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A CLASS OF *I*-FUNCTIONS

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Abstract

The *I*-Function introduced by the author [4] is the ultimate generalization of all the hypergeometric functions series or complex integrals (like *E* and *H*-Functions). This function was based on the generalized Hardy-Titchmarsh theorems for symmetric and unsymmetric Fourier kernels. However, there is a class of *I*-Functions which have not come to the light in spite of their potential applications in science, engineering and artificial intelligence (*AI*). The purpose of this paper is to introduce such functions. This investigation may be useful in finding solutions of differential and Integral equations arising in mixed boundary value problems of Physics, Engineering and Biology.

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1 Introduction

Several researchers have studied and analyzed the *I*-Function and its trivial extensions. Some of them have investigated properties and characteristics including algebraic aspects [5]. Some applications of these functions have also attracted applied mathematicians. In particular, solutions of certain class of single and dual integral equations involving special functions results in terms of *I*-Functions.

As we know the kernels in Integral transforms and Integral equations play important roles. These classified as symmetric and unsymmetric Fourier kernels as per the following definitions

$$f(x) = \int_a^b K(u, x)h(u)du, \quad (1.1)$$

$$h(u) = \int_a^b H(u, x)f(x)dx. \quad (1.2)$$

Here $f(x)$ and $h(u)$ are known and unknown functions respective. $K(u, x)$ are called kernels (known) and if both are same then known as symmetrical kernels otherwise unsymmetrical.

The emergence of Saxena's *I*-Function was dependent on the generalizations of HardyTitchmarsh theorems for both types brought out by the author [1,7] during his *Ph.D.* thesis work [8] which was published subsequently. This was an extension of the theory of linear integral equations. The theorems are given below:

Theorem 1.1. *Generalized Hardy-Titchmarsh Theorem*

Let

$$g(x) = \int_0^\infty K(ux)f(u)du, \quad (1.3)$$

where $K(x)$ be a kernel function expressed in a finite series of kernels

$$K(x) = \sum_{r=1}^n K_r(x) \quad (1.4)$$

and suppose $\overline{K_r}(\xi)(\xi = \sigma + it)$ are Mellin transforms of $K_r(x)(r = 1, 2, 3, \dots, n)$, which satisfy the conditions

$$\overline{K_r}(\xi) = \left\{ A_{1,r} + \frac{B_{1,r}}{\xi} + O\left(\frac{1}{|\xi|^2}\right) \right\} \Gamma(\xi) \cos \frac{1}{2}\xi\pi, \quad \text{as } t \rightarrow \infty$$

$$= \left\{ A_{2,r} + \frac{B_{2,r}}{\xi} + O\left(\frac{1}{|\xi|^2}\right) \right\} \Gamma(\xi) \cos \frac{1}{2}\xi\pi, \quad \text{as } t \rightarrow -\infty,$$

where $\mathbf{A}_{i,r}$ ($i = 1, 2$) are constants. Then

$$\frac{1}{2}[f(x+0) + f(x-0)] = \frac{1}{2\pi i} \int_L^\infty \left[\sum_{r=1}^n K_r(\xi) \right]^{-1} G(1-\xi)x^{-\xi} d\xi, \quad (1.5)$$

where $G(\xi)$, the Mellin transform of $G(x)$, belong to $L_2(\sigma - i\infty, \sigma + i\infty)$.

Proof. Let $K(x)$ be a kernel function expressed as

$$K(x) = \sum_{r=1}^n K_r(x),$$

where n is finite and $\bar{K}(\xi)$ the Mellin transform of $K_r(x)$; $\bar{L}_r(\xi) = \frac{1}{\bar{K}_r(1-\xi)}$ satisfy the following conditions:

- (i) $\mathbf{K}_r(\mathbf{x})$ and $\bar{\mathbf{L}}_r(\xi), \xi = \sigma + it$ (σ and t are real) are regular in the strips $\sigma_1 < \sigma < \sigma_2$, where $\sigma_1 < 0, \sigma_2 > 1$ except, perhaps for a finite number of simple poles in the imaginary axis.
- (ii)

$$\begin{aligned} \bar{K}_r(\xi) &= \left\{ A_{1,r} + \frac{B_{1,r}}{\xi} + O\left(\frac{1}{|\xi|^2}\right) \right\} \Gamma(\xi) \cos \frac{1}{2}\xi\pi, \quad \text{as } t \rightarrow \infty \\ &= \left\{ A_{2,r} + \frac{B_{2,r}}{\xi} + O\left(\frac{1}{|\xi|^2}\right) \right\} \Gamma(\xi) \cos \frac{1}{2}\xi\pi, \quad \text{as } t \rightarrow -\infty \\ \bar{L}_r(\xi) &= \left\{ C_{1,r} + \frac{D_{1,r}}{\xi} + O\left(\frac{1}{|\xi|^2}\right) \right\} \Gamma(\xi) \cos \frac{1}{2}\xi\pi, \quad \text{as } t \rightarrow \infty \\ &= \left\{ C_{2,r} + \frac{D_{2,r}}{\xi} + O\left(\frac{1}{|\xi|^2}\right) \right\} \Gamma(\xi) \cos \frac{1}{2}\xi\pi, \quad \text{as } t \rightarrow -\infty, \end{aligned}$$

where $A_{1,r}, B_{1,r}, A_{2,r}, B_{2,r}, C_{1,r}, D_{1,r}, C_{2,r}$ and $D_{2,r}$ ($r = 1, 2, \dots, n$) are constants.

- (iii) $f(u)$ and $h(u)$ functions of bounded variation near $u = x, f(x) \in L(0, \infty)$ and $\bar{f}(\xi) \in L(\frac{1}{2} - i\infty, \frac{1}{2} + i\infty)$. Then solution of integral equation

$$f(x) = \int_0^\infty K(ux)h(u)du$$

is given by

$$\frac{1}{2}[h(x+0) + h(x-0)] = \lim_{t \rightarrow \infty} \frac{1}{2\pi i} \int_{\frac{1}{2}-it}^{\frac{1}{2}+it} \Phi(\xi) \bar{f}(1-\xi)x^{-\xi} d\xi \quad (1.6)$$

where

$$\Phi(\xi) = \left[\sum_{r=1}^n \frac{1}{\bar{L}_r(\xi)} \right]^{-1}$$

and $\bar{f}(\xi)$ is Mellin transform of $f(x)$.

Above generalized Hardy-Titchmarsh theorem evolves variety of functions which nontrivial generalizations of functions which may fall under the category of symmetric and unsymmetric Fourier kernels. The class of functions which are generated from hypergeometric functions or their special cases may be called 'A class of I -Functions'. This will include Saxena's I -Function. An extensive use of Mellin Transforms has been carried out in these derivations which are given in the next section.

The Mellin transform of a function $f(x)$ is defined as, if we take kernel

$$\begin{aligned} K(u, x) &= u^{\xi-1}, \\ \bar{f}(\xi) &= M[f(u)] = \int_0^\infty u^{\xi-1} f(u) du. \end{aligned} \quad (1.7)$$

The inversion formula of Mellin transform is given by

$$M^{-1}[\bar{f}(\xi)] = f(u) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} u^{-\xi} \bar{f}(\xi) d\xi. \quad (1.8)$$

The Parseval's theorem (convolution theorem) of the Mellin transform [6] is

$$M[f(u)] = \bar{f}(\xi), M[g(u)] = \bar{g}(\xi), \quad (1.9)$$

$$\int_0^\infty f(u)g(x)du = \lim_{t \rightarrow \infty} \frac{1}{2\pi i} \int_{\sigma_0 - it}^{\sigma_0 + it} x^{-\xi} \bar{g}(\xi) \bar{f}(1 - \xi) d\xi, \quad (1.10)$$

$$M[f(ux)] = x^{-\xi}, f(\xi) = M[f(u)], \quad (1.11)$$

then,

$$\int_0^\infty f(u)g(ux)du = \lim_{t \rightarrow \infty} \frac{1}{2\pi i} \int_{\sigma_0 - it}^{\sigma_0 + it} x^{-\xi} \bar{g}(\xi) \bar{f}(1 - \xi) d\xi. \quad (1.12)$$

Validity conditions of the equation are,

$$f(\xi) \in L_p(\sigma - i\infty, \sigma + i\infty) \text{ and } x^{1-\sigma}g(x) \in L_p(0, \infty), p \geq 1.$$

□

2 A Class of I -Functions

On the basis of the generalized Hardy-Titchmarsh theorem we can introduce several new functions and call them as a new class of I -Functions. Some of such functions are related to classical special functions and denoted as listed below

$$I_J, \quad I_Y, \quad I_W, \quad I_F, \quad I_E, \quad I_G, \quad I_{BM}, \quad I_H/I.$$

These functions can be generated from Bessel, Modified Bessel, Whittaker, Generalized Hypergeometric, Mac-Robert's- E , Meijer's- G , Bessel-Maitland and Fox's H -Functions [5,6] respectively. The I_H/I is Saxena's I -Function. These functions are defined as:

(i) **Bessel I -Function:**

$$I_J(z) = \frac{1}{2\pi i} \int_L \frac{\Gamma(1 - 1/2(a_0 + v_0) + \xi) z^\xi d\xi}{\sum_{i=1}^r \Gamma(1/2(a_i - v_i - \xi))}, \quad (2.1)$$

$a_i \geq 0, \text{Re}(\xi) > \text{Re}(v_i) > \frac{3}{2}$ for $i = 0, 2, \dots, r$; L is Barne's contour parallel to imaginary axis.

(ii) **Modified Bessel I -Function:**

$$I_Y(z) = \frac{1}{2\pi i} \int_L \frac{\Gamma(1 - 1/2(a_0 - v_0 - 1) + \xi) \Gamma(1/2(a_0 - v_0) - \xi) z}{\sum_{i=1}^r \Gamma(1 - 1/2(a_i + v_i) + \xi) \Gamma(1 + 1/2(a_i + v_i) - \xi)}, \quad (2.2)$$

$a_i \geq 0, |\text{Re}(v_i)| < \text{Re}(\xi) < \frac{3}{2}$ for $i = 0, 2, \dots, r$

(iii) **Whittaker I -Function:**

$$I_W(z) = \frac{1}{2\pi i} \int_L \frac{\Gamma(-a_0 + \lambda_0 + \xi) \Gamma(a_0 - \mu_0 + \frac{1}{2} - \xi) z^\xi d\xi}{\sum_{i=1}^r \Gamma(\frac{1}{2} - a_i + \mu_i) + \xi}, \quad (2.3)$$

$$|\text{Re}(\mu_i)| - \frac{1}{2} < \text{Re}(\xi) < -\text{Re}(\lambda_i) \text{ for } i = 0, 2, \dots, r$$

(iv) **Hypergeometric I -Function:**

$$I_F(z) = \frac{1}{2\pi i} \int_L \frac{\Gamma(1 + \xi) z^\xi d\xi}{\sum_{i=1}^r \prod_{j=1}^p \Gamma(1 - a_{ir} - \xi) \prod_{j=1}^q \Gamma(b_{ir} + \xi)}, \quad (2.4)$$

Where $p < q, a_{ir}, b_{ir} \geq 0$.

(v) **E - I -Function:**

$$I_E(z) = \frac{1}{2\pi i} \int_L \frac{\Gamma(\xi) \prod_{j=1}^q \Gamma(b_j - \xi) z z^\xi d\xi}{\sum_{i=1}^r \prod \Gamma(a_{ij} - \xi)}, \quad (2.5)$$

where L is Barne's contour on complex plane parallel to imaginary axis running from $\sigma - i\infty$ to $\sigma + i\infty$, where σ is appropriate real number. In this particular section $\sigma = 0$ and all the poles of $\Gamma(b_j - \xi)$ ($j = 1, 2, \dots, p$) are to the right of L and of $\Gamma(\xi)$ to the left; $p_i < q + 1$ and $|\arg(z)| < (m + n - \frac{1}{2}p_j - q)\pi$ for $i = 1, 2, \dots, r$.

(vi) **$G - I$ -Function:**

$$I_G(z) = \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^m \Gamma(b_j - \xi) \prod_{j=1}^n \Gamma(1 - a_j + \xi) z^\xi d\xi}{\sum_{i=1}^r \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} + \xi) \prod_{j=n+1}^{p_i} \Gamma(a_{ji} - \xi)}. \quad (2.6)$$

As earlier L is Barne's contour such that all the poles of $\Gamma(b_j - \xi)$ ($j = 1, 2, \dots, m$) are to the right of L and all the poles of $\Gamma(1 - a_j + \xi)$ ($j = 1, 2, \dots, n$) are to the left.

(vii) **Bessel-Maitland-I-Function:**

$$I_{BM}(z) = \frac{1}{2\pi i} \int_L \frac{\Gamma(\xi) z^\xi d\xi}{\sum_{i=1}^r \Gamma(1 + v_i + \lambda_i \xi)}, \quad (2.7)$$

$v_i, \lambda_i > 0 (i = 1, 2, \dots, r)$.

(viii) **$H - I$ -Function (Saxena's I -Function):**

$$I(z) = I_{p_i, q_i; r}^{m, n} \left[z / \begin{matrix} (a_j, \alpha_j), \dots, (a_p, \alpha_p) \\ (b_j, \beta_j), \dots, (b_p, \beta_p) \end{matrix} \right] \quad (2.8)$$

$$= \frac{1}{2\pi i} \int_L \varphi(s) z^s ds, \quad (2.9)$$

where

$$\varphi(s) = \frac{\prod_{j=1}^m \Gamma(b_j - \beta_j s) \prod_{j=1}^m \Gamma(1 - a_j + \alpha_j s)}{\sum_{i=1}^r \left\{ \prod_{j=m+1}^{q_i} (1 - b_{ji} + \beta_{ji} s) \prod_{j=n+1}^{p_i} (a_{ji} - \alpha_{ji} s) \right\}}, \quad (2.10)$$

$p_i (i = 1, 2, 3, \dots, r), q_i (i = 1, 2, 3, \dots, r), m$ and n are integers satisfying $0 < n < p_i$ and $0 < m < q_i, r$ is finite and $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji}$, are complex numbers.

For I -function, there are three different paths L of integration

- L is a contour which runs from $\sigma - i\infty$ to $\sigma + i\infty$ (σ is real), so that all poles of $\Gamma(b_j - \beta_j s)$, $j = 1, 2, 3, \dots, m$ are to the right and all poles of $\Gamma(1 - a_j + \alpha_j s)$, $j = 1, 2, 3, \dots, n$ are to the left of L .
- L is a loop starting and ending at $\sigma + i\infty$ and encircling all the poles of $\Gamma(b_j - \beta_j s)$, $j = 1, 2, 3, \dots, m$, once in the negative direction but none of the pole of $\Gamma(1 - a_j + \alpha_j s)$, $j = 1, 2, 3, \dots, n$. The integral converges if $q > 1$ and either $p_i < q_i$ or $p_i = q_i$ and $|z| < 1, i = 1, 2, 3, \dots, r$.
- L is a loop starting and ending at $\sigma + i\infty$ and encircling all the poles of $\Gamma(1 - a_j + \alpha_j s)$, once in positive direction, but none of the poles of $\Gamma(b_j - \beta_j s)$, $j = 1, 2, 3, \dots, m$.

On specializing the parameters in I -Function we can arrive at G and H functions. Thus G and H functions are particular cases of I -Function.

3 Example: [5]

The definitions (1.1) and (1.2) are motivated and derived from the theory of symmetrical and unsymmetrical Fourier kernel and supported by generalized Hardy-Titchmarsh theorem. Other than the above functions there are several other functions defined in terms of Gamma functions can generate more I -Functions. For examples like Bessel-Maitland I -Functions can be evolved as given below, in which use of Erdélyi-Kober operators \Re and \Im made extensively [5].

Theorem 3.1. *If $f(x)$ is solution of the integral equation*

$$\int_0^\infty \sum_{j=1}^N u^{\alpha_j} J_{\mu_j}^{\lambda_j}(ux) f(u) du = g(x), \quad x > 1, \quad (3.1)$$

where α, λ and μ are arbitrary real numbers. Then

$$f(x) = \int_0^\infty I_{m+N-1, m+N+1; r}^{N, m} \left[ux \left| \begin{matrix} (A_j, \gamma_j)_{1, m'} & (A_{ji}, \gamma_{ji})_{1, N-1} \\ (B_j, \delta_j)_{1, N'} & (B_{ji}, \delta_{ji})_{1, m+1} \end{matrix} \right. \right] g_1(u) du, \quad (3.2)$$

where

$$g_1(x) = \prod_{k=1}^m \{ \Re_k[g(x)] \}, \Re \left[b_k - e_k, \frac{\mu_k}{f_k} : \frac{1}{f_k} \right] g(x) = \Re_k[g(x)], x > 1.$$

$A_j = 1 - b_k - f_k, \gamma_j = f_k, (j, k = 1, 2, \dots, m); B_j = 1 - \lambda_j + \mu_j - \lambda_j \alpha_j, \delta_j = \lambda_j, (j = 1, 2, \dots, N); A_{ji} = 1 - \lambda_{ji} + \mu_{ji} - \lambda_{ji} \alpha_{ji}, \gamma_{ji} = \lambda_{ji} (j = 1, 2, \dots, N-1; i = 1, 2, \dots, r) B_{ji} = 1 - e_{ki} - f_{ki}, \delta_{ji} = f_{ki}, (j, k = 1, 2, \dots, m; i = 1, 2, \dots, r); B_{m+1, i} = -\alpha_i, \delta_{m+1, i} = 1, (i = 1, 2, \dots, r)$ provided

- $\lambda' > 0, |\arg x| < \frac{1}{2} \lambda' \pi,$
- $\lambda' \geq 0, |\arg x| \leq \frac{1}{2} \lambda' \pi, \operatorname{Re}(\mu' + 1) < 0,$

where

$$\lambda' = \sum_{j=1}^m \gamma_j + \sum_{j=1}^N \delta_j - \max_{1 \leq i \leq r} \left[\sum_{j=1}^{N-1} \gamma_{ji} + \sum_{j=1}^{m+1} \delta_{ji} \right] \quad (3.3)$$

and

$$\mu' = \sum_{j=1}^N B_j - \sum_{j=1}^m A_j - \min_{1 \leq i \leq r} \left[\sum_{j=1}^{N-1} A_{ji} - \sum_{j=1}^{m+1} B_{ji} - 1 \right]. \quad (3.4)$$

Proof. Applying Parseval's theorem of the Mellin transforms in (3.1), we obtain

$$\lim_{t \rightarrow \infty} \frac{1}{2\pi i} \int_{\sigma_0 - it}^{\sigma_0 + it} \sum_{j=1}^N \frac{\Gamma(\alpha_j + \xi)}{\Gamma(1 + \mu_j - \lambda_j \alpha_j - \lambda_j \xi)} x^{-\xi} F(1 - \xi) d\xi = g(x), x > 1, \quad (3.5)$$

where $\xi = \sigma_0 + it$ and $F(\xi)$ is Mellin transform of $f(u)$.

Proceeding on the same lines of above and apply the second fractional integral operator \mathfrak{R} . We obtain the transformation of the equation (3.1) as

$$\int_0^\infty \sum_{j=1}^N H_{m+2,m}^{0,m+1} \left[ux \left| \begin{matrix} (\dots), (b_k, f_k)_{1,m}, (\dots) \\ (\alpha_j, 1), (e_k, f_k)_{1,m}, (1 + \mu_j - \lambda_j \alpha, \lambda_j) \end{matrix} \right. \right] f(u) du = g_1(x), \quad (3.6)$$

where $g_1(x)$ is known. Proceeding on the similar lines as above, we obtain

$$f(x) = \frac{1}{2\pi i} \int_L \frac{\prod_{j=1}^N \Gamma(1 + \mu_j - \lambda_j \alpha_j - \lambda_j + \lambda_j s) \prod_{k=1}^m \Gamma\{b_k + f_k - f_k s\} x^{-s} G(1 - s) ds}{\sum_{i=1}^r \left[\Gamma(\alpha_i + 1 - s) \prod_{j=1}^{N-1} \Gamma(1 + \mu_{ji} - \lambda_{ji} \alpha_{ji} - \lambda_{ji} + \lambda_{ji} s) \prod_{k=1}^m \Gamma\{e_{ki} + f_{ki} - f_{ki} s\} \right]}. \quad (3.7)$$

Again, applying Parseval's theorem defined in section-1, we finally obtain R.H.S of (1.8). \square

Corollary 3.1. If α, λ and μ are arbitrary positive real numbers ($0 < \lambda_j < 1$) for $j = 1, \dots, N$, and if

$$\int_0^\infty \sum_{j=1}^N u^{\alpha_j} J_{\mu_j}^{\lambda_j}(ux) f(u) du = g(x), 0 < x < 1. \quad (3.8)$$

Then

$$f(x) = \int_0^\infty I_{m+N-1, m+N+1; r}^{m+N, 0} \left[ux \left| \begin{matrix} (a_k - g_k, g_k)_{1,m}, (1 - \lambda_{ji} + \mu_{ji} - \lambda_{ji} \alpha_{ji}, \lambda_{ji})_{1, N-1} \\ (1 - \lambda_j + \mu_j - \lambda_j \alpha_j, \lambda_j)_{1, N}, (\tau_k - g_k, g_k)_{1, m}, (-\alpha_j, 1) \end{matrix} \right. \right] h_1(u) du, \quad (3.9)$$

where

$$h_1(x) = \prod_{k=1}^m \{\mathfrak{I}_k[g(x)]\}, \mathfrak{I}_k[g(x)] = \mathfrak{I}[a_k - \tau_k, \tau_k g_k^{-1} - 1 : g_k^{-1}] g(x), \quad 0 < x < 1.$$

provided $a_k > \tau_k$ and $\frac{\tau_k}{g_k} > c$, ($\xi = c + it$), $m > 0, N > 0, \text{Re}(\tau_k - \min a_k) > 0, (k = 1, 2, \dots, m)$ and other conditions of I -Function are same as given earlier.

Proof. Now, to establish the next inversion, we shall use the operator \mathfrak{I} , and Beta function. Further we replace x by v in (3.5) and multiply both the sides by

$$\left(x^{\frac{1}{g_1}} - v^{\frac{1}{g_1}} \right)^{a_1 - \tau_1 - 1} v^{\frac{\tau_1}{g_1} - 1},$$

and integrate under the integral sign with respect to 0 to x and apply the same process as above, we obtain the transformations of the integral equation (3.8), is given as

$$\int_0^\infty \sum_{j=1}^N H_{j_m, m+2}^{1, m} \left[ux \left| \begin{matrix} (\dots), (\tau_k, g_k)_{1, m}, (\dots) \\ (\alpha_j, 1), (a_k, g_k)_{1, m}, (1 + \mu_j - \lambda_j \alpha, \lambda_j) \end{matrix} \right. \right] f(u) du = h_1(x) \quad (3.10)$$

where

$$h_1(x) = \prod_{k=1}^m \{\mathfrak{I}_k[g(x)]\}, \quad 0 < x < 1. \quad (3.11)$$

By proceeding on similar lines, we will obtain R.H.S of (3.9).

The I -function thus created may be useful in solving problems arising in forms of integral equations including mixed boundary problems of science, engineering and biology. \square

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