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APPLICATIONS OF FRACTIONAL GABOR TRANSFORM

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Abstract

Gabor transform is useful tool in applied mathematics to solve number of problems in various fields. Our Fractional Gabor transform adds one more parameter α which removes complexity of problems. Seeing this importance this paper is devoted to some applications of Fractional Gabor transform. We first give Fractional Gabor transform of first order derivative of $f(x)$ and n^{th} order derivative of $f(x)$ with respect to x. Some results which are required for solving Cauchy's linear differential equation are given here. Solution of Cauchy's linear differential equation is given. This paper provides some important and basic applications to the solution of some differential equations.

2020 Mathematical Sciences Classification: 46F12. **Keywords and Phrases:** Fourier transform, Gabor transform, fractional Gabor transform, testing function space, signal processing.

1 Introduction

An integral transform is the most useful technique in function transformation. It is an operator that maps function from one space to another, and the practical motivation for an integral transform is to reduce the complexity of the problem that is the mathematical operations will be much easier to handle in the image space. It arises quite commonly not only in mathematics but also in optic, signal processing and many other areas of science and engineering. It provides new aspects to many mathematical disciplines such as transforms theory, functional analysis, differential equation etc.

The theory of integral transform is flourished since the work of great mathematician having lot of applications. Bhosale [1] studied application of generalized fractional Fourier transform for solving particular types of partial differential equations. Khairnar *et al.* [4] explained bilateral Laplace-Melline transform and its application. Some new applications of Laplce-Weirstrass transform explained in [5]. Integral transformation of generalized functions and its applications are given by Pathak [6]. Diffusion of solid particles confined in viscous fluid are prepared and written by Ursell [10]. Application of the Gabor transform for analysis of Electromyographic signals of the intestine in the low frequency region given by [12]. Application of Gabor transform in the classification of myoelectric signal given by [8]. For Laplace, Fourier, Sumudu and Shehu transforms have been introduced on time scales and have served as a powerful tool for modeling and solving problems that bridge the gap between continuous and discrete dynamic systems [2]. A New α-Laplace Transform on time scale explained in [9].

License plate character segmentation based on the Gabor transform and vector quantization discussed in [3]. A convolution and product theorem for the Fractional Gabor transform given by [7]. Motivated by above, Fractional Gabor transform adds one more parameter and reduces complexity of the problem, so here we have given some applications of Fractional Gabor transform.

This paper is summarized as follows: in section 2, preliminary results are given. Fractional Gabor transform of derivatives are given in section 3. In section 4 solution of Cauchy's linear differential equation is given. An application of the Fractional Gabor transform to differential equation is given in section 5 . Lastly section 6 concluded the paper. Notations and terminology used as in Zemanian [11].

2 Preliminary Results

The Gabor transform with parameter u of $f(x)$ denoted by $G[f(x)] = G(u)$ and is given by,

$$G(u) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-\frac{(x-t)^2}{2}} e^{-iux} dx \tag{2.1}$$

for parameter $u > 0$.

The fractional Gabor transform is defined as,

$$\left[G_\alpha f(x)\right](u) = G_\alpha(u,t) = \int_{-\infty}^{\infty} f(x) K_\alpha(x,u,t) dx \tag{2.2}$$

where

$$K_\alpha(x,u,t) = \sqrt{\frac{1 - i\cot\alpha}{2\pi}} e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}} e^{-\frac{(x-t)^2 \operatorname{cosec}\alpha}{2}} e^{-iux\operatorname{cosec}\alpha} \tag{2.3}$$

3 Fractional Gabor Transform of Derivatives

Result 3.1. *Fractional Gabor transform of first order derivative of $f(x)$ with respect to x.*

Suppose that $f(x)$ is continuous for all $x \geq 0$ also its derivative $f'(x)$ is piecewise continuous for all $x \geq 0$. If the Fractional Gabor transform of $f(x)$ is

$$\left[G_\alpha f(x)\right](u) = \sqrt{\frac{1 - i\cot\alpha}{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}} e^{-\frac{(x-t)^2 \operatorname{cosec}\alpha}{2}} e^{-iux\operatorname{cosec}\alpha} dx,$$

then

$$\left[G_\alpha f_x(x)\right](u) = k\left[G_\alpha(xf(x))\right](u) + k_1\left[G_\alpha(f(x))\right](u).$$

where $k = \operatorname{cosec}\alpha - i\cot\alpha$, $k_1 = iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha$.

Proof.

$$\left[G_\alpha f_x(x)\right](u) = \sqrt{\frac{1 - i\cot\alpha}{2\pi}} \int_{-\infty}^{\infty} f_x(x) e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}} e^{-\frac{(x-t)^2 \operatorname{cosec}\alpha}{2}} e^{-iux\operatorname{cosec}\alpha} dx$$

Integarating with respect to $'x'$,

$$\begin{aligned}
\left[G_\alpha f_x(x)\right](u) = &-(i\cot\alpha - \operatorname{cosec}\alpha)\sqrt{\frac{1 - i\cot\alpha}{2\pi}} \int_{-\infty}^{\infty} x f(x) e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2} - \frac{(x-t)^2 \operatorname{cosec}\alpha}{2} - iux\operatorname{cosec}\alpha} dx \\
&+ (iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha)\sqrt{\frac{1 - i\cot\alpha}{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2} - \frac{(x-t)^2 \operatorname{cosec}\alpha}{2} - iux\operatorname{cosec}\alpha} dx \\
&= (\operatorname{cosec}\alpha - i\cot\alpha)\left[G_\alpha(xf(x))\right](u) + (iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha)\left[G_\alpha(f(x))\right](u)
\end{aligned}$$

$$\left[G_\alpha f_x(x)\right](u) = k\left[G_\alpha(xf(x))\right](u) + k_1\left[G_\alpha(f(x))\right](u), \tag{3.1}$$

where

$$k = \operatorname{cosec}\alpha - i\cot\alpha, k_1 = iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha \tag{3.2}$$

□

Result 3.2. *Fractional Gabor transform of n^{th} order derivative of $f(x)$ with respect to x.*

Suppose that $f(x)$ is continuous for all $x \geq 0$ also its derivatives up to n^{th} order are piecewise continuous for all $x \geq 0$. If $\left[G_\alpha f(x)\right](u)$ is the Fractional Gabor transform of $f(x)$ then
$\left[G_\alpha f_{x^n}(x)\right](u) = k\left[G_\alpha\left(xf_{x^n-1}(x)\right)\right](u) + k_1\left[G_\alpha\left(xf_{x^n-1}(x)\right)\right](u)$,
where $k = \operatorname{cosec}\alpha - i\cot\alpha, k_1 = iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha$.

Proof.

$$\begin{aligned}
\left[G_\alpha f_{xx}(x)\right](u) &= \sqrt{\frac{1 - i\cot\alpha}{2\pi}} \int_{-\infty}^{\infty} f_{xx}(x) e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2} - \frac{(x-t)^2 \operatorname{cosec}\alpha}{2} - iux\operatorname{cosec}\alpha} dx \\
&= \sqrt{\frac{1 - i\cot\alpha}{2\pi}} \left\{ (\operatorname{cosec}\alpha - i\cot\alpha) \int_{-\infty}^{\infty} x f_x(x) e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2} - \frac{(x-t)^2 \operatorname{cosec}\alpha}{2} - iux\operatorname{cosec}\alpha} dx \right. \\
&\left. + (iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha) \int_{-\infty}^{\infty} f_x(x) e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2} - \frac{(x-t)^2 \operatorname{cosec}\alpha}{2} - iux\operatorname{cosec}\alpha} dx \right\}
\end{aligned}$$

$$= (\operatorname{cosec}\alpha - i\cot\alpha)\left[G_\alpha\left(xf_x(x)\right)\right](u) + (iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha)\left[G_\alpha\left(f_x(x)\right)\right](u). \tag{3.3}$$

(using (3.1)).

Now,

$$\left[G_\alpha f_{xxx}(x)\right](u) = \sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty} f_{xxx}(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2} - \frac{(x-t)^2\operatorname{cosec}\alpha}{2} - iux\operatorname{cosec}\alpha}dx$$

$$= -(i\cot\alpha - \operatorname{cosec}\alpha)\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty} xf_{xx}(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2} - \frac{(x-t)^2\operatorname{cosec}\alpha}{2} - iux\operatorname{cosec}\alpha}dx$$

$$+ (iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha)\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty} f_{xx}(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2} - \frac{(x-t)^2\operatorname{cosec}\alpha}{2} - iux\operatorname{cosec}\alpha} \left(\text{By integrating w.r.t } {}^6x'\right)$$

$$= (\operatorname{cosec}\alpha - i\cot\alpha)\left[G_\alpha\left(xf_{xx}(x)\right)\right](u) + (iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha)\left[G_\alpha\left(f_{xx}(x)\right)\right](u)$$

$$= k\left[G_\alpha\left(xf_{xx}(x)\right)\right](u) + k_1\left[G_\alpha\left(f_{xx}(x)\right)\right](u).$$

$$\left[G_\alpha f_x(x)\right](u) = k\left[G_\alpha(xf(x))\right](u) + k_1\left[G_\alpha(xf(x))\right](u), \tag{3.4}$$

$$\left[G_\alpha f_{xx}(x)\right](u) = k\left[G_\alpha\left(xf_x(x)\right)\right](u) + k_1\left[G_\alpha\left(xf_x(x)\right)\right](u),$$

$$\left[G_\alpha f_{xxx}(x)\right](u) = k\left[G_\alpha\left(xf_{xx}(x)\right)\right](u) + k_1\left[G_\alpha\left(xf_{xx}(x)\right)\right](u),$$

$$\left[G_\alpha f_{x^n}(x)\right](u) = k\left[G_\alpha\left(xf_{x^{n-1}}(x)\right)\right](u) + k_1\left[G_\alpha\left(xf_{x^{n-1}}(x)\right)\right](u),$$

□

Result 3.3. *Fractional Gabor transform of n^{th} order derivative of $f(x)$ with respect to x :*

Suppose that $f(x)$ is continuous for all $x \geq 0$ also its derivative $f'(x)$ is piecewise continuous for all $x \geq 0$. If $\left[G_\alpha f(x)\right](u)$ is the Fractional Gabor transform of $f(x)$ then
$\left[G_\alpha f_{x^n}(x)\right](u) = k\sum_{j=1}^{n} k_1^{j-1}\left[G_\alpha\left(xf_{x^n-j}(x)\right)\right](u) + k_1^n\left[G_\alpha f(x)\right](u)$,
where $k = \operatorname{cosec}\alpha - i\cot\alpha$, $k_1 = iu\operatorname{cosec}\alpha - t\operatorname{cosec}\alpha$.

Proof. Equation (3.1) is

$$\left[G_\alpha f_x(x)\right](u) = k\left[G_\alpha(xf(x))\right](u) + k_1\left[G_\alpha f(x)\right](u).$$

Equation (3.3) is

$$\left[G_\alpha f_{xx}(x)\right](u) = k\left[G_\alpha\left(xf_x(x)\right)\right](u) + k_1\left[G_\alpha f_x(x)\right](u).$$

An applying to equation (3.1) in equation (3.3) gives

$$\begin{aligned}\left[G_\alpha f_{xx}(x)\right](u) &= k\left[G_\alpha\left(xf_x(x)\right)\right](u) + k_1\left\{k\left[G_\alpha(xf(x))\right](u) + k_1\left[G_\alpha f(x)\right](u)\right\}\\ &= k\left[G_\alpha\left(xf_x(x)\right)\right](u) + kk_1\left[G_\alpha(xf(x))\right](u) + k_1^2\left[G_\alpha f(x)\right](u)\end{aligned}$$

Equation (3.4) is

$$\left[G_\alpha f_{xxx}(x)\right](u) = k\left[G_\alpha\left(xf_{xx}(x)\right)\right](u) + k_1\left[G_\alpha\left(f_{xx}(x)\right)\right](u)$$

Using above equation in it

$$\begin{aligned}\left[G_\alpha f_{xxx}(x)\right](u) =& k\left[G_\alpha\left(xf_{xx}(x)\right)\right](u)\\ &+ k_1\left\{k\left[G_\alpha\left(xf_x(x)\right)\right](u) + kk_1\left[G_\alpha(xf(x))\right](u) + k_1^2\left[G_\alpha(f(x))\right](u)\right\}\\ =& k\left[G_\alpha\left(xf_{xx}(x)\right)\right](u) + kk_1\left[G_\alpha\left(xf_x(x)\right)\right](u) + kk_1^2\left[G_\alpha(xf(x))\right](u)\\ &+ k_1^3\left[G_\alpha(f(x))\right](u)\\ \left[G_\alpha f_{x^n}(x)\right](u) =& k\sum_{j=1}^{n} k_1^{j-1}\left[G_\alpha\left(xf_{x^{n-j}}(x)\right)\right](u) + k_1^n\left[G_\alpha f(x)\right](u).\end{aligned}$$

□

4 Application of Fractional Gabor transform to solve Cauchy's linear differential equation

First some results required for solving Cauchy's linear differential equation are proved.

4.1 Fractional Gabor transform of $xf_x(x)$

$$\left[G_\alpha\left(xf_x(x)\right)\right](u)=k\left[G_\alpha\left(x^2f(x)\right)\right](u)+k_1\left[G_\alpha(xf(x))\right](u)-\left[G_\alpha(f(x))\right](u),$$

where $k=\operatorname{cosec}\alpha-i\cot\alpha, k_1=iu\operatorname{cosec}\alpha-t\operatorname{cosec}\alpha$.

Proof.

$$\left[G_\alpha\left(xf_x(x)\right)\right](u)=\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}xf_x(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}}e^{-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}}e^{-iux\operatorname{cosec}\alpha}dx.$$

Integarating with respect to ’ x'

Integarating with respect to ’ x'

$$\begin{aligned}
&=-(i\cot\alpha-\operatorname{cosec}\alpha)\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}x^2f(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}-iux\operatorname{cosec}\alpha}dx\\
&-(t\operatorname{cosec}\alpha-iu\operatorname{cosec}\alpha)\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}xf(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}-iux\operatorname{cosec}\alpha}dx\\
&-\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}f(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}-iux\operatorname{cosec}\alpha}dx\\
&=k\left[G_\alpha\left(x^2f(x)\right)\right](u)+k_1\left[G_\alpha(xf(x))\right](u)-\left[G_\alpha(f(x))\right](u),
\end{aligned}$$

where

$$k=\operatorname{cosec}\alpha-i\cot\alpha, k_1=iu\operatorname{cosec}\alpha-t\operatorname{cosec}\alpha.$$

□

4.2 Fractional Gabor transform of $x^2f_{xx}(x)$

$$\left[G_\alpha\left(x^2f_{xx}(x)\right)\right](u)=k\left[G_\alpha\left(x^3f_x(x)\right)\right](u)+k_1\left[G_\alpha\left(x^2f_x(x)\right)\right](u)-2\left[G_\alpha\left(xf_x(x)\right)\right](u),$$

where $k=\operatorname{cosec}\alpha-i\cot\alpha, k_1=iu\operatorname{cosec}\alpha-t\operatorname{cosec}\alpha$.

Proof.

$$\begin{aligned}
\left[G_\alpha\left(x^2f_{xx}(x)\right)\right](u)&=\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}x^2f_{xx}(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}}e^{-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}}e^{-iux\operatorname{cosec}\alpha}dx\\
&=-\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}\Bigg[x^2e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}-iux\operatorname{cosec}\alpha}\\
&\times\left(x(i\cot\alpha-\operatorname{cosec}\alpha)+(t\operatorname{cosec}\alpha-iucsc\alpha)\right)\\
&\times e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}}e^{-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}}e^{-iux\operatorname{cosec}\alpha}2x\Bigg]f_x(x)dx
\end{aligned}$$

(On integarating with respect to ’ x')

$$\begin{aligned}
&=-(i\cot\alpha-\operatorname{cosec}\alpha)\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}x^3f_x(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}-iux\operatorname{cosec}\alpha}dx\\
&-(t\operatorname{cosec}\alpha-iu\operatorname{cosec}\alpha)\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}x^2f_x(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}-iux\operatorname{cosec}\alpha}dx\\
&-\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}2xf_x(x)e^{i\frac{\left(x^2+u^2\right)\cot\alpha}{2}-\frac{(x-t)^2\operatorname{cosec}\alpha}{2}-iux\operatorname{cosec}\alpha}dx\\
&=\ k\left[G_\alpha\left(x^3f_x(x)\right)\right](u)+k_1\left[G_\alpha\left(x^2f_x(x)\right)\right](u)-2\left[G_\alpha\left(xf_x(x)\right)\right](u),
\end{aligned}$$

where $k=\operatorname{cosec}\alpha-i\cot\alpha, k_1=iu\operatorname{cosec}\alpha-t\operatorname{cosec}\alpha$. □

4.3 Cauchy's linear differential equation by using Fractional Gabor transforms:

Here, the solution of Fractional Gabor transform by using above two results was given.

The Cauchy's linear differential equation is $x^2 f_{xx}(x)+xf_x(x)+f(x)=0$.

The Fractional Gabor transform of $x^2 f_{xx}(x)+xf_x(x)+f(x)=0$ is

$$\left[G_\alpha\left(x^2 f_{xx}(x)\right)\right](u)+\left[G_\alpha\left(xf_x(x)\right)\right](u)+\left[G_\alpha(f(x))\right](u)=0. \tag{4.1}$$

Using results (4.1) and (4.2) equation (4.1) becomes,

$$\begin{aligned}&\left\{k\left[G_\alpha\left(x^3 f_x(x)\right)\right](u)+k_1\left[G_\alpha\left(x^2 f_x(x)\right)\right](u)-2\left[G_\alpha\left(xf_x(x)\right)\right](u)\right\}\\ &+\left\{k\left[G_\alpha\left(x^2 f(x)\right)\right](u)+k_1\left[G_\alpha(xf(x))\right](u)-\left[G_\alpha(f(x))\right](u)\right\}+\left[G_\alpha(f(x))\right](u)=0.\\ &k\left[G_\alpha\left(x^3 f_x(x)\right)\right](u)+k_1\left[G_\alpha\left(x^2 f_x(x)\right)\right](u)-2\left[G_\alpha\left(xf_x(x)\right)\right](u)\\ &+k\left[G_\alpha\left(x^2 f(x)\right)\right](u)+k_1\left[G_\alpha(xf(x))\right](u)=0,\end{aligned}$$

where $k=\operatorname{cosec}\alpha-i\cot\alpha,\ k_1=iu\operatorname{cosec}\alpha-t\operatorname{cosec}\alpha$.

5 Application of Fractional Gabor transform to differential equation

Consider the differential equation

$$P\left(D_x\right)(u(x))=f(x) \tag{5.1}$$

where $f\in E'$ and $P\left(D_x\right)=\sum_{|\beta|\le m}a_\beta D_x^\beta$ is a linear differential operator of order m with constant coefficients a_β.

Suppose that the equation (5.1) possesses a solution u.

Applying the Fractional Gabor transform to (5.1)

$$\begin{aligned}\left[G_\alpha\left(P\left(D_x\right)u\right)\right]&=\left[G_\alpha(f)\right]\\ &=\left[G_\alpha\left(\sum_{|\beta|\le m}a_\beta D_x^\beta\right)u\right]\\ &=\left\{\sum a_\beta\left(\sum_{j=o}^{\beta}C_{\alpha,j}[i(2x\cos\alpha-2u)-2(x-t)]^{\beta-2j}\right)\right\}G_\alpha(u),\end{aligned}$$

where $C_{\alpha,j}$ are constant depends on α, j and β is order of derivative.

$$\left\{\sum a_\beta\left(\sum_{j=o}^{\beta}C_{\alpha,j}[i(2x\cos\alpha-2u)-2(x-t)]^{\beta-2j}\right)\right\}G_\alpha(u)=G_\alpha(f). \tag{5.2}$$

Now

$$\left|P\left(a_\beta\sum_{j=o}^{\beta}C_{\alpha,j}[i(2x\cos\alpha-2u)-2(x-t)]^{\beta-2j}\right)\right|>0,\ \text{for } u=\left(u_1,u_2,\ldots,u_n\right). \tag{5.3}$$

Equation (5.2) gives,

$$G_\alpha(u)=P\left(a_\beta\sum_{j=o}^{\beta}C_{\alpha,j}[i(2x\cos\alpha-2u)-2(x-t)]^{\beta-2j}\right)^{-1}G_\alpha(f) \tag{5.4}$$

Applying inversion of Fractional Gabor transform $\left[G_\alpha\right]^{-1}$ to equation (5.4) we get,

$$u=\left[G_\alpha\right]^{-1}\left[\frac{G_\alpha(f)}{P\left(a_\beta\sum_{j=o}^{\beta}C_{\alpha,j}[i(2x\cos\alpha-2u)-2(x-t)]^{\beta-2j}\right)}\right] \tag{5.5}$$

Next we show that if $f\in E'$ and P satisfies equation (5.3) then equation (5.5) defines a generalized function which is a solution of equation (5.1).

Indeed, since $f\in E'$ then for $0<\alpha<\pi, G_\alpha(f)\in E'$ and hence by equation (5.3) and the definition that if $\theta\in E$ and $f\in E'$ then the product ' θf ' is defined by $\langle\theta,f,\psi\rangle=,\langle f,\theta,\psi\rangle,\forall\psi\in E$.

Hence we have $\frac{G_\alpha(f)}{P\left(a_\beta\sum_{j=o}^{\beta}C_{\alpha,j}[i(2x\cos\alpha-2u)-2(x-t)]^{\beta-2j}\right)}\in E'$ and so $u\in E'$.

To show that u satisfies equation (5.1) we apply G_α to both side of equation (5.5) and get (5.4). Since generalized functions admit multiplication by polynomial then we obtain equality (5.2). Finally applying $\left[G_\alpha\right]^{-1}$ to (5.2) we get (5.1) that is given differential equation.

6 Conclusion

Since Fractional Gabor transform has found numerous applications in various fields. We tried to develop a new Gabor type of transform, Fractional Gabor transform on the same lines. The solutions of Cauchy's linear differential equation were obtained. The Fractional Gabor transform is usually used to simplify a differential equation into a simple and solvable algebra problem. Even when the algebra becomes a little complex, it is still easier to solve than solving a differential equation.

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