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# A COMMON FIXED-POINT THEOREM IN PERTURBED METRIC SPACES WITH RATIONAL CONTRACTIVE CONDITION

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**Abstract**

In the present paper, we establish a common fixed point theorem in perturbed metric spaces for two pair of weakly compatible mappings using rational contraction.Additionally, an application and an example are provided to support our generalization.
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## 1 Introduction and Preliminaries

The existence and uniqueness of fixed points are the main concerns of fixed point theory, which is widely acknowledged to be one of the most studied areas of nonlinear functional analysis today. In order to ensure that fixed points exist and are unique, Banach [2] obtained the first fundamentals result in this area. In short, there is a unique fixed point in a complete metric space for every contraction mapping. This result is called the principle of Banach contraction. Since the introduction of the Banach principle, this topic has become more significant than ever before due to the fixed point theory's limitless potential for usage in a wide range of scientific fields, such as physics, chemistry, economics, some branches of engineering, and numerous branches of mathematics. Because of this, several authors have explored for more fixed point conclusions using the well-known Banach principle. They have also successfully published new fixed point results that were created by combining or utilizing two extremely potent approaches. Substituting a more universal space for the idea of a metric space is one method to achieve this. Some generalizations of metric spaces, such as quasi-metric spaces, partial metric spaces, G-metric spaces, fuzzy metric spaces, b-metric spaces, perturbed metric spaces, multiplicative metric spaces, etc., could be considered as replacements. Because of its significance and uses in various scientific domains, the well-known Banach Contraction Principle has been expanded upon and developed by a number of authors over time by introducing rational contractions in complete metric spaces. Jaggi is credited with one of these attempts [9].

Assume that ( $\Upsilon, \Sigma$ ) be a complete metric space and let $\mathcal{N} : \Upsilon \to \Upsilon$ be a self-mapping. If there exist $\alpha, \beta \in [0, 1)$ with $\alpha + \beta < 1$ such that A self-mapping $\mathcal{N}$ on a complete metric spaces ( $\Upsilon, \Sigma$ ) admits a unique fixed point if $\Sigma(\mathcal{N}a, \mathcal{N}b) \leq \alpha\Sigma(a, b) + \frac{\beta\Sigma(b,\mathcal{N}b)[1+\Sigma(a,\mathcal{N}a)]}{1+\Sigma(a,b)}$ for all $a, b \in Y$ with $a \neq b$, then $\mathcal{N}$ has a unique fixed point p $\in \Upsilon$.

In 1975, Dass and Gupta [2] gave the following fixed point theorem of new rational contraction to generalize the Banach contraction principle

Let $(\Upsilon, \Sigma$ ) be a complete metric space, and let $\mathcal{N} : \Upsilon \to \Upsilon$ be a self mapping. If there exist $\alpha, \beta \in [0, 1)$ with $\alpha + \beta < 1$ such that

$$\Sigma(\mathcal{N}(a), \mathcal{N}(b)) \leqslant \alpha\Sigma(a, b) + \beta\frac{[1 + \Sigma(a, \mathcal{N}(a))]\Sigma(b, \mathcal{N}(b))}{1 + \Sigma(a, b)} \tag{1.1}$$

then $\mathcal{N}$ has a unique fixed point $x^* \in \Upsilon$.

For more details one can see [1, 3, 5, 7, 8, 11-21].

Perturbed metric spaces, one of the most fascinating of abstract spaces, are an exciting generalization of a metric space that was proposed in 2025 by Jleli and Samet [10]. The recently suggested idea of perturbed metric spaces is defined as follows:

**Definition 1.1.** ([10]). *Assume that the two provided mappings are $\nabla, \aleph : \Upsilon \times \Upsilon \to [0, \infty)$. Accordingly, if $\nabla - \aleph : \Upsilon \times \Upsilon \to [0, \infty)$ is defined as follows, then $\nabla$ represents a perturbed metric on $\Upsilon$ with regard to $\aleph$.*
$(\nabla - \aleph)(a, b) = \nabla(a, b) - \aleph(a, b)$
*that is, for all $a, b, o \in X$,*
*(i) $(\nabla - \aleph)(a, b) \geq 0$;*
*(ii) $(\nabla - \aleph)(a, b) = 0 \Leftrightarrow a = b$;*
*(iii) $(\nabla - \aleph)(a, b) = (\nabla - \aleph)(a, b)$;*
*(iv) $(\nabla - \aleph)(a, b) \leq (\nabla - \aleph)(a, b) + (\nabla - \aleph)(a, b)$,*
*where $\Sigma = \nabla - \aleph$ produces a standard metric, the mapping $\aleph$ is referred to as a perturbed mapping, and ( $\Upsilon, \nabla, \mathcal{X}$ ) is referred to as a perturbed metric space. For a standard metric space, we use ( $\Upsilon, \Sigma$ ). Keep in mind that a metric on $\Upsilon$ is not always a perturbed metric on $\Upsilon$.*

For other examples and preliminary work on perturbed metric space, refer to Jleli & Samet [10]. Several topological concepts in the layout of perturbed metric spaces are now presented:

**Definition 1.2.** ([10]). *Let $(\Upsilon, \nabla, \aleph)$ form a perturbed metric space. Take into consideration a self-mapping $\mathcal{N}$ defined on $\Upsilon$ and a sequence $\{p_n\}$ in $\Upsilon$.*
*(i) If the metric d is defined as $\Sigma = \nabla - \aleph$, and $\{p_n\}$ is a convergent sequence in a standard metric space ( $Y, \Sigma$ ), then $\{p_n\}$ is a convergent sequence in the "perturbed sense in ( $\Upsilon, \nabla, \aleph$ ).*
*(ii) The sequence $\{p_n\}$ is a perturbed Cauchy sequence in ( $\Upsilon, \nabla, \aleph$ ) if it is a Cauchy in the context of a standard metric space ( $\Upsilon, \Sigma$ ).*
*(iii) ( $\Upsilon, \nabla, \aleph$ ) is referred to as a complete perturbed metric space when ( $\Upsilon, \Sigma$ ) is a standard complete metric space. This means that each perturbed Cauchy sequence in ( $\Upsilon, \nabla, \aleph$ ) converges in the 'perturbed sense'.*
*(iv) A mapping $\mathcal{N}$ is perturbed continuous if it is continuous in the standard metric space $(\Upsilon, \Sigma)$.*

**Definition 1.3.** . *Let $\mathfrak{B}$ and $\mathfrak{C}$ be two self-mappings of perturbed metric spaces ( $\Upsilon, \nabla, \aleph$ ).*
*Commutative mappings are those in which $\mathfrak{B}\mathfrak{C}a = \mathfrak{C}\mathfrak{B}a$ for all $a \in \Upsilon$.*

**Definition 1.4.** . *Let $\mathfrak{B}$ and $\mathfrak{C}$ be two self-mappings of perturbed metric spaces ( $\Upsilon, \nabla, \aleph$ ).*
*Weak commutative mappings are defined as $\nabla(\mathfrak{B}\mathfrak{C}a, \mathfrak{C}\mathfrak{B}a) \leq \nabla(\mathfrak{B}a, \mathfrak{C}a)$ for any $a$ in $\Upsilon$.*

**Remark 1.1.** . *Weakly commuting is implied by commuting: $\mathfrak{B}\mathfrak{C}a = \mathfrak{C}\mathfrak{B}a$ for all $a \in \Upsilon$ if two mappings $\mathfrak{B}$ and $\mathfrak{C}$ commute. This indicates that $\nabla(\mathfrak{B}\mathfrak{C}a, \mathfrak{C}\mathfrak{B}a) = 0$. The weak commutativity criterion, $\nabla(\mathfrak{B}\mathfrak{C}a, \mathfrak{C}\mathfrak{B}a) \leq \nabla(\mathfrak{B}a, \mathfrak{C}a)$, is always met since standard distance is always nonnegative. The opposite is untrue: It is possible for two mappings to commute weakly without really commuting. This occurs when there is at least one $a_0 \in \Upsilon$ for which $\mathfrak{B}\mathfrak{C}a_0 \neq \mathfrak{C}\mathfrak{B}a_0$, yet $\nabla(\mathfrak{B}\mathfrak{C}a, \mathfrak{C}\mathfrak{B}a) \leq \nabla(\mathfrak{B}a, \mathfrak{C}a)$ holds for all $a \in \Upsilon$.*

**Example 1.1.** For an interval $\Upsilon = [0, 2]$, we shall define $\nabla : \Upsilon \times \Upsilon \to [0, \infty)$ be the mapping defined by

$$\nabla(\kappa, t) = (\kappa - t)^2$$

for all $\kappa, t \in \Upsilon$. Then $\nabla$ is a perturbed metric on $\mathbb{Y}$, where the perturbed mapping $\mathbb{N}$ is given by

$$\aleph(\kappa, t) = (\kappa - t)^2 - |\kappa - t|,$$

$\kappa, t \in \Upsilon$, and the exact metric $\Sigma$ is given by

$$\Sigma(\kappa, t) = |\kappa - t|$$

$\kappa, t \in \Upsilon$. Clearly, $(\Upsilon, \nabla, \aleph)$ is a complete perturbed metric space.
Define mappings $\mathfrak{B}$ and $\mathfrak{C} : \Upsilon \to \Upsilon$ by

$$\mathfrak{B}a = 3 - a \text{ and } \mathfrak{C}a = \begin{cases} a & \text{if} \quad a \in \quad [0, 1) \\ 3 - a & \text{if} \quad a \in \quad [1, 2]. \end{cases}$$

Then $\mathfrak{B}$ and $\mathfrak{C}$ are weakly commuting but not commuting.

**Definition 1.5.** . *Consider two self-mappings of perturbed metric spaces ( $\Upsilon, \nabla, \aleph$ ) with values $\mathfrak{B}$ and $\mathfrak{C}$. If two mappings $\mathfrak{B}$ and $\mathfrak{C}$ commute at coincidence points, meaning that $\mathfrak{B}a = \mathfrak{C}a$ implies $\mathfrak{B}\mathfrak{C}a = \mathfrak{C}\mathfrak{B}a$ for $a \in \Upsilon$, then they are said to be weakly compatible.*

## 2 Main Result

Ghaziyah, Karapinar, and Shahi [6] demonstrated the following outcome in a complete perturbed metric space in 2025:

*Let $\mathcal{N}$ be a perturbed continuous mapping on a complete perturbed metric space ( $\Upsilon, \nabla, \aleph$ ). If there exists $\psi \in \Psi$ for all $\kappa, \eta \in \Upsilon$ satisfying*
$\nabla(\mathcal{N}\kappa, \mathcal{N}\eta) \leq \psi\left(\max\left\{\nabla(\kappa,\eta), \nabla(\kappa,\mathcal{N}\kappa), \nabla(\eta,\mathcal{N}\eta), \frac{\nabla(\eta,\mathcal{N}\eta)[1+\nabla(\kappa,\mathcal{N}\kappa)]}{1+\nabla(\kappa,\eta)}\right\}\right)$.
*Then $\mathcal{N}$ contain a fixed point .*

Now we extend and generalize the above result in complete perturbed metric spaces as follow:

**Theorem 2.1.** . *Consider a complete perturbed metric space ( $\Upsilon, \nabla, \aleph$ ). Assume that self-maps $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}$, and $\mathfrak{E} : \Upsilon \to \Upsilon$ are continuous mappings that satisfy an additional requirement:*
$(C_1)\, \nabla(\mathfrak{B}\kappa, \mathfrak{C}\eta) \leq k_1 \nabla(\mathfrak{D}\eta, \mathfrak{E}\kappa) + k_2 \max\left\{\nabla(\mathfrak{D}\eta, \mathfrak{E}\kappa), \frac{\nabla(\mathfrak{E}\kappa,\mathfrak{B}\kappa)\nabla(\mathfrak{D}\eta,\mathfrak{C}\eta)}{1+\nabla(\mathfrak{B}\kappa,\mathfrak{C}\eta)}\right\} + k_3 \min\{\nabla(\mathfrak{E}\kappa, \mathfrak{C}\eta), \nabla(\mathfrak{D}\eta, \mathfrak{B}\kappa)\}$
*for all $\kappa, \eta \in \Upsilon$,*
*where $k_1 + k_2 + k_3 < 1, k_i \geq 0, i = 1, 2, 3,$* (2.1)
*$(C_2)\, \mathfrak{B}Y \subseteq \mathfrak{D}Y$ and $\mathfrak{C}Y \subseteq \mathfrak{E}Y$,*
*$(C_3)\, (\mathfrak{B}, \mathfrak{E})$ and $(\mathfrak{C}, \mathfrak{D})$ are weakly compatible,*
*$(C_4)$ perturbed function $\aleph(b, b) = 0$.*
*Then, for $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}$ and $\mathfrak{E}$ in $\Upsilon$, there is only one common fixed point.*

*Proof.* Let $\kappa_0^p := \kappa$ be an arbitrary point in $\Upsilon$, and using condition $(C_2)$, we establish the sequences $\{\kappa_{\mathrm{n}}^p\}$ and $\{\eta_{\mathrm{n}}^p\}$ in $\Upsilon$ as follows:

$$\eta_{2n}^p = \mathfrak{D}\kappa_{2n+1}^p = \mathfrak{B}\kappa_{2n}^p \text{ and } \eta_{2n+1}^p = \mathfrak{E}\kappa_{2n+2}^p = \mathfrak{C}\kappa_{2n+1}^p \text{ for all } n = 0, 1, 2, \ldots\cdot$$

If $\eta_{2n}^p = \eta_{2n+1}^p$, for some $n$, then $\kappa_{2n+1}^p$ is a coincident point of $\mathfrak{D}$ and $\mathfrak{E}$.
Similarly, if $\eta_{2n+1}^p = \eta_{2n+2}^p$, for some $n$, then $\mathfrak{B}\kappa_{2n+2}^p = \mathfrak{E}\kappa_{2n+2}^p$, implying that $\kappa_{2n+2}^p$ is a coincident point of $\mathfrak{B}$ and $\mathfrak{E}$.
Assume that $\eta_{2n}^p \neq \eta_{2n+1}^p$, for all $n$.
Then, using ( $C_1$ ), by putting $\kappa = \kappa_{2n}^p$ and $\eta = \kappa_{2n+1}^p$, we have
$\nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right) = \nabla\left(\mathfrak{B}\kappa_{2n}^p, \mathfrak{C}\kappa_{2n+1}^p\right) \leq k_1 \nabla\left(\mathfrak{D}\kappa_{2n+1}^p, \mathfrak{E}\kappa_{2n}^p\right) +$
$k_2 \max\left\{\nabla\left(\mathfrak{D}\kappa_{2n+1}^p, \mathfrak{E}\kappa_{2n}^p\right), \frac{\nabla\left(\mathfrak{E}\kappa_{2n}^p, \mathfrak{B}\kappa_{2n}^p\right)\nabla\left(\mathfrak{D}\kappa_{2n+1}^p, \mathfrak{U}\kappa_{2n+1}^p\right)}{1+\nabla\left(\mathfrak{B}\kappa_{2n}^p, \mathfrak{C}\kappa_{2n+1}^p\right)}\right\} +$
$k_3 \min\left\{\nabla\left(\mathfrak{E}\kappa_{2n}^p, \mathfrak{C}\kappa_{2n+1}^p\right), \nabla\left(\mathfrak{D}\kappa_{2n+1}^p, \mathfrak{B}\kappa_{2n}^p\right)\right\}$.
That is, $\nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right) = \nabla\left(\mathfrak{B}\kappa_{2n}^p, \mathfrak{C}\kappa_{2n+1}^p\right) \leq k_1 \nabla\left(\eta_{2n}^p, \eta_{2n-1}^p\right) +$
$k_2 \max\left\{\nabla\left(\eta_{2n}^p, \eta_{2n-1}^p\right), \frac{\nabla\left(\eta_{2n-1}^p, \eta_{2n}^p\right)\nabla\left(\eta_{2n}^p, \kappa_{2n+1}^p\right)}{1+\nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right)}\right\} + k_3 \min\left\{\nabla\left(\eta_{2n-1}^p, \eta_{2n+1}^p\right), \nabla\left(\eta_{2n}^p, \eta_{2n}^p\right)\right\}$
i.e., $\nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right) \leq (k_1 + k_2)\nabla\left(\eta_{2n}^p, \eta_{2n-1}^p\right) +$

$$k_3 \min\left\{\nabla\left(\eta_{2n-1}^p, \eta_{2n+1}^p\right), \nabla\left(\eta_{2n}^p, \eta_{2n}^p\right)\right\}. \tag{2.2}$$

By definition, $\Sigma = \nabla - \mathbb{N}$ is the exact metric. In view of (2.2), we deduce that

$$\Sigma\left(\eta_{2n}^p, \eta_{2n+1}^p\right) + \mathrm{R}\left(\eta_{2n}^p, \eta_{2n+1}^p\right)$$
$$\leq (k_1 + k_2)\nabla\left(\eta_{2n}^p, \eta_{2n-1}^p\right) + k_3 \min\left\{\nabla\left(\eta_{2n-1}^p, \eta_{2n+1}^p\right), \nabla\left(\eta_{2n}^p, \eta_{2n}^p\right)\right\}$$
$$\Sigma\left(\eta_{2n}^p, \eta_{2n+1}^p\right) \leq (k_1 + k_2)\left(\Sigma\left(\eta_{2n}^p, \eta_{2n-1}^p\right) + \kappa\left(\eta_{2n}^p, \eta_{2n-1}^p\right)\right) +$$
$$k_3 \min\left\{\left(\mathrm{d}\left(\eta_{2n-1}^p, \eta_{2n+1}^p\right) + \mathrm{R}\left(\eta_{2n-1}^p, \eta_{2n+1}^p\right)\right), \Sigma\left(\eta_{2n}^p, \eta_{2n}^p\right) + \kappa\left(\eta_{2n}^p, \eta_{2n}^p\right)\right\}$$
$$\Sigma\left(\eta_{2n}^p, \eta_{2n+1}^p\right) \leq (k_1 + k_2)\nabla\left(\eta_{2n}^p, \eta_{2n-1}^p\right).$$

Similarly,
$\Sigma\left(\eta_{2n-1}^p, \eta_{2n}^p\right) \leq (k_1 + k_2)\nabla\left(\eta_{2n-2}^p, \eta_{2n-1}^p\right)$.
Thus, for any $n \in \mathbb{N}$,
$\Sigma\left(\eta_n^p, \eta_{n+1}^p\right) \leq (k_1 + k_2)\nabla\left(\eta_{n-1}^p, \eta_n^p\right) \leq (k_1 + k_2)^2 \nabla\left(\eta_{n-2}^p, \eta_{n-1}^p\right)$
$\leq \ldots \leq (k_1 + k_2)^n \nabla\left(\eta_0^p, \eta_1^p\right)$.

Now, considering $n, m \in \mathbb{N}$ where $m > n$, we have:

$$\Sigma\left(\eta_n^p, \eta_{n+m}^p\right) \leq \Sigma\left(\eta_n^p, \eta_{n+1}^p\right) + \Sigma\left(\eta_{n+1}^p, \eta_{n+m}^p\right)$$
$$\leq \Sigma\left(\eta_n^p, \eta_{n+1}^p\right) + \Sigma\left(\eta_{n+1}^p, \eta_{n+2}^p\right) + \Sigma\left(\eta_{n+2}^p, \eta_{n+m}^p\right)$$
$$\leq \Sigma\left(\eta_n^p, \eta_{n+1}^p\right) + \Sigma\left(\eta_{n+1}^p, \eta_{n+2}^p\right) + \Sigma\left(\eta_{n+2}^p, \eta_{n+3}^p\right) + \Sigma\left(\eta_{n+3}^p, \eta_{n+m}^p\right)$$
$$\leq \cdots \leq \Sigma\left(\eta_n^p, \eta_{n+1}^p\right) + \Sigma\left(\eta_{n+1}^p, \eta_{n+2}^p\right) + \Sigma\left(\eta_{n+2}^p, \eta_{n+3}^p\right) + \cdots + \Sigma\left(\eta_{n+m-2}^p, \eta_{n+m-1}^p\right)$$
$$+ \Sigma\left(\eta_{n+m-1}^p, \eta_{n+m}^p\right),$$
$$\Sigma\left(\eta_n^p, \eta_{n+m}^p\right) \leq (k_1+k_2)^n \nabla\left(\eta_0^p, \eta_1^p\right) + (k_1+k_2)^{n+1} \nabla\left(\eta_0^p, \eta_1^p\right) +$$
$$(k_1+k_2)^{n+2} \nabla\left(\eta_0^p, \eta_1^p\right) + \cdots + (k_1+k_2)^{n+m-2} \nabla\left(\eta_0^p, \eta_1^p\right) + (k_1+k_2)^{n+m-1} \nabla\left(\eta_0^p, \eta_1^p\right)$$
$$\leq (k_1+k_2)^n \left[1 + (k_1+k_2) + (k_1+k_2)^2 + .. + (k_1+k_2)^{m-2} + (k_1+k_2)^{m-1}\right].$$

$\Sigma\left(\eta_n^p, \eta_{n+m}^p\right) \leq \frac{(k_1+k_2)^n}{1-(k_1+k_2)} \nabla\left(\eta_0^p, \eta_1^p\right)$.
Since $(k_1+k_2) < 1$, therefore, $\lim_{m,n\to\infty} \Sigma\left(\eta_n^p, \eta_{n+m}^p\right) = 0$.
Now, following standard reasoning, the above inequality establishes that the sequence $\{\eta_n^p\}$ forms Cauchy in the layout of the standard metric space $(\Upsilon, \Sigma)$. It yields that the constructed sequence $\{\eta_n^p\}$ is a perturbed Cauchy in a perturbed metric space $(\Upsilon, \nabla, X)$. Since $\Upsilon$ is a complete therefore, $\lim_{n\to\infty} \Sigma\left(\eta_{2n}^p, \eta^*\right) = 0$.
Since $\mathfrak{B}\Upsilon \subseteq \mathfrak{D}\Upsilon$ there exists $\kappa^* \in \Upsilon$ such that $\eta^* = \mathfrak{D}\kappa^*$.
We claim that $\eta^* = \mathbb{C}\kappa^*$.
Now from triangle inequality of a standard metric space, we have

$$\Sigma\left(\mathfrak{C}\kappa^*, \eta_{2n+1}^p\right) \leq \Sigma\left(\mathfrak{C}\kappa^*, \eta_{2n}^p\right) + \Sigma\left(\eta_{2n}^p, \eta_{2n+1}^p\right).$$

By definition, $\Sigma = \nabla - \aleph$ is the exact metric, so we have

$$\nabla\left(\mathfrak{C}\kappa^*, \eta_{2n+1}^p\right) - \aleph\left(\mathfrak{C}\kappa^*, \eta_{2n+1}^p\right)$$
$$\leq \nabla\left(\mathfrak{C}\aleph^*, \eta_{2n}^p\right) - \aleph\left(\mathfrak{C}\kappa^*, \eta_{2n}^p\right) + \nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right) - \aleph\left(\eta_{2n}^p, \eta_{2n+1}^p\right),$$
$$\Sigma\left(\mathbb{C}\kappa^*, \eta_{2n+1}^p\right) \leq \nabla\left(\mathbb{C}\kappa^*, \eta_{2n}^p\right) + \nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right). \tag{2.3}$$

Again, from triangle inequality of a standard metric space, we have
$\Sigma\left(\mathbb{C}\kappa^*, \eta_{2n}^p\right) \leq \Sigma\left(\mathbb{C}\kappa^*, \eta^*\right) + \Sigma\left(\eta^*, \eta_{2n}^p\right)$,
$\Sigma\left(\mathbb{C}\kappa^*, \eta_{2n}^p\right) \leq \nabla\left(\mathbb{C}\kappa^*, \eta^*\right) + \nabla\left(\eta^*, \eta_{2n}^p\right)$.
Now (2.3) becomes

$$\Sigma\left(\mathbb{C}\kappa^*, \eta_{2n+1}^p\right) \leq \nabla\left(\mathbb{C}\kappa^*, \eta_{2n}^p\right) + \nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right)$$
$$\leq \Sigma\left(\mathbb{C}\kappa^*, \eta_{2n}^p\right) + \aleph\left(\mathbb{C}\kappa^*, \eta_{2n}^p\right) + \nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right)$$
$$\Sigma\left(\mathbb{C}\kappa^*, \eta_{2n+1}^p\right) \leq \nabla\left(\mathbb{C}\kappa^*, \eta^*\right) + \nabla\left(\eta^*, \eta_{2n}^p\right) + \nabla\left(\eta_{2n}^p, \eta_{2n+1}^p\right) + \aleph\left(\mathbb{C}\kappa^*, \eta_{2n}^p\right).$$

Taking $n \to \infty$, we have
$\Sigma\left(\widetilde{C}\kappa^*, \eta^*\right) \leq \nabla\left(\mathbb{C}\kappa^*, \eta^*\right) + \nabla\left(\eta^*, \eta^*\right) + \nabla\left(\eta^*, \eta^*\right) \leq \nabla\left(\mathbb{C}\kappa^*, \eta^*\right) + 0 + 0 + \aleph\left(\mathbb{C}\kappa^*, \eta^*\right)$
$\Sigma\left(\mathbb{C}\kappa^*, \eta^*\right) \leq \nabla\left(\mathbb{C}\kappa^*, \eta^*\right) + \aleph\left(\mathbb{C}\kappa^*, \eta^*\right)$
implies that $\eta^* = \mathbb{C}\kappa^*$.
Now
$\Sigma\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right) \leq \Sigma\left(\mathfrak{D}\kappa^*, \eta_{2n}^p\right) + \Sigma\left(\eta_{2n}^p, \mathfrak{C}\kappa^*\right)$,
$\Sigma\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right) \leq \Sigma\left(\mathfrak{D}\kappa^*, \eta_{2n}^p\right) + \nabla\left(\mathfrak{B}\kappa_{2n}^p, \mathfrak{C}\kappa^*\right) - \aleph\left(\mathfrak{B}\kappa_{2n}^p, \mathfrak{C}\kappa^*\right)$,
$\Sigma\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right) \leq \Sigma\left(\mathfrak{D}\kappa^*, \eta_{2n}^p\right) + \nabla\left(\mathfrak{B}\kappa_{2n}^p, \mathfrak{C}\kappa^*\right)$,
$\Sigma\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right) \leq \Sigma\left(\mathfrak{D}\kappa^*, \eta_{2n}^p\right) + k_1 \nabla\left(\mathfrak{D}\kappa^*, \mathfrak{E}\kappa_{2n}^p\right) + k_2 \max\left\{\nabla\left(\mathfrak{D}\kappa^*, \mathfrak{E}\kappa_{2n}^p\right), \frac{\nabla\left(\mathfrak{E}\kappa_{2n}^p, \mathfrak{B}\kappa_{2n}^p\right)\nabla\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right)}{1+\nabla\left(\mathfrak{B}\kappa_{2n}^p, \mathfrak{C}\kappa^*\right)}\right\} +$
$k_3 \min\left\{\nabla\left(\mathfrak{E}\kappa_{2n}^p, \mathfrak{C}\kappa^*\right), \nabla\left(\mathfrak{D}\kappa^*, \mathfrak{B}\kappa_{2n}^p\right)\right\}$.
Taking n $\to \infty$, we have
$\Sigma\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right) \leq \Sigma\left(\mathfrak{D}\kappa^*, \eta^*\right) + k_1 \nabla\left(\mathfrak{D}\kappa^*, \eta^*\right) + k_2 \max\left\{\nabla\left(\mathfrak{D}\kappa^*, \eta^*\right), \frac{\nabla(\eta^*, \eta^*)\nabla(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*)}{1+\nabla(\eta^*, \mathfrak{C}\kappa^*)}\right\}$
$+k_3 \min\left\{\nabla\left(\eta^*, \mathbb{C}\kappa^*\right), \nabla\left(\mathfrak{D}\kappa^*, \eta^*\right)\right\}$.
i.e., $\Sigma\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right) \leq 0 + 0 + k_2 \max\{0, 0\} + k_3 \min\{0, 0\}$.
We obtain $\lim_{n\to\infty} \Sigma\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right) = 0$.
Hence $\Sigma\left(\mathfrak{D}\kappa^*, \mathfrak{C}\kappa^*\right) = 0$, implying $\eta^* = \mathfrak{D}\kappa^* = \mathfrak{C}\kappa^*$.

Again, since $\mathbb{C}\Upsilon \subseteq \mathbb{E}\Upsilon$, there exists a point $w \in \Upsilon$ such that $\eta^* = \mathbb{E}w$.
Furthermore, using ( $C_1$ ), on putting $\kappa = w$ and $\eta = \kappa^*$, we have
$\nabla(\mathfrak{B}w, \eta^*) = \nabla(\mathfrak{B}w, \mathfrak{C}\kappa^*) \leq k_1\nabla(\mathfrak{D}\kappa^*, \mathfrak{C}w) + k_2 \max\left\{\nabla(\mathfrak{D}\kappa^*, \mathfrak{C}w), \frac{\nabla(\mathfrak{C}w,\mathfrak{B}w)\nabla(\mathfrak{D}\kappa^*,\mathfrak{C}\kappa^*)}{1+\nabla(\mathfrak{B}w,\mathfrak{C}\kappa^*)}\right\} +$
$k_3 \min\{\nabla(\mathfrak{E}w, \mathfrak{C}\kappa^*), \nabla(\mathfrak{D}\kappa^*, \mathfrak{B}w)\}$.
$\nabla(\mathfrak{B}w, \mathfrak{C}\kappa^*) \leq k_1\nabla(\eta^*, \mathfrak{G}w) + k_2 \max\left\{\nabla(\eta^*, \eta^*), \frac{\nabla(\eta^*,\mathfrak{B}w)\nabla(\eta^*,\eta^*)}{1+\nabla(\mathfrak{B}w,\eta^*)}\right\} + k_3 \min\{\nabla(\eta^*, \eta^*), \nabla(\eta^*, \mathfrak{B}w)\}$.
$\nabla(\mathfrak{B}w, \mathfrak{C}\kappa^*) \leq k_1\nabla(\eta^*, \mathfrak{E}w) + k_2 \max\{0, 0\} + k_3 \min\{0, \nabla(\eta^*, \mathfrak{B}w)\} = k_1\nabla(\eta^*, \mathfrak{E}w)$.
$\nabla(\mathfrak{B}w, \eta^*) \leq 0$.
Hence, $\Sigma(\mathfrak{B}w, \eta^*) = 0$, which implies that $\eta^* = \mathfrak{B}w$.
Consequently, this leads to $\eta^* = \mathfrak{B}w = \mathfrak{E}w$; hence, $\eta^* = \mathfrak{D}\kappa^* = \mathfrak{C}\kappa^* = \mathfrak{B}w = \mathfrak{E}w$.
Since ( $\mathfrak{C}, \mathfrak{D}$ ) is weakly compatible, it follows that $\mathfrak{C}\mathfrak{D}\kappa^* = \mathfrak{D}\mathfrak{C}\kappa^*$, thereby implying $\mathfrak{C}\eta^* = \mathfrak{D}\eta^*$.
Now, let us prove that $\eta^*$ is a fixed-point of $\mathbb{C}$.
If $\mathbb{C}\eta^* \neq \eta^*$, then according to ( $C_1$ ): on putting $\kappa = w$ and $\eta = \eta^*$, we have
$\nabla(\eta^*, \mathfrak{C}\eta^*) = \nabla(\mathfrak{B}w, \mathfrak{C}\eta^*) \leq k_1\nabla(\mathfrak{D}\eta^*, \mathfrak{C}w) + k_2 \max\left\{\nabla(\mathfrak{D}\eta^*, \mathfrak{C}w), \frac{\nabla(\mathfrak{C}w,\mathfrak{B}w)\nabla(\mathfrak{D}\eta^*,\mathfrak{C}\eta^*)}{1+\nabla(\mathfrak{B}w,\mathfrak{C}\eta^*)}\right\} +$
$k_3 \min\{\nabla(\mathfrak{C}w, \mathfrak{C}\eta^*), \nabla(\mathfrak{D}\eta^*, \mathfrak{B}w)\}$.
That is, $\nabla(\eta^*, \mathbb{C}\eta^*) \leq k_1\nabla(\mathbb{C}\eta^*, \eta^*) + k_2 \max\{\nabla(\mathbb{C}\eta^*, \eta^*), 0\} +$
$k_3 \min\{\nabla(\eta^*, \mathbb{C}\eta^*), \nabla(\mathbb{C}\eta^*, \eta^*)\} \leq (k_1 + k_2 + k_3)\nabla(\mathsf{C}\eta^*, \eta^*)$.
This implies that $\nabla(\eta^*, \mathbb{C}\eta^*) < \nabla(\eta^*, \circlearrowleft \eta^*)$, which is a contradiction.
Therefore, $\eta^* = \mathcal{C}\eta^*$.
Hence, $\eta^* = \mathfrak{C}\eta^* = \mathfrak{D}\eta^*$.
Similarly, due to the weak compatibility of ( $\mathfrak{B}, \mathfrak{E}$ ), we observe that $\eta^* = \mathfrak{B}\eta^* = \mathfrak{E}\eta^*$.
Now from ( $C_1$ ) by putting $\kappa = \eta^*$ and $\eta = \eta^*$, we get
$\nabla(\mathfrak{B}\eta^*, \eta^*) = \nabla(\mathfrak{B}\eta^*, \mathfrak{C}\eta^*) \leq k_1\nabla(\mathfrak{D}\eta^*, \mathfrak{E}\eta^*) + k_2 \max\left\{\nabla(\mathfrak{D}\eta^*, \mathfrak{E}\eta^*), \frac{\nabla(\mathfrak{C}\eta^*,\mathfrak{B}\eta^*)\nabla(\mathfrak{D}\eta^*,\mathfrak{C}\eta^*)}{1+\nabla(\mathfrak{B}\eta^*,\mathfrak{C}\eta^*)}\right\}$
$+k_3 \min\{\nabla(\mathfrak{E}\eta^*, \mathfrak{C}\eta^*), \nabla(\mathfrak{D}\eta^*, \mathfrak{B}\eta^*)\}$
$\nabla(\mathfrak{B}\eta^*, \eta^*) \leq k_1\nabla(\eta^*, \mathfrak{B}\eta^*) + k_2 \max\{\nabla(\eta^*, \mathfrak{B}\eta^*), 0\} + k_3 \min\{\nabla(\mathfrak{B}\eta^*, \eta^*), \nabla(\eta^*, \mathfrak{B}\eta^*)\}$
$\leq (k_1 + k_2 + k_3)\nabla(\mathfrak{B}\eta^*, \eta^*)$.
This implies that $\nabla(\mathfrak{B}\eta^*, \eta^*) < \nabla(\mathfrak{B}\eta^*, \eta^*)$, a contradiction.
Thus, $\eta^* = \mathfrak{B}\eta^* = \mathfrak{C}\eta^* = \mathfrak{D}\eta^* = \mathfrak{E}\eta^*$, and $\eta^*$ is a common fixed-point of $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}$, and $\mathfrak{E}$.
Uniqueness of $\eta^*$, let us suppose that $\eta_1^*$ and $\eta_2^*; \eta_1^* \neq \eta_2^*$ are common fixed-points of $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}$ and $\mathfrak{E}$.
Using $(C_1)$, on putting $\kappa = \eta_1^*$ and $\eta = \eta_2^*$, we obtain
$\nabla(\eta_1^*, \eta_2^*) = \nabla(\mathfrak{B}\eta_1^*, \mathfrak{C}\eta_2^*) \leq k_1\nabla(\mathfrak{D}\eta_2^*, \mathfrak{E}\eta_1^*) + k_2 \max\left\{\nabla(\mathfrak{D}\eta_2^*, \mathfrak{E}\eta_1^*), \frac{\nabla(\mathfrak{E}\eta_1^*,\mathfrak{B}\eta_1^*)\nabla(\mathfrak{D}\eta_2^*,\mathfrak{C}\eta_2^*)}{1+\nabla(\mathfrak{B}\eta_1^*,\mathfrak{C}\eta_2^*)}\right\} +$
$k_3 \min\{\nabla(\mathfrak{E}\eta_1^*, \mathfrak{C}\eta_2^*), \nabla(\mathfrak{D}\eta_2^*, \mathfrak{B}\eta_1^*)\}$
$\nabla(\eta_1^*, \eta_2^*) \leq k_1\nabla(\eta_2^*, \eta_1^*) + k_2 \max\{\nabla(\eta_2^*, \eta_1^*), 0\} + k_3 \min\{\nabla(\eta_1^*, \eta_2^*), \nabla(\eta_2^*, \eta_1^*)\}$.
That is, $\Sigma(\eta_1^*, \eta_2^*) = 0$.
Therefore, $\eta_1^* = \eta_2^*$.
This concludes the proof of our theorem. □

**Remark 2.1.** . *In Theorem* 2.1*, the following inequalities hold:* $(k_1 + k_2) < 1, k_1 < 1, k_2 < 1$.
*Following Remark* 2.1*, we can derive the subsequent corollaries from Theorem* 2.1.

**Corollary 2.1.** . *Let ( $\Upsilon, \nabla, \aleph$ ) be a complete perturbed metric space. Let $\mathfrak{B}, \mathbb{C} : \Upsilon \to \Upsilon$ be continuous mappings satisfying:*

$$\nabla(\mathfrak{B}\kappa, \mathfrak{C}\eta) \leq k_1\nabla(\kappa, \eta) + k_2 \max\left\{\nabla(\kappa, \eta), \frac{\nabla(\kappa, \mathfrak{B}\kappa)\nabla(\eta, \mathfrak{C}\eta)}{1 + \nabla(\mathfrak{B}\kappa, \mathfrak{C}\eta)}\right\} + k_3 \min\{\nabla(\kappa, \mathfrak{C}\eta), \nabla(\eta, \mathfrak{B}\kappa)\},$$

*for all $\kappa, \eta \in \Upsilon$, where $k_i \in [0, 1)$, i=1,2,3 satisfy $(k_1 + k_2) + k_3 < 1$.*
*Then $\mathfrak{B}$ and $\mathfrak{C}$ possess a unique common fixed point.*

**Corollary 2.2.** ([6]). *Suppose that ( $\Upsilon, \nabla, \aleph$ ) be a complete perturbed metric space. Let $\mathfrak{B} : \Upsilon \to \Upsilon$ be continuous mappings satisfying:*

$$\nabla(\mathfrak{B}\kappa, \mathfrak{B}\eta) \leq k_1\nabla(\kappa, \eta), \text{ for all } \kappa, \eta \in \Upsilon,$$

*$k_1 \in$ (0,1), then $\mathfrak{B}$ has a unique fixed-point.*
*This result signifies a Banach fixed-point theorem in a complete perturbed metric space.*

**Example 2.1.** For an interval $\Upsilon = [0,1]$, we shall define $\nabla : \Upsilon \times \Upsilon \to [0,\infty)$ be the mapping defined by

$$\nabla(\kappa, t) = (\kappa - t)^2$$

for all $\kappa, t \in \Upsilon$.
Then $\nabla$ is a perturbed metric on $\Upsilon$, where the perturbed mapping $\aleph$ is given by

$$\aleph(\kappa, t) = (\kappa - t)^2 - |\kappa - t|,$$

$\kappa, t \in \Upsilon$, and the exact metric $\Sigma$ is given by

$$\Sigma(\kappa, t) = |\kappa - t|,$$

$\kappa, t \in \Upsilon$. Clearly, ( $\Upsilon, \nabla, \aleph$ ) is a complete perturbed metric space.
Define the mappings $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}, \mathfrak{E} : \Upsilon \to \Upsilon$ be continuous mappings as follows:

$$\mathfrak{B}\kappa = \frac{\kappa}{9}, \mathfrak{C}\kappa = \frac{\kappa}{18}, \mathfrak{D}\kappa = \frac{\kappa}{6}, \mathfrak{E}\kappa = \frac{\kappa}{3}.$$

We observe that $\mathfrak{B}\Upsilon \subseteq \mathfrak{D}\Upsilon$ and $\mathfrak{C}\Upsilon \subseteq \mathfrak{E}\Upsilon$. For each $\kappa \in \Upsilon$, we have $\mathfrak{B}\mathfrak{E}\kappa = \frac{\kappa}{27} = \mathfrak{E}\mathfrak{B}\kappa$ and $\mathfrak{C}\mathfrak{D}\kappa = \frac{\kappa}{108} = \mathfrak{D}\mathfrak{C}\kappa$. Therefore, the pairs ( $\mathfrak{B}, \mathfrak{C}$ ) and ( $\mathfrak{C}, \mathfrak{D}$ ) are commuting mapping and hence are weakly compatible. Therefore, all conditions of Theorem 2.1 are satisfied, and $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}$ and $\mathfrak{E}$ have a unique common fixed-point at 0 in $\Upsilon$.

## 3 Application

*Let $\Upsilon = C[a,b]$ be the set of continuous functions on $\Omega$ whose square is integrable on $\Omega$ where $\Omega = [0,1]$ is a standard metric space.*
*Consider the integral equations*

$$\begin{aligned} \kappa(t) &= \int_\Omega q_1(t, b, \kappa(b))db + \eta(t), \\ \eta(t) &= \int_\Omega q_2(t, b, \eta(b))db + \eta(t), \end{aligned} \tag{3.1}$$

*where* $q_1, q_2 : \Omega \times \Omega \times \mathbb{R} \to \mathbb{R}$ *and* $\eta : \Omega \to \mathbb{R}_+$ *are given continuous mappings.*
$C[a,b] = \{f : [a,b] \to \mathbb{R} \mid f$ *is continuous on* $[a,b]\}$.
*The mapping is defined by the relation:*

$$\nabla(z, w) = \max_{t\in[a,b]} |z(t) - w(t)| + \tau(z(a) - w(a)), \tau \geq -1, \tag{3.2}$$

$z, w \in C[a,b]$.
*The mapping $\nabla$ is a perturbed metric on $C[a,b]$ with respect to the perturbing function $\aleph$, defined by the relation:*

$$\aleph(z, w) = \tau(z(a) - w(a)), \tau \geq -1, z, w \in C[a,b]. \tag{3.3}$$

*The function $\Sigma C[a,b] \times C[a,b] \to [0,+\infty)$ defined by the relation:*

$$\Sigma(z, w) = \max_{t\in[a,b]} |z(t) - w(t)|, z, w \in C[a,b]. \tag{3.4}$$

*is an exact metric on $C[a,b]$.*
*So, $(C[a,b], \nabla, \aleph)$ is a perturbed metric space and $(C[a,b], \Sigma)$ is an standard metric space. Suppose that the following conditions hold:*
*(i) For each $b, t \in \Omega$, we have*

$$\kappa_1(t) \leq \int_\Omega q_1(t, b, \kappa_1(b))\, db$$

*and*

$$\kappa_2(t) \leq \int_\Omega q_2(t, b, \kappa_2(b))\, db.$$

*(ii) There exists $p : \Omega \to \Omega$ satisfying*

$$\int_\Omega |q_1(t, b, \kappa(b)) - q_2(t, b, \eta(b))|\, db \leq p(t)|\kappa(t) - \eta(t)|$$

*for each $b, t \in \Omega$ with $\sup_{t\in\Omega} p(t) \leq k$ where $k \in [0,1)$.*
*Then the integral equations (3.1) have a common solution in $C[a,b]$.*

*Proof.* $(\mathfrak{B}\kappa)(t) = \int_\Omega q_1(t,b,\kappa(b))db + \eta(t)$ and $(\mathfrak{C}\kappa)(t) = \int_\Omega q_2(t,b,\kappa(b))db + \eta(t)$.
From (i), we have
Now, for all comparable $\kappa, \eta \in \Upsilon$, we have

$$\begin{aligned}
\nabla(\mathfrak{B}\kappa, \mathfrak{C}\eta) &= \sup_{t\in\Omega} |(\mathfrak{B}\kappa)(t) - (\mathfrak{C}\eta)(t)| \\
&= \sup_{t\in\Omega} \left| \int_\Omega q_1(t,b,\kappa(\mathfrak{b}))d\mathfrak{b} - \int_\Omega q_2(t,b,\eta(b))db \right| \\
&\le \sup_{t\in\Omega} \int_\Omega |q_1(t,b,\kappa(b)) - q_2(t,b,\eta(b))|\, db \\
&\le \sup_{t\in\Omega}(t)|\kappa(t) - \eta(t)| \\
&\le k \sup_{t\in\Omega} |\kappa(t) - \eta(t)| \\
&= k\nabla(\kappa,\eta) \\
&= k\nabla_{a,b}(\mathfrak{B},\mathfrak{C}),
\end{aligned}$$

where

$$\begin{aligned}
\nabla_{\kappa,\eta}(\mathfrak{B},\mathfrak{C}) &= \nabla(\kappa,\eta) \\
&\le \left\{ k_1\nabla(\kappa,\eta) + k_2 \max\left\{ \nabla(\kappa,\eta), \frac{\nabla(\kappa,\mathfrak{B}\kappa)\nabla(\eta,\mathfrak{C}\eta)}{1+\nabla(\mathfrak{B}\kappa,\mathfrak{C}\eta)} \right\} \right. \\
&\left. +k_3 \min\{\nabla(\kappa,\mathfrak{C}\eta), \nabla(\eta,\mathfrak{B}\kappa)\} \right\}.
\end{aligned}$$

where $k_i \in [0,1), i = 1,2,3$ satisfy $k_1 + k_2 + k_3 < 1$.
Now we can apply Corollary 2.1 to obtain the common solutions of integral equations (3.1) in $C[a,b]$. □

## 4 Conculsion

This paper defines commutative, weakly commutative, and weakly compatible mappings in perturbed metric spaces, which is a natural and recent extension of classical metric spaces. We also obtained a common fixed point result for self-mappings that satisfy the rational contractive condition. In addition to providing enhancements and improvements over previous work, our results also generalize several of the well-known fixed-point theorems in [2,4,6,9,10]. This demonstrates the value of perturbed metric spaces in expanding the field of fixed-point theory and offers a solid basis for further research in this area.

## References


[1] O Acar, Fixed point theorems for rational type -contraction, *Carpathian Math. Publ.* , **13**(1)(2021), 39-47.

[2] S. Banach, Sur les oprations dans les ensembles abstraits et leur application aux quations intgrales, *Fundamenta Mathematicae*,**3**(1922),133-181.

[3] Cabrera, J. Harjani and K. Sadarangani, A fixed point theorem for contractions of rational type in partially ordered metric spaces, *Annali Dell Universita Di Ferrara.*, **59**(2) (2013), 251-258.

[4] B. K. Dass, and S. Gupta, An extension of Banach contraction principle through rational expressions, *Indian Journal of Pure and Applied Mathematics*,**6**(1975), 1455-1458.

[5] K. S. Eke and J. G. Oghonyon, Some fixed point theorems for rational-type contractive maps in ordered metric spaces, *Computer Science.*, **15**(1)(2020), 65-72.

[6] A. Ghaziyah, E.Karapinar and P. Shahi, On rational contractions in perturbed metric spaces, *Research in Mathematics*,**12**(2025), 1-8 .

[7] K. Habib Alam, Y. Rohen and A. Tomar, On fixed point and its application to the spread of infectious diseases model in Mvbmetric space, *Mathematical Methods in the Applied Sciences*, **47**(7)(2024), 6489-6503.

[8] J. Harjani, B. Lopez and K. Sadarangani, A fixed point theorem for mappings satisfying a contractive condition of rational type on a partially ordered metric space, *Abstract Appl.Anal.*, **2010** (2010), 1-9.

[9] D. S. Jaggi, Some unique fixed point theorems, *Indian J. Pure Appl. Math.*,**8**(1977), 223-230.

[10] M. Jleli and B. Samet, On Banach's fixed-point theorem in perturbed metric spaces, *Journal of Applied Analysis and Computation*, **15**(2025), 993-1001.

[11] P. Joginder and U. C. Gairola, Fixed Point for generalized rational type contraction in partially ordered metric spaces, *Jñānābha*, **52**(1) (2022), 162-166.

[12] A. Muraliraj and R. Thangathamizh, Rational Type Intuitionistic Fuzzy Contraction Results in Intuitionistic Fuzzy Metric Spaces With An Application, *Jñānābha*, **54**(1)(2024), 122-133.

[13] N. S. Rao, K. Kalyani and T. Gemechu, Fixed Point Theorems in Partially Ordered Metric Spaces with Rational Expressions, *Information Sciences Letters.*, **10**(3) (2021), 1-8.

[14] N. Seshagiri Rao and K. Kalyani, On some coupled fixed point theorems with rational expressions in partially ordered metric spaces, *Sahand Commun Math Anal (SCMA)*, **18** (1)(2021),12336.

[15] N. Seshagiri Rao and K. Kalyani, Generalized fixed point results of rational type contractions in partially ordered metric spaces, *BMC Research Notes*, **390** (2021), 1-13.

[16] A. Tomar and R. Sharma, Some Coincidence and common fixed point theorems concerning F-contraction and applications, *J. Internat. Math. Virtual Institute*, **8** (2018), 181-198.

[17] A. Tomar, Giniswamy, C. Jeyanthi and P. G. Maheshwari, Coincidence and common fixed point of F contractions via CLRST property, *Surveys in Mathematics and its Applications*, **11**(2016), 21-31.

[18] A. Tomar and E. Karapinar, On variants of continuity and existence of fixed point via Meir-Keeler contractions in MC spaces, *J. Adv. Math. Stud.*, **9**(2)(2016), 348-359.

[19] A. Tomar, G. Giniswamy,C. Jeyanthi and P.G. Maheshwari, On Coincidence and Common Fixed Point of Six Maps Satisfying F-Contractions, *TWMS J. App. Eng. Math.*, **6**(2)(2016), 224-231.

[20] A. Tomar,S. Beloul, R. Sharma and S. Upadhyay, Common fixed point theorems via generalized condition (B) in quasi-partial metric space and applications, *Demonstr. Math.*, **50** (01)(2017), 278-298.

[21] A. Tomar, M.Joshi, S. Kumar Padaliya and B. Joshi, Fixed Point under Set-Valued Relation-Theoretic Nonlinear Contractions and Application, *Filomat*, **33**(14)(2019), 4655-4664.