining a consistent temperature gradient ΔT and salinity gradient ΔS are maintained across the porous layer. We are assuming that chemical equilibrium is maintained at the boundaries. We assume that the Oberbeck-Boussinesq approximation is applied to account for the effect of density variations.

2.2 Governing equations

The modified Darcy law for viscoelastic fluid of the Oldroyd type is used to model the momentum equation [12]. The balance equations for mass, momentum, and energy can be cast in dimensional form as

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\epsilon}\frac{\partial \mathbf{q}}{\partial t}-\mu_m(\mathbf{H}.\nabla)\mathbf{H}\right)+\left(1+\lambda_2\frac{\partial}{\partial t}\right)\mu K^{-1}.\mathbf{q}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(-\nabla P+\rho\mathbf{g}\right), \tag{2.1}$$

$$\gamma\frac{\partial T}{\partial t}+\mathbf{q}.\nabla T=\nabla(\kappa_T\nabla T), \tag{2.2}$$

$$\frac{\partial \mathbf{H}}{\partial t}+\mathbf{q}.\nabla\mathbf{H}-\mathbf{H}.\nabla\mathbf{q}=\Lambda\nabla^2\mathbf{H}, \tag{2.3}$$

$$\epsilon\frac{\partial S}{\partial t}+(\mathbf{q}.\nabla)S=\epsilon\kappa_S\nabla^2S+k(S_{eq}(T)-S), \tag{2.4}$$

$$\nabla\cdot\mathbf{q}=\mathbf{0}, \tag{2.5}$$

$$\nabla\cdot\mathbf{H}=0. \tag{2.6}$$

Here $\mathbf{q} = (u, v, w)$ is velocity of the fluid, $\mathbf{H} = (H_x, H_y, H_z)$ is magnetic field, μ_m is magnetic permeability, λ_1 is relaxation time, λ_2 is retardation time, P is pressure, ρ is fluid density, ϵ is porosity, T is temperature, S is solute, κ_T is thermal diffusivity, κ_S is solutal diffusivity and k is lumped effect reaction, respectively. The relation between the reference density, temperature and salinity is given by

$$\rho=\rho_0\left[1-\beta_T(T-T_0)+\beta_S(S-S_0)\right]. \tag{2.7}$$

The appropriate boundary conditions for temperature, solute, and magnetic field are:

$$T=T_0+\Delta T \text{ at } z=0 \text{ and } T=T_0 \text{ at } z=d. \tag{2.8}$$

$$S=S_0+\Delta S \text{ at } z=0 \text{ and } S=S_0 \text{ at } z=d. \tag{2.9}$$

$$\mathbf{H}\times\hat{\mathbf{k}}=0 \text{ at } z=0,\ d. \tag{2.10}$$

2.3 Basic state

Following [23], we have considered the equilibrium solute concentration as a linear function of temperature i.e. $S_{eq}(T) = S_0 + \varphi\,(T - T_0)$, where $\varphi = \frac{S_l - S_u}{T_l - T_u} = \frac{\triangle S}{\triangle T}$. The value of φ may be positive i.e. the solubility increases with temperature (*i.e.* $\varphi > 0$) or negative i.e. the solubility decreases with temperature (*i.e.* $\varphi < 0$). Here, we have taken the case ($\varphi > 0$).

The basic state of the fluid is assumed to be quiescent, and is given by

$$\mathbf{q}_b = (0,0,0)\,,\ P = P_b(z), T = T_b(z), S = S_b(z) = S_{eq}(T_b), \rho = \rho_b(z),\ \mathbf{H}_b = H_b\hat{\mathbf{k}}. \tag{2.11}$$

Using Eq. (2.11) in Eqs. (2.1) − (2.7) yield

$$\frac{dp_b}{dz} = -\rho_b g, \frac{d^2T_b}{dz^2} = 0, \frac{d^2S_b}{dz^2} = 0. \tag{2.12}$$

$$\rho_b = \rho_0\left[1 - \beta_T(T_b - T_0) + \beta_S(S_b - S_0)\right]. \tag{2.13}$$

The basic state solution for temperature and solutal fields are given by

$$T_b(z) = T_l - \Delta T\frac{z}{d},\ S_b(z) = S_l - \Delta S\frac{z}{d}. \tag{2.14}$$

In the basic state S_b=$S_{eq}(T_b)$. As mentioned above that S_{eq} is linear in T, this allows the existence of a steady basic state in which the solute is everywhere in chemical equilibrium with the solid matrix and therefore the vertical flux of solute is constant in space.

2.4 Perturbed state

On the basic state, we superpose a perturbation in the form

$$\mathbf{q} = \mathbf{q}'(x,y,z,t), T = T_b(z) + T'(x,y,z,t), S = S_b(z) + S'(x,y,z,t), P = P_b(z) + P'(x,y,z,t),$$
$$\rho = \rho_b(z) + \rho'(x,y,z,t), \mathbf{H} = \mathbf{H}_b + \mathbf{H}'(x,y,z,t), \tag{2.15}$$

where primes indicate perturbations. Introducing Eq. (2.15) in Eqs. (2.1) − (2.6), and using basic state from Eq. (2.12), we obtain

$$\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\left[\frac{\rho_0}{\epsilon}\frac{\partial \mathbf{q}'}{\partial t} - \mu_m H_b\frac{\partial \mathbf{H}'}{\partial z} + \nabla P' - \rho_0\mathbf{g}\left[T'\beta_T - S'\beta_S\right]\right] + \left(1 + \lambda_2\frac{\partial}{\partial t}\right)\mu\mathbf{K}^{-1}\cdot\mathbf{q}' = 0, \tag{2.16}$$

$$\gamma\frac{\partial T'}{\partial t} + (\mathbf{q}'\cdot\nabla)T' - w'\frac{\Delta T}{d} = (\eta\nabla_1^2 + \frac{\partial^2}{\partial z^2})T', \tag{2.17}$$

$$\frac{\partial \mathbf{H}'}{\partial t} + (\mathbf{q}'\cdot\nabla)\,\mathbf{H}' - (\mathbf{H}'\cdot\nabla)\,\mathbf{q}' - H_b\frac{\partial \mathbf{q}'}{\partial z} = \Lambda\nabla^2\mathbf{H}, \tag{2.18}$$

$$\epsilon\frac{\partial S'}{\partial t} + (\mathbf{q}'\cdot\nabla)S' - w'\frac{\Delta S}{d} = \epsilon\kappa_s\nabla^2 S' + k(S_{eq}(T') - S'), \tag{2.19}$$

$$\nabla\cdot\mathbf{q}' = 0, \tag{2.20}$$

$$\nabla\cdot\mathbf{H}' = 0. \tag{2.21}$$

Using the following transformations

$$(x',y',z') = d(x^*,y^*,z^*), t' = \frac{d^2}{\kappa_{Tz}}t^*, \mathbf{q}' = (u',v',w') = \frac{\epsilon\kappa_{Tz}}{d}\,(u^*,v^*,w^*)\,, P' = \frac{\mu\kappa_{Tz}}{K_z}P^*,$$
$$\lambda_1 = \frac{d^2}{\kappa_{Tz}}\lambda_1^*, \lambda_2 = \frac{d^2}{\kappa_{Tz}}\lambda_2^*, \mathbf{H}' = H_b\mathbf{H}^*, T^{'} = (\Delta T)\,T^*, S^{'} = (\Delta S)\,S^*, \tag{2.22}$$

we non-dimensionalized the Eqs. (2.16) − (2.21), and obtained the non-dimensional governing equations (after dropping the asterisks for simplicity) as

$$\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\frac{\partial \mathbf{q}}{\partial t} - QP_m\frac{\partial \mathrm{H}}{\partial z} + \frac{\nabla P}{\epsilon} - Ra_T T\hat{\mathbf{k}} + Ra_S S\hat{\mathbf{k}}\right] + \left(1 + \lambda_2\frac{\partial}{\partial t}\right)\mathbf{q}_a = 0, \tag{2.23}$$

$$\gamma\frac{\partial T}{\partial t} + \epsilon(\mathbf{q}\cdot\nabla)T - \epsilon w = \left(\eta\nabla_1^2 + \frac{\partial^2}{\partial z^2}\right)T, \tag{2.24}$$

$$\frac{\partial S}{\partial t} + (\mathbf{q}\cdot\nabla)S - w = \frac{1}{Le}\nabla^2 S + \chi(T - S), \tag{2.25}$$

$$\frac{1}{\epsilon}\frac{\partial \mathbf{H}}{\partial t}+(\mathbf{q}\cdot\nabla)\mathbf{H}-(\mathbf{H}\cdot\nabla)\mathbf{q}-\frac{\partial \mathbf{q}}{\partial z}=P_m\nabla^2\mathbf{H}, \tag{2.26}$$

with $\nabla.\mathbf{q}=\nabla.\mathbf{H}=0$, where the non-dimensional parameters, $Va=\frac{\epsilon Pr}{Da}$ is Vadasz number, $Ra_T=\frac{\beta_T g\Delta T K_z}{\nu\kappa_{T_z}}$ is Rayleigh number, $Ra_S=\frac{\beta_S g\Delta S K_z}{\nu\kappa_{T_z}}$ is solutal Rayleigh number, $Q=\frac{\mu_m H_b^2 K_z}{\rho_0\nu\Lambda}$ is Chandershekhar number, $P_m=\frac{\lambda}{\kappa_{T_z}}$ is magnetic Prandtl number, $\chi=\frac{kd^2}{\epsilon\kappa_{T_z}}$ is Damköhler number, $Le=\frac{\kappa_{T_z}}{\kappa_s}$ is Lewis number and $\mathbf{q}_a=\left(\frac{1}{\xi}u,\frac{1}{\xi}v,w\right)$ is the anisotropic modified velocity vector. Eliminating the pressure term from Eq. (2.23) by applying the curl operator twice, we obtain

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\frac{\partial(\nabla^2\mathbf{q})}{\partial t}-QP_m\nabla^2\left(\frac{\partial \mathbf{H}}{\partial z}\right)-\hat{\mathbf{i}}Ra_T\frac{\partial^2 T}{\partial y\partial z}-\hat{\mathbf{j}}Ra_T\frac{\partial^2 T}{\partial x\partial z}+\hat{\mathbf{k}}Ra_T\nabla_1^2 T\right.$$
$$\left.+\hat{\mathbf{i}}Ra_S\frac{\partial^2 S}{\partial y\partial z}+\hat{\mathbf{j}}Ra_S\frac{\partial^2 S}{\partial x\partial z}-\hat{\mathbf{k}}Ra_S\nabla_1^2 T\right]+\left(1+\lambda_2\frac{\partial}{\partial t}\right)C=0, \tag{2.27}$$

where $C=(C_1,C_2,C_3)$, $C_1=\frac{1}{\xi}\frac{\partial^2 v}{\partial y\partial x}+\frac{\partial^2 w}{\partial x\partial z}-\frac{1}{\xi}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$, $C_2=\frac{1}{\xi}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}-\frac{1}{\xi}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right)$
$C_3=-\left(\nabla_1^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w$.

The boundaries of the porous medium can be considered as either free or rigid. Although free-free surfaces are artificial as they cannot be realized in laboratory, but in real situations like geothermal regions, one cannot avoid the penetration of the fluid into porous medium; so it is appropriate to take free-free surfaces. Therefore, Eqs. (2.24)-(2.27) are solved for stress-free, isothermal, impermeable boundary conditions

$$w=\frac{\partial^2 w}{\partial z^2}=\frac{\partial H_z}{\partial z}=T=S=0 \text{ at } z=0,\ 1. \tag{2.28}$$

3 Linear Stability Analysis

Taking vertical component and eliminating non-linear terms from Eqs. (2.24)-(2.27), we get linear form as:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\frac{\partial(\nabla^2 w)}{\partial t}-QP_m\nabla^2\frac{\partial H_z}{\partial z}-Ra_T\nabla_1^2 T+Ra_S\nabla_1^2 S\right]+\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\nabla_1^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w=0, \tag{3.1}$$

$$\left(\gamma\frac{\partial}{\partial t}-\eta\nabla_1^2-\frac{d^2}{dz^2}\right)T-w\epsilon=0, \tag{3.2}$$

$$\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2+\chi\right)S-\chi T-w=0, \tag{3.3}$$

$$\left(\frac{1}{\epsilon}\frac{\partial}{\partial t}-P_m\nabla^2\right)H_z=\frac{\partial w}{\partial z}. \tag{3.4}$$

Combining Eqs. (3.1) and (3.4), we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\left(\frac{1}{\epsilon}\frac{\partial}{\partial t}-P_m\nabla^2\right)\frac{\partial(\nabla^2 w)}{\partial t}-QP_m\nabla^2\frac{\partial^2 w}{\partial z^2}\right]-\left(\frac{1}{\epsilon}\frac{\partial}{\partial t}-P_m\nabla^2\right)(Ra_T\nabla_1^2 T-$$
$$Ra_S\nabla_1^2 S)+\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{1}{\epsilon}\frac{\partial}{\partial t}-P_m\nabla^2\right)\left(\nabla_1^2+\frac{1}{\epsilon}\frac{\partial^2}{\partial z^2}\right)w=0. \tag{3.5}$$

We seek a normal mode technique for the linear stability of the basic flow, which is of the form;

$$\begin{pmatrix} w \\ T \\ S \end{pmatrix}=\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix}e^{i(lx+my)+\sigma t}, \tag{3.6}$$

where l and m are horizontal wave numbers and σ(a complex quantity) is the growth rate. W, Θ and Φ are the amplitudes of the stream function, temperature, and concentration field, respectively. Substituting Eq. (3.6) in Eqs. (3.5), (3.3), and (3.4), we get

$$\left[\frac{\sigma}{Va}(D^2-a^2)-\frac{QP_m(D^2-a^2)D^2}{\frac{\sigma}{\epsilon}-P_m(D^2-a^2)}+\frac{(1+\sigma\lambda_2)}{(1+\sigma\lambda_1)}\left(\frac{D^2}{\xi}-a^2\right)\right]W+Ra_T a^2\Theta-Ra_S a^2\Phi=0, \tag{3.7}$$

$$-\epsilon W+(\gamma\sigma+\eta a^2-D^2)\Theta=0, \tag{3.8}$$

$$-W-\chi\Theta+\left(\sigma-\frac{(D^2-a^2)}{Le}+\chi\right)\Phi=0, \tag{3.9}$$

where $D=\frac{d}{dz}$ and $a^2=l^2+m^2$. The corresponding boundary condition (2.28) will be of the form:

$$W=D^2W=\Theta=\Phi=0 \text{ at } z=0,\ 1, \tag{3.10}$$

We consider the solution of Eqs. (3.7)(3.9) that satisfies the boundary conditions corresponding to the free-free case.

$$[W(z),\Theta(z),\Phi(z)]=[W_0,\Theta_0,\Phi_0]\sin(n\pi z),\quad (n=1,2,3,...). \tag{3.11}$$

Substituting Eq. (3.11) into Eqs. (3.7)-(3.9), we obtain a matrix equation considering $n=1$

$$\begin{bmatrix} \frac{\sigma\delta^2}{Va}+\frac{QP_m\delta^2\pi^2}{\frac{\sigma}{\epsilon}+P_m\delta^2}+\frac{(1+\lambda_2\sigma)}{(1+\lambda_1\sigma)}\delta_1^2 & -a^2Ra_T & a^2Ra_S \\ -\epsilon & \gamma\sigma+\delta_2^2 & 0 \\ -1 & -\chi & \sigma+\delta_3^2+\chi \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}. \tag{3.12}$$

where $\delta^2=a^2+\pi^2$, $\delta_1^2=a^2+\frac{\pi^2}{\xi}$, $\delta_2^2=\eta a^2+\pi^2$ and $\delta_3^2=\frac{\delta^2}{Le}$.
For non-trivial solution of W, Θ and Φ, we need to make the determinant of the above matrix as zero, we get

$$Ra_T=\frac{Ra_S(\epsilon\chi+\delta_2^2+\gamma\sigma)}{\epsilon(\sigma+\delta_3^2+\chi)}+\left(\frac{\gamma\sigma+\delta_2^2}{\epsilon a^2}\right)\left[\frac{\sigma\delta^2}{Va}+\frac{QP_m\delta^2\pi^2}{\frac{\sigma}{\epsilon}+P_m\delta^2}+\frac{(1+\lambda_2\sigma)}{(1+\lambda_1\sigma)}\delta_1^2\right]. \tag{3.13}$$

3.1 Stationary state

For the direct bifurcation (i.e., steady onset), we have $\sigma=0$ at the the margin of stability. Then, the Rayleigh number at which marginally stable steady mode exists, becomes

$$Ra_T^{st}=\left(\frac{Q\pi^2}{\epsilon a^2}+\frac{\delta_1^2}{\epsilon a^2}\right)\delta_2^2+Ra_S\frac{\epsilon\chi+\delta_2^2}{\epsilon(\delta_3^2+\chi)}. \tag{3.14}$$

In the absence of $Q=0$ the Eq. (3.14) reduces to

$$Ra_T^{st}=\frac{\delta_1^2\delta_2^2}{\epsilon a^2}+Ra_S\frac{\epsilon\chi+\delta_2^2}{\epsilon(\delta_3^2+\chi)}. \tag{3.15}$$

For the case of single component fluid saturated anisotropic porous media i.e. when $Ra_S=0$, Eq. (3.15) reduces to

$$Ra_T^{st}=\frac{\pi^2}{a^2}(\eta a^2+1)\left(a^2+\frac{1}{\xi}\right). \tag{3.16}$$

Eq. (3.16) con-sides with that of Storesletten [36]. Further if porous media is isotropic in mechanical and thermal properties, i.e. $\eta=\xi=1$, then Eq. (3.16) becomes

$$Ra_T^{st}=\frac{\pi^2\left(a^2+1\right)^2}{a^2}, \tag{3.17}$$

which has the critical value $Ra_{T,c}^{st}=4\pi^2$ for $a_c=\pi$, as obtained by Horton and Rogers [10] and Lapwood [16].

3.2 Oscillatory motion

The growth rate σ is in general a complex quantity such that $\sigma=\sigma_r+i\omega$. The system with $\sigma_r<0$ is always stable, while for $\sigma_r>0$ it will become unstable. For neutral stability state $\sigma_r=0$.
We put $\sigma=i\omega(\omega$ is real) in Eq. (3.13) and obtain

$$Ra_T=\Pi_1+(i\omega)\Pi_2. \tag{3.18}$$

The expression for Π_1 is given by

$$\Pi_1=D_1+D_2+D_3+D_4,$$

where $D_1=\frac{Ra_S(\delta_3^2\epsilon\chi+Q\chi^2+\delta_2^2\delta_3^2+\chi\delta_2^2-\omega^2\gamma)}{\epsilon((\delta^2+\chi)^2+\omega^2)}$, $\quad D_2=-\frac{\omega^2\delta^2\gamma}{\epsilon Vaa^2}$, $\quad D_3=\frac{QP_m^2\epsilon\pi^2\delta^4\gamma-\delta_2^2QP_m\pi^2\delta^2}{\epsilon^2P_m^2\delta^4a^2+\omega^2a^2}$,
$D_4=\frac{\delta_1^2\delta_2^2\omega^2\lambda_1\lambda_2+\delta_2^2\delta_3^2-\omega^2\delta_1^2\gamma(\lambda_2-\lambda_1)}{\epsilon a^2(1+\omega^2\lambda_1^2)}$.
Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (3.18) it follows that either $\omega=0$ (steady

onset) or $\Pi_2 = 0$ ($\omega \neq 0$, oscillatory onset). For oscillatory onset $\Pi_2 = 0(\omega \neq 0)$ and this gives a dispersion relation of the form

$$G_1 \left(\omega^2\right)^3 + G_2 \left(\omega^2\right)^2 + G_3 \left(\omega^2\right) + G_4 = 0, \tag{3.19}$$

where the constants $G_1 = M_1M_7\epsilon a^2 + M_5\epsilon a^2, G_2 = M_1M_3M_7\epsilon + M_1M_6\epsilon a^2 + M_1M_7M_9a^2 + M_2M_7\epsilon + M_5M_9a^2 + M_4\epsilon a^2 + M_3M5\epsilon + M_7M_8a^2, G_3 = M_1M_3M_6\epsilon + M_1M_3M_7M_9 + M_1M_6M_9a^2 + M_2M6\epsilon + M_2M_7M_9 + M_4M_9a^2 + M_3M_5M_9 + M_3M_4\epsilon + M_6M_8a^2 and G_4 = M_1M_3M_6M_9 + M_2M_6M_9 + M_3M_4M_9 + M_3M_6M_8$
Now Eq. (3.18) with $\Pi_2 = 0$, gives oscillatory Rayleigh number Ra_T^{osc} at the margin of stability as

$$Ra_T^{osc} = \Pi_1. \tag{3.20}$$

Also for the oscillatory convection to occur, ω^2 must be positive. The symbols $M_1, M_2, M_3, M_4, M_5, M_6, M_7, M_8, M_9$ and Π_2 are defined below in Appendix-I.

Appendix-I

$M_1 = \frac{\delta^2\delta_2^2}{\epsilon a^2 Va}$, $M_2 = QP_m^2\epsilon\pi^2\delta^4\gamma - \delta_2^2QP_m\pi^2\delta^2$, $M_3 = a^2\epsilon^2P_m^2\delta^4$, $M_4 = \delta_1^2\delta_2^2(\lambda_2 - \lambda_1) + \gamma\delta_1^2$, $M_5 = \delta_1^2\delta_1\delta_2\gamma$, $M_6 = \epsilon a^2$, $M_7 = \epsilon a^2\lambda_1^2$, $M_8 = -Ra_S(\gamma\delta_3^2 + \gamma\chi + \delta_2^2 + Q\chi)$, $M_9 = \epsilon(\delta_3^2 + \chi)^2$, $\Pi_2 = M_1 + \frac{M_2}{M_3 + a^2\omega^2} + \frac{M_4 + M5\omega^2}{M_6 + M_7\omega^2} + \frac{M_8}{M_9 + \epsilon\omega^2}$.

4 A weak nonlinear theory

In this section, we consider the nonlinear analysis using a truncated representation of Fourier series considering two terms. Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding eigen-functions describing qualitatively the convective flow, it cannot provide information about the values of the convection amplitudes, nor regarding the rate of heat and mass transfer. In order to obtain this additional information, we perform the non-linear analysis, which is useful to understand the physical mechanism with minimum amount of mathematical analysis and is a step forward toward understanding full non-linear problem.
Introducing stream function ψ and ϕ as $u = \frac{\partial\psi}{\partial z}$, $w = -\frac{\partial\psi}{\partial x}$ and $H_x = \frac{\partial\phi}{\partial z}$, $H_z = -\frac{\partial\phi}{\partial x}$ into vertical component of Eqs. (2.24)-(2.27), we obtain

$$\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{Va}\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\psi - QP_m\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\phi + Ra_T\frac{\partial T}{\partial x}Ra_S\frac{\partial S}{\partial x}\right]$$
$$+\left(1 + \lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\psi = 0, \tag{4.1}$$

$$\epsilon\frac{\partial\psi}{\partial x} + \left(\gamma\frac{\partial}{\partial t} - \eta\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2}\right)T - \epsilon\frac{\partial\left(\psi, T\right)}{\partial\left(x, z\right)} = 0, \tag{4.2}$$

$$\frac{\partial\psi}{\partial x} + \left(\frac{\partial}{\partial t} - \frac{1}{Le}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) + \chi\right)S - \chi T - \frac{\partial(\psi, S)}{\partial(x, z)} = 0, \tag{4.3}$$

$$\frac{1}{\epsilon}\frac{\partial\phi}{\partial t} - \frac{\partial\left(\psi, \phi\right)}{\partial\left(x, z\right)} - \frac{\partial\psi}{\partial z} - P_m\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\phi = 0. \tag{4.4}$$

A minimal double Fourier series which describes the finite amplitude steady-state convection is given by

$$\psi = A_1(t)\sin(\pi ax)\sin\left(\pi z\right), \tag{4.5}$$

$$T = B_1\left(t\right)\cos\left(\pi ax\right)\sin\left(\pi z\right) + B_2\left(t\right)\sin\left(2\pi z\right), \tag{4.6}$$

$$S = C_1\left(t\right)\cos\left(\pi ax\right)\sin\left(\pi z\right) + C_2\left(t\right)\sin\left(2\pi z\right), \tag{4.7}$$

$$\phi = D_1(t)\sin\left(\pi ax\right)\cos\left(\pi z\right) + D_2(t)\sin\left(2\pi ax\right), \tag{4.8}$$

where the amplitudes $A_1(t)$, $B_1(t)$, $B_2(t)$, $C_1(t)$, $C_2(t)$, $D_1(t)$ and $D_2(t)$ are to be determined from the dynamics of the system. Substituting Eqs. (4.5) – (4.8) into Eqs. (4.1)-(4.4) and equating the coefficients of like terms, we obtain the following non-linear autonomous system of differential equations

$$M_1'\lambda_1\frac{dE_1}{dt} = F_1E_1 + F_2A_1 + F_3B_1 + F_4C_1 + F_5D_1 + F_6A_1B_2 + F_7A_1C_2 + F_8A_1D_2, \tag{4.9}$$

$$\frac{dA_1}{dt} = E_1, \tag{4.10}$$

$$\frac{dB_1}{dt} = -\left(\frac{\pi\epsilon a}{\gamma}\right)A_1 - \left(\frac{\eta(\pi^2a^2+\pi^2)}{\gamma}\right)B_1 - \frac{2\pi\epsilon a}{\gamma}A_1B_2, \quad (4.11)$$

$$\frac{dB_2}{dt} = -\left(\frac{4\pi^2}{\gamma}\right)B_2 + \frac{\epsilon\pi^2 a}{2\gamma}A_1B_1, \quad (4.12)$$

$$\frac{dC_1}{dt} = \chi B_1 - 2\pi^2 aA_1C_2 - \left(\frac{\pi^2a^2+\pi^2}{Le}+\chi\right)C_1 - \pi aA_1, \quad (4.13)$$

$$\frac{dC_2}{dt} = -\left(\frac{4\pi^2}{Le}+\chi\right)C_2 + \left(\frac{\pi^2 a}{2}\right)A_1C_1 + \chi B_2, \quad (4.14)$$

$$\frac{dD_1}{dt} = -\epsilon P_m\left(\pi^2a^2+\pi^2\right)D_1 + \epsilon\pi^2 aA_1D_2 + \pi\epsilon A_1, \quad (4.15)$$

$$\frac{dD_2}{dt} = -4\epsilon P_m\pi^2a^2D_2 - \frac{\epsilon\pi^2 a}{2}A_1D_1, \quad (4.16)$$

where, $M_1' = \left(\frac{\pi^2a^2+\pi^2}{Va}\right)$, $F_1 = -\left(\frac{\pi^2a^2+\pi^2}{Va}\right) - \left(\frac{\pi^2}{\xi}+\pi^2a^2\right)\lambda_2$, $F_2 = -\pi^2\epsilon^2\lambda_1 QP_m(\pi^2a^2+\pi^2) - \left(\frac{\pi^2}{\xi}+\pi^2a^2\right) + \frac{\epsilon\pi^2a^2\lambda_1 Ra_T}{\gamma} - \pi^2a^2\lambda_1 Ra_S$, $F_3 = -\pi aRa_T + \frac{\eta(\pi^2a^2+\pi^2)\lambda_1\pi aRa_T}{\gamma} + \chi\lambda_1\pi aRa_S$, $F_4 = \pi aRa_S - \pi a\lambda_1 Ra_S\left(\frac{\pi^2a^2+\pi^2}{Le}+\chi\right)$, $F_5 = P_m^2\epsilon Q\pi\lambda_1(\pi^2a^2+\pi^2)^2 - QP_m\pi(\pi^2a^2+\pi^2)$, $F_6 = \frac{2\pi^2a^2\epsilon Ra_T\lambda_1}{\gamma}$, $F_7 = -2\pi^3a^2\lambda_1 Ra_S$, $F_8 = -QP_m\pi^3 a\epsilon\lambda_1(\pi^2a^2+\pi^2)$.

Qualitative predictions of above autonomous differential equations are discussed and stated as. Eqs. (4.9) – (4.16) represent a system which is uniformly bounded in time and possesses many properties of the full problem. They are same as the original Eqs. (2.16) – (2.21), and therefore Eqs. (4.9) – (4.16) must be dissipative. This shows that volume in the phase space must contract. For proving this statement that, the volume contraction, it is necessary to show that velocity field has a constant negative divergence. Indeed,

$$\begin{aligned}\nabla.V &= \frac{\partial\dot{A}_1}{\partial A_1} + \frac{\partial\dot{B}_1}{\partial B_1} + \frac{\partial\dot{B}_2}{\partial B_2} + \frac{\partial\dot{C}_1}{\partial C_1} + \frac{\partial\dot{C}_2}{\partial C_2} + \frac{\partial\dot{D}_1}{\partial D_1} + \frac{\partial\dot{D}_2}{\partial D2} + \frac{\partial\dot{E}_1}{\partial E_1} \\ &= -\Big[\frac{\eta P_1}{\gamma} + \frac{4\pi^2}{\gamma} + P_3 + P_4 + \epsilon P_m P_1 + 4\epsilon P_m\pi^2a^2 + \frac{1}{\lambda_1} + \frac{P_2\lambda_2 Va}{P_1\lambda_1}\Big], \end{aligned} \quad (4.17)$$

where, $P_1 = \pi^2a^2+\pi^2$; $P_2 = \left(\frac{\pi^2}{\xi}+\pi^2a^2\right)$; $P_3 = \left(\frac{\pi^2a^2+\pi^2}{Le}+\chi\right)$; $P_4 = \left(\frac{4\pi^2}{Le}+\chi\right)$.

Here dot represent a time derivative. All physical parameter used in above expression inside the square bracket is non-negative therefore overall right hand quantity is negative and therefore system is bounded and dissipative. So the trajectories are attracted to a set of measure zero in the phase space, or anyone say that they may be attracted to a fixed point, a limit cycle or, a strange attractor.

4.1 Heat and mass transports

It is known fact that for higher value of Rayleigh number the onset of convection is generally governed by heat and mass transfer within system. Consequently, here we are defining the Nusselt number and Sherwood number(following Srivastava and Bera [32]) as below The Nusselt number and Sherwood number is defined by,

$$Nu = \frac{h}{\kappa_{T_z}\Delta T/d} = (1-2\pi B_2), \quad (4.18)$$

$$Sh = \frac{J}{\kappa_S\Delta S/d} = (1-2\pi C_2)\,, \quad (4.19)$$

where B_2, C_2 are found in terms of A_1. Calculating B_2 and C_2, in the steady case, which is independent of viscoelastic parameters, so complete calculation is not given in this paper.

5 Result and Discussion

Using linear and weakly non-linear theory of stability analysis, we have trying to draw a sketch that how magnetic field and temperature-dependent mineral solubility (i.e. chemical reaction) affects the onset of thermal convection in viscoelastic fluid saturated porous medium. The porous medium is assumed to be both hydro-dynamically and thermally anisotropic. Expressions for the stationary and oscillatory Rayleigh numbers are obtained in terms of the non-dimensional parameters Q, λ_1, λ_2, χ, and Ra_S in linear stability theory. The non-linear theory, which is based on the Fourier series minimal representation, quantifies heat and mass transports and explains the possibility of finite amplitude motions.

5.1 Linear Stability Analysis

Linear stability analysis is divided into two subsections: The first part is dedicated to a comparative study of stationary and oscillatory cases for different values of χ, and in the second part, we focus on oscillatory cases for the F/F boundary conditions.

5.1.1 Comparison Between Stationary (limiting case) and Oscillatory Convection

The impact of the chemical reaction parameter (χ) on the evolution of the stationary (limiting case-absence of viscoelastic behavior) and oscillatory neutral stability curves for $Le = 100$, $Va = 100$, $Ra_S = 50$, $P_m = 0.4$, $\lambda_1 = 0.8$, $\lambda_2 = 0.4$, $\xi = 1.5$, $\eta = 0.3$, $\phi = 0.4$, $\gamma = 1.0$ and $Q = 10$ is depicted in Figures 5.3 (a)(b). It is found that a single critical Rayleigh number is sufficient to fully indicate the instability of the system and that the oscillatory neutral curve is beneath the stationary neutral curve. Additionally, these figures demonstrate that when χ increases, the minimum of the critical Rayleigh number falls, indicating that χ advances the beginning of both stationary and oscillatory convection. This can be explained with a quick glance at the solute equation, which now includes an extra reaction term. The solute concentration of a displaced fluid particle will shift and swiftly equilibrate with the surrounding fluid as a result of raising the value of chemical processes. Thus, it diminishes and finally removes the energy source needed for oscillatory and stationary instability to start.

5.1.2 Oscillatory Convection

An attempt is made to present comparative stability criteria for Maxwell and Oldroyd B-fluids through Table 5.1 and Table 5.2. The critical Rayleigh number and critical frequency are calculated for various values of Chandrashekhar number Q in the absence and presence of Damköhler number (χ) in Table 5.1 and Table 5.2 for the oscillatory state. Ra_{T_C} is observed to be positive or negative for the considered governing parameters, indicating a stabilizing or destabilizing influence on the viscoelastic fluid-saturated porous layer. In the case of Oldroyd-B fluids, the Chandrashekhar number can either stabilize or destabilize the porous layer. For Maxwell fluids, the porous layer is always destabilizing.

Figs.5.3(a)-(b) illustrate how the critical Rayleigh number varies with relaxation parameter (λ_1) in the (λ_2, Ra_T) and (a, Ra_T) planes, respectively. It is observed that with an increase in the value of relaxation parameter (λ_1), the critical oscillatory Rayleigh number decreases. Additionally, it is clear from Figure 5.3(a) that, compared to its absence, the values of Ra_T are lower when the Damköhler number (χ) is present. Increasing the relaxation and retardation parameters results in a faster and slower commencement of oscillatory convection, respectively. It can be seen from Fig. 5.3(b) that for the same values of the relaxation parameter and fixed values of the other parameters chosen for this figure, the curves corresponding to the anisotropic case lie below the curves corresponding to the isotropic case.

Figs. 5.4(a)–(c) depict neutral curves for various controlling parameters in (a, Ra_T) plane for oscillatory convection. Fig. 5.4(a) depicts the influence of Chandrashekhar number (Q) on the stability of the system. The minimum of the Rayleigh number increases when, Q is increased, suggesting that increasing the value of Q, amplifies the stability of the system. This can be explained as, when the magnetic field strength permeating the medium is considerably strong, it induces viscosity into the fluid, and the magnetic lines are distorted by convection. Then these magnetic lines hinder the growth of disturbances, leading to the delay in the onset of instability. From this figure, it is also seen that for the same values of the Q and fixed values of the other parameters, the curves corresponding to the anisotropic case lie above the curves to the isotropic case. It can be understood by the fact that Chandrasekhar number ($Q=\left(\frac{\mu_m H_b^2 K_z}{\rho_0 \nu \Lambda}\right)$) is inversely proportional to thermal diffusivity. A decrease in thermal diffusivity implies an increase in buoyancy, which implies an increase in the Rayleigh number. Furthermore, we discover that as, Q increases, the minimum Rayleigh number shifts toward larger wave number values, indicating that the cell width decreases.

The effect of the retardation parameter (λ_2) on the stability is shown in Fig. 5.4(b). The minimum of the Rayleigh number increases when the retardation parameter λ_2 is increased, suggesting that increasing λ_2 amplifies the stability of the system. The behaviour of viscoelastic parameter is evident and similar to that reported by [13].

In Fig. 5.4(c), the effect of Damköhler number (χ) on the oscillatory Rayleigh number is demonstrated. The minimum of the Rayleigh number decreases as χ increases, indicating that χ, advances the onset of oscillatory convection. The physics behind this behaviour, is that increasing the value of $\chi=\left(\frac{kd^2}{\epsilon \kappa_{T_z}}\right)$ increases the chemical reaction rate, and because of that, the Rayleigh number decreases. It is also observed that for the same values of χ and fixed values of the other parameters chosen, the curves corresponding to the anisotropic case lie above the curves corresponding to the isotropic case.

5.2 Weakly Nonlinear Stability Analysis

5.2.1 Unsteady Case

The transient behaviour of heat and mass transfer is investigated numerically by solving an autonomous system of differential equations using the RungeKutta method with appropriate initial conditions. The Nusselt and Sherwood numbers are then calculated as a function of time.

The Figs. 5.4$(a)-(d)$, Table 5.3 and Figs. 5.4 (a_1)-(d_1), Table 5.3, show the response of the time t corresponding to the Nusselt number (Nu) and Sherwood number (Sh) by varying in one of the parameters, while the others are held constant at their respective values: $Q=20.0$, $\lambda_1=0.8$, $\lambda_2=0.3$, $\chi=0.7$, $\xi=1.5$, $\eta=0.5$, $\gamma=0.5$, $P_m=0.4$, $Ra_t=1000$, $Ra_S=10.0$, $Va=100.0$ and $Le=10$. The figures show that the initial value of the Nusselt number (Nu) is 1 at $t=0$. It rises and oscillates at an intermediate value of time t, then becomes nearly constant and approaches the steady state with increasing value of time. The effects of various parameters on the Nu and Sh in the unsteady case are found to be identical to those in the linear stability analysis.

Table 5.4 and Table 5.6 illustrate the variation in Nusselt and Sherwood numbers for Maxwell, Oldroyd-B, and Rivlin-Ericksen fluids for different values of Chandrashekhar number Q, both in the presence and absence of Damköhler number (χ). We can infer from these tables that, in the case of Oldroyd-B fluid, there is higher mass and heat transport, respectively. Additionally, in increasing order variation of Nu and Sh is as follows:

$(Nu)_{Oldroyd-B} > (Nu)_{Maxwell} > (Nu)_{Rivlin-Ericksen}$,

$(Sh)_{Oldroyd-B} > (Sh)_{Maxwell} > (Sh)_{Rivlin-Ericksen}$.

Figs. 5.6 − 5.9 are drawn to explain how the pattern of streamlines, isotherms, isohalines, and magnetic streamlines changes for unsteady cases over various short times (0.01,0.006, 0.001). The streamlines pattern is shown in Figs. 5.6$(a)-(c)$ for various times. It is shown from the figure that, how streamlines do not have contours for a short period of time before developing as the passage of time does. The spread of streamlines with time also shows an improvement in convection. The isotherms are shown in Figs. 5.7$(a)-(c)$ for various times. These figures make it quite evident that as time goes on, the convection state manifests itself as a contour. Figs. 5.8$(a)-(c)$ displays the isohalines for various time periods. These figures exhibit isothermal behaviour. The magnetic streamlines for the unsteady case are displayed at various times in Figs. 5.9$(a)-(c)$.

Table 5.1: *Comparisons between critical frequency and critical Rayleigh numbers for viscoelastic fluids for different values of Chandrashekhar number Q when $\chi=0$.*

oscillatory onset					
Maxwell fluid $(\lambda_1=2,\lambda_2=0)$			OldroydB fluid $(\lambda_1=2,\lambda_2=1)$		
Q	ω_c	Ra_{T_C}	Q	ω_c	Ra_{T_C}
0	42.8275	-250.0138	0	20.1725	-120.1958
5	49.5483	-237.7710	5	23.4025	-72.8240
10	55.3606	-225.7247	10	26.2380	-26.3292
20	65.7084	-201.8833	20	31.1440	65.4473
30	74.5609	-178.1899	30	35.3761	156.77

Table 5.2: *Comparisons between critical frequency and critical Rayleigh numbers for viscoelastic fluids for different values of Chandrashekhar number Q when $\chi = 0.7$.*

oscillatory onset					
Maxwell fluid ($\lambda_1 = 2, \lambda_2 = 0$)			OldroydB fluid ($\lambda_1 = 2, \lambda_2 = 1$)		
Q	ω_c	Ra_{T_C}	Q	ω_c	Ra_{T_C}
0	44.0747	-256.9995	0	20.7619	-119.3270
5	55.7516	-283.0433	5	26.3708	-82.0305
10	65.3749	-309.1726	10	30.9794	-45.1103
20	81.2718	-361.5081	20	38.5784	28.3950
30	94.5321	-413.8788	30	44.9093	101.7442

Table 5.3: *Variation of thermal Nusselt number with time t for different value of χ.*

t	Nu			
	$\chi = 0.7$	$\chi = 7.0$	$\chi = 70.0$	$\chi = 700.0$
0.002	1.00126	1.00126	1.00126	1.00126
0.008	1.03809	1.03809	1.03811	1.0382
0.014	1.39453	1.39462	1.39539	1.39823
0.02	3.57866	3.57918	3.58364	3.59891
0.026	4.82665	4.82642	4.82441	4.8176
0.032	3.33017	3.33012	3.32985	3.33129
0.038	3.88244	3.88307	3.88874	3.91082
0.044	4.19711	4.19686	4.19467	4.185
0.05	3.77111	3.77107	3.77082	3.77166
0.056	3.92736	3.92778	3.93158	3.94684
0.062	4.00189	4.00182	4.00138	3.99891
0.068	3.89735	3.89739	3.89772	3.89998
0.074	3.93049	3.93071	3.93273	3.94098
0.08	3.94254	3.94257	3.94296	3.94429

Table 5.4: *Variation of thermal Sherwood number with time t for different value of χ.*

t	Sh			
	$\chi = 0.7$	$\chi = 7.0$	$\chi = 70.0$	$\chi = 700.0$
0.002	1.0001	1.0001	1.0001	1.0001
0.008	1.00378	1.00378	1.00378	1.00379
0.014	1.04389	1.0439	1.04399	1.04429
0.02	1.34375	1.34383	1.34452	1.3469
0.026	1.93063	1.93068	1.93104	1.93181
0.032	1.80913	1.80907	1.80853	1.80703
0.038	1.49082	1.49071	1.4897	1.48622
0.044	1.45057	1.45063	1.45115	1.45315
0.05	1.62115	1.62128	1.62261	1.6277
0.056	1.69147	1.69151	1.69184	1.69312
0.062	1.62667	1.62656	1.62548	1.62104
0.068	1.55461	1.55455	1.55391	1.55146
0.074	1.56729	1.56737	1.56812	1.57146
0.08	1.61536	1.61544	1.61632	1.61985

Table 5.5: *Comparisons between Nusselt and Sherwood numbers for viscoelastic fluids for different values of Chandrashekhar number Q when $\chi = 0$.*

Maxwell fluid			OldroydB fluid			Rivlin-Ericksen fluid		
Q	Nu	Sh	Q	Nu	Sh	Q	Nu	Sh
0	1.5814	2.6125	0	1.7428	2.8015	0	1.4621	2.3012
5	1.5911	2.6232	5	1.7599	2.8111	5	1.4761	2.3196
10	1.6082	2.6410	10	1.7628	2.9015	10	1.4888	2.3368
20	1.6321	2.6696	20	1.7795	2.9368	20	1.4967	2.5616
30	1.6596	2.6789	30	1.7901	2.9594	30	1.5628	2.5981

Table 5.6: *Comparisons between Nusselt and Sherwood numbers for viscoelastic fluids for different values of Chandrashekhar number Q when $\chi = 0.7$.*

Maxwell fluid			OldroydB fluid			Rivlin-Ericksen fluid		
Q	Nu	Sh	Q	Nu	Sh	Q	Nu	Sh
0	1.3422	2.1159	0	1.4865	2.3980	0	1.2631	2.0065
5	1.3956	2.1369	5	1.4899	2.4678	5	1.2798	2.0196
10	1.5623	2.2617	10	1.5627	2.4955	10	1.2859	2.0280
20	1.7864	2.3498	20	1.5906	2.5849	20	1.2911	2.0308
30	1.8976	2.5321	30	1.6521	2.5997	30	1.3125	2.0397

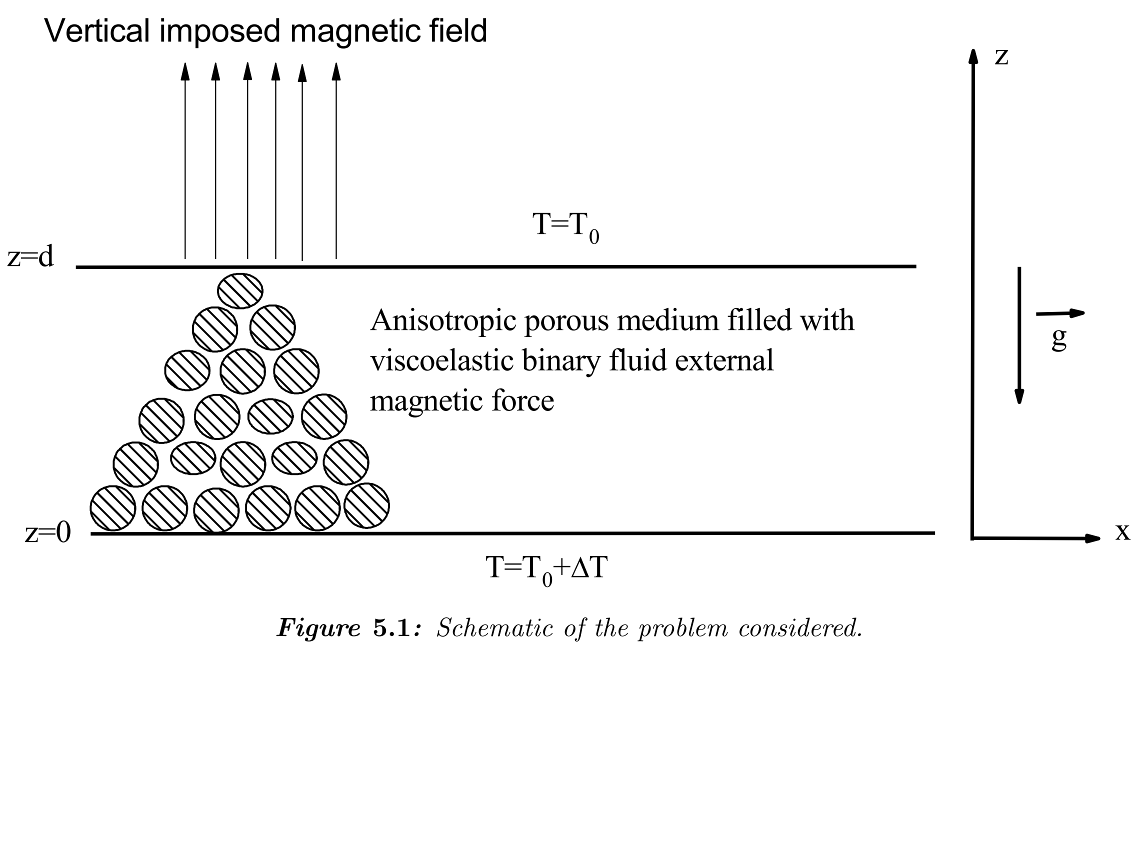

Figure 5.1: *Schematic of the problem considered.*

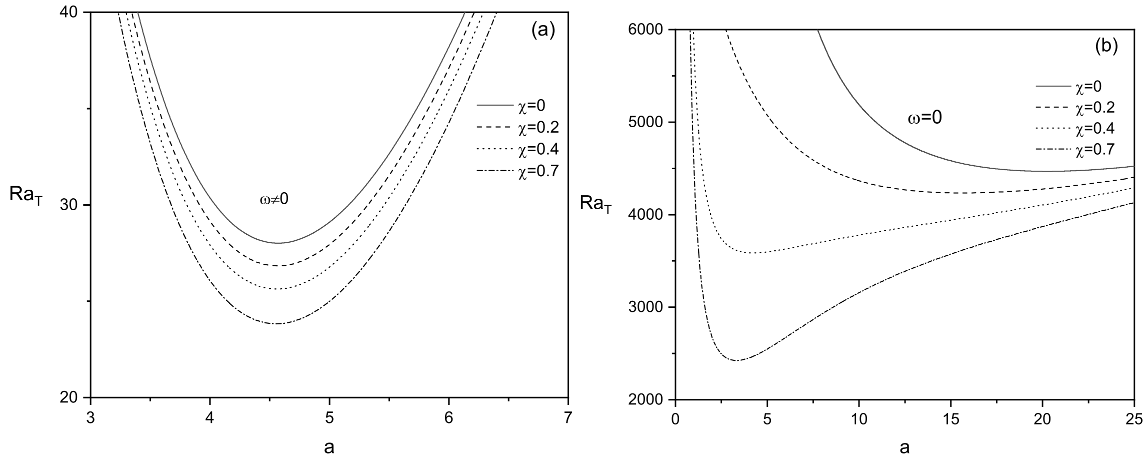

Figure 5.2: *Oscillatory and Stationary Neutral stability curves for different values of χ on (a,Ra_T) plane for (a) Oscillatory convection, (b) Stationary convection.*

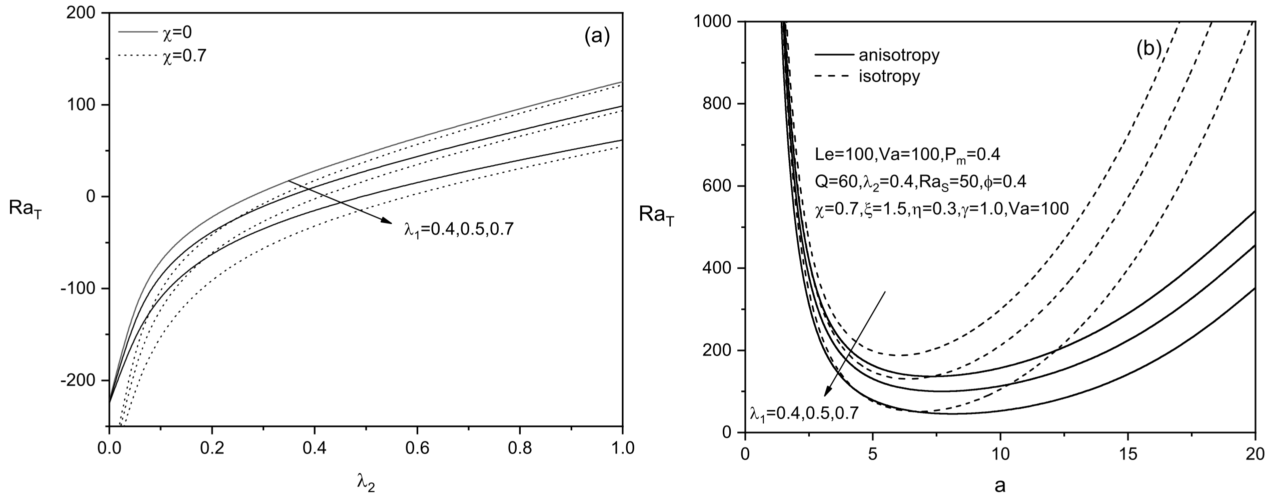

Figure 5.3: *Oscillatory neutral stability curves for different values of (λ_1) on (a) (λ_2,Ra_T) plane, (b) comparative study in isotropic and anisotropic cases (a, Ra_T) plane.*

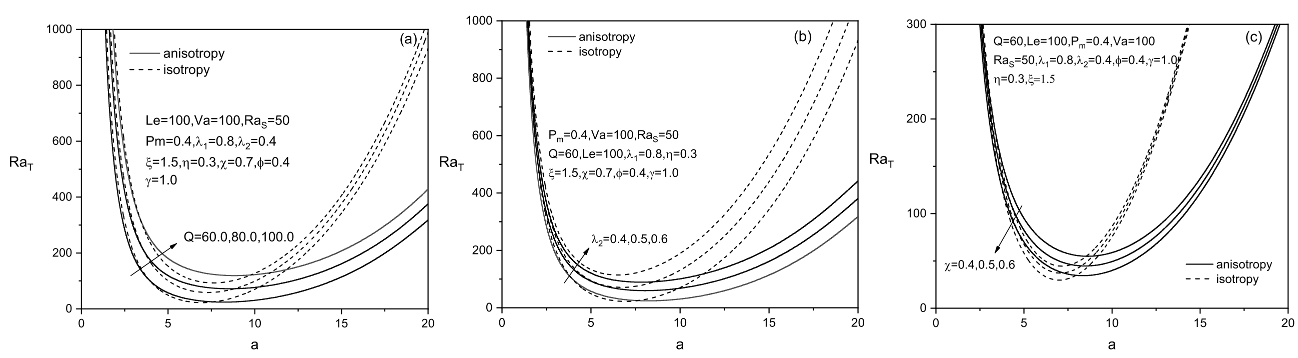

Figure 5.4: *Oscillatory neutral stability curves for different values of (a) Chandrashekhar number (Q), (b) retardation parameter (λ_2), (c) Damköhler number (χ).*

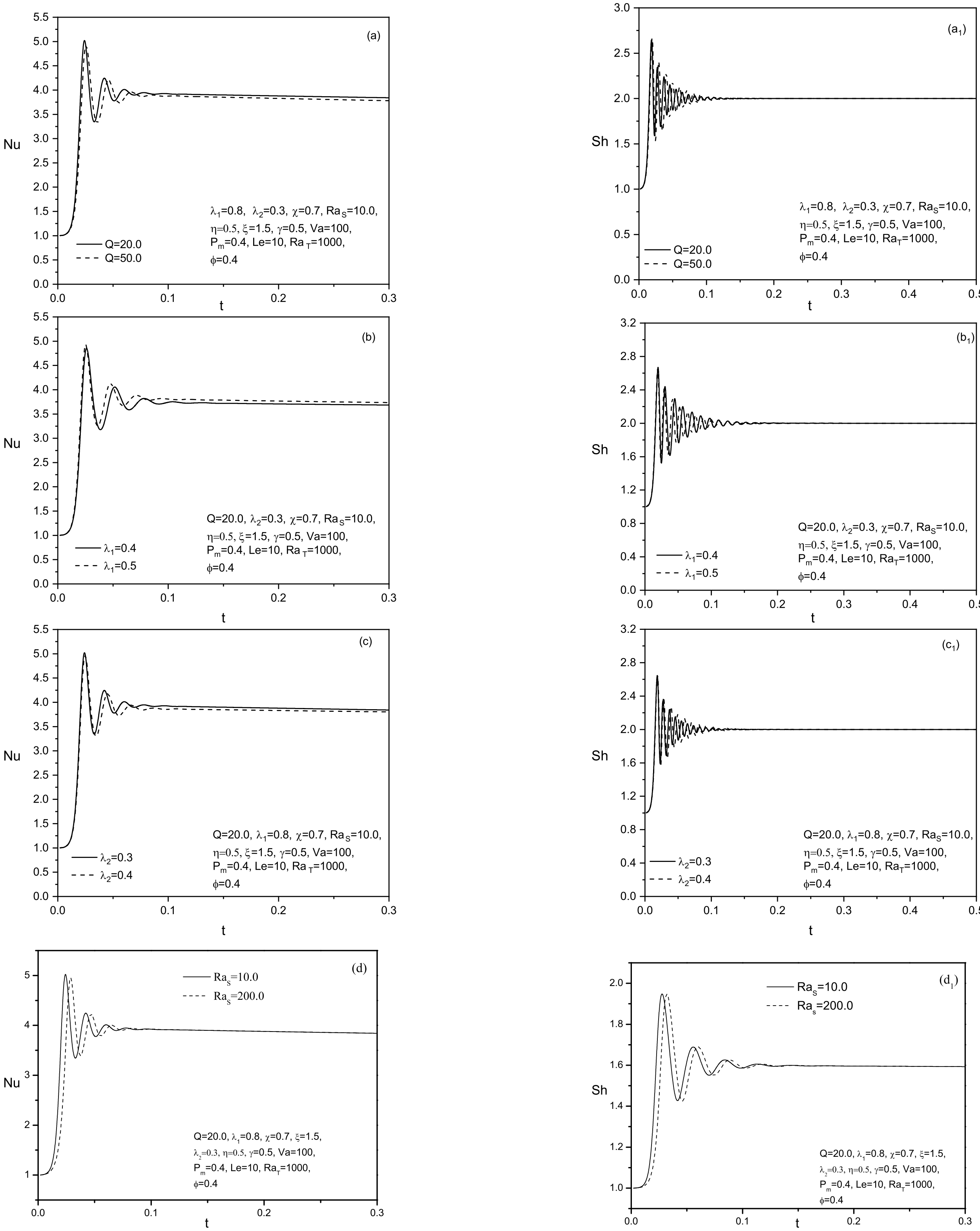

Figure 5.5: *Variation of Nusselt number (Nu) and Sherwood number (Sh) with time (t) for different values of (a) Q, (b) λ_1, (c) λ_2, (d) Ra_S on (t, Nu) plane and (a_1) Q, (b_1) λ_1, (c_1) λ_2, (d_1) Ra_S on (t, Sh) plane.*

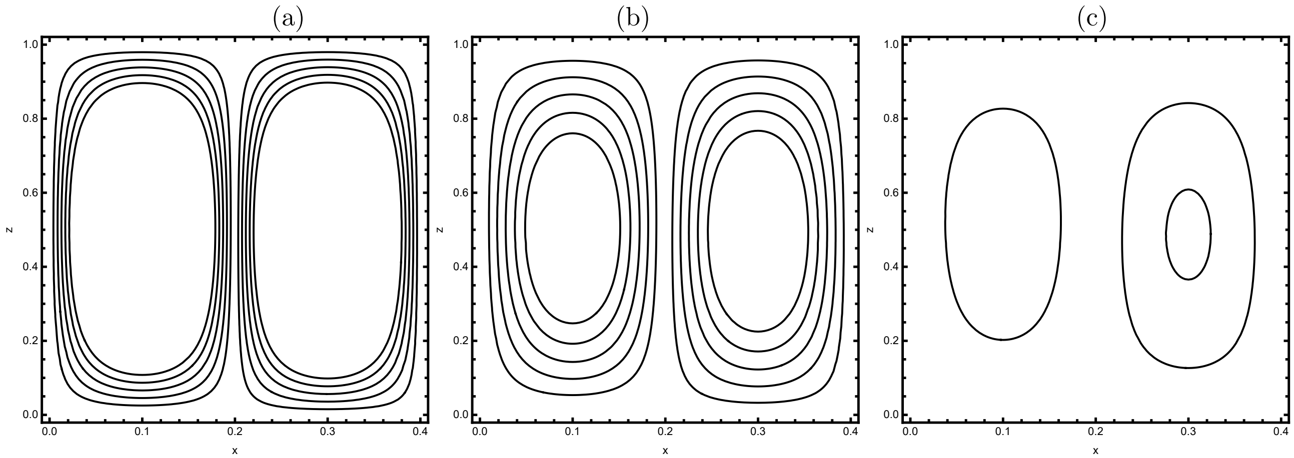

Figure 5.6: *Unsteady streamlines for different small time (a)*$t = 0.01$*, (b)* $t = 0.006$*, (c)* $t = 0.001$.

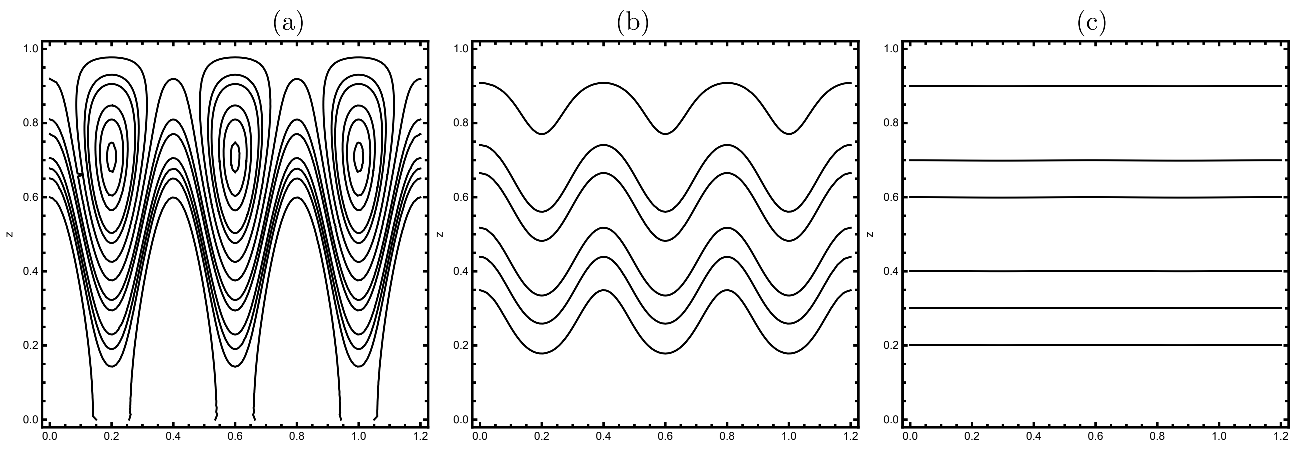

Figure 5.7: *Unsteady isotherms for different small time (a)*$t = 0.01$*, (b)* $t = 0.006$*, (c)* $t = 0.001$.

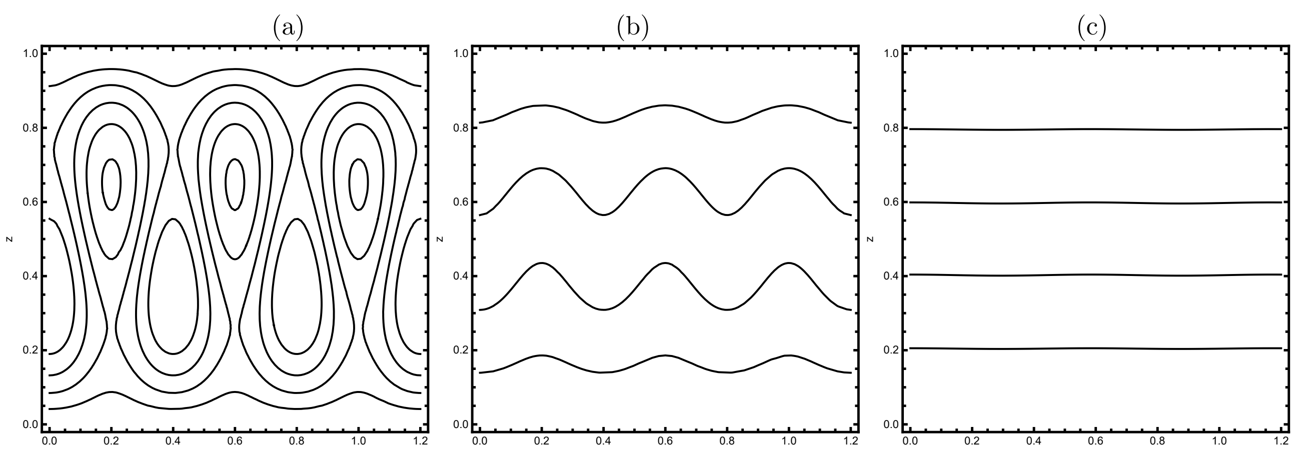

Figure 5.8: *Unsteady isohalines for different small time (a)*$t = 0.01$*, (b)* $t = 0.006$*, (c)* $t = 0.001$.

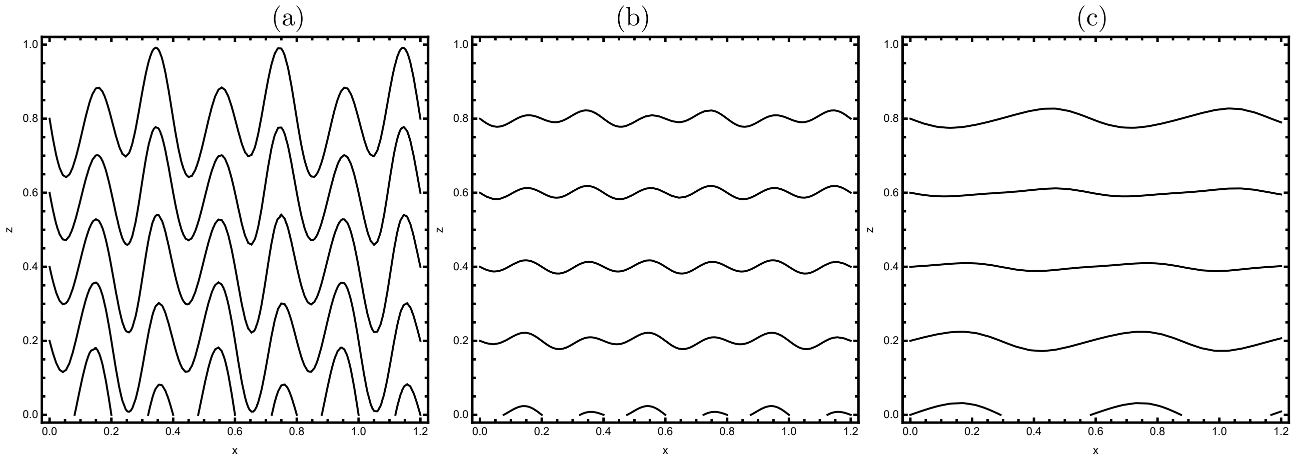

***Figure* 5.9:** *Unsteady magnetic streamlines for different small time (a)*$t = 0.01$*, (b)* $t = 0.006$*, (c)* $t = 0.001$*.*

6 Conclusion

Using linear and weakly non-linear stability analyses, we attempted to investigate the effect of the chemical reaction on magneto-convection in viscoelastic fluid saturated anisotropic porous medium. For stationary and oscillatory convection, the onset criterion has been calculated analytically. The following conclusions are drawn:

1. It is obvious that $Ra_T^{osc} < Ra_T^{st}$.
2. The Chandrashekhar number (Q) in Oldroyd-B fluids has the potential to either stabilize or destabilize the porous layer for oscillatory convection. The porous layer is always destabilizing for Maxwell fluids.
3. The effect of increasing Q and λ_2 is found to delay the onset of oscillatory convection.
4. On increasing the value of λ_1 and χ, the value of the Rayleigh number corresponding to oscillatory convection decreases, thus it advances the onset of convection.
5. These results are also verified by studying the effect of different controlling parameters on heat and mass transfer.

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