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EOQ MODEL UNDER FINANCING AGREEMENTS WITH STOCKBROKERS AND TRADERS FOR EXPONENTIAL TIME-SENSITIVE AND STOCK-ASSOCIATED DEMAND RATE

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Abstract

The demand rate was assumed to be invariable when previous academics created inventory models under trade credit. However, during the maturity period, the demand rate remains relatively stable. During the expansion stage, the demand function for high-tech merchandise raises with time. We expand the constant demand in this study to include exponential time-dependent and stock-level sensitive variables. The mathematical algorithm for determining the best outcomes is created. Managerial insights and numerical examples are provided.

2020 Mathematical Sciences Classification:
Keywords and Phrases: Financing, stockbrokers, customer, decay, demand.

1 Introduction

The demand rate is consistently measured in the conventional economic order quantity (EOQ). In contrast, the demand rate fluctuates constantly in real-world observations. The traits of high-tech products include (i) a shorter product life cycle, (ii) faster response times, (iii) a growing demand for internationalization, and (iv) extensive customization. Furthermore, as a result of technical advancements and the global recession, inventory cost components typically decline over time.

When shortfalls are unsatisfactory, Soni [23] proposed optimal replenishment rules that permit (i) a finishing inventory to be non-zero and (ii) the maximum amount of exhibited space is limited. Teng *et al.* [26] created an EOQ model with rising demand under allowable delay. Yang [33] introduced an EOQ with relaxed incurable conditions under stock-linked demand rate and stock-sensitive carrying cost. In order to help traders find out their optimal probable replenishment cycle time, Chen and Teng [5] developed an EOQ model that incorporates the following facts: (i) decaying products have the longest life spans, and (ii) suppliers frequently provide trade credits to entice additional customers. Chen *et al.* [5] devised a simple arithmetic-geometric-mean-variation technique to use a tentatively allowed delay to obtain the best outcome for the inventory problem.

Urban [30] looked into extending EOQ models to include finance conformances with clients and providers. A lot-sizing model for fading commodities with an existing stock-sensitive demand was presented by Min, Zhou, and Zhao [17]. Under the two stages of trade credit strategy encircled by the EPQ structure, Huang [10] suggested a highly selective dealer's replenishment decision. An applicable EPQ model was presented by Teng *et al.* [25], wherein the producer receives trade credit from the supplier and grants trade credit to the buyer jointly. The most advantageous pricing and lot-sizing strategies for producers under trade credit were discussed by Chung *et al.* [8]. Several methods have been developed by Chung and Liao [7] to enhance the integrated inventory model using two-stage trade credit. An effect of supplier credit policy that links credit periods to order quantity was suggested by Khouja and Mehrez [14]. To address the EOQ model with a temporary selling value, Wee *et al.* [32] created a most advantageous result for the EOQ strategy in which the best answers can be expressed algebraically. A simple deterministic EOQ or EPQ problem with partial backordering was proposed by Pentico and Drake [19]. The difference between the selling and procurement costs was addressed by Teng *et al.* [24], who also examined the best price and ordering practices with allowable payment delays. An EOQ model for linearly time-associated failing products with time-sensitive demand was created by Tripathi and Kaur [27]. An inventory structure for commodities with an invariable fall rate is discussed, as noted by Scillia *et al.* [20].

A review of the trade credit literature was published by Seifert *et al.* [21], indicating room for further investigation. In response to various supply chain rivalry and the effect of trade credit on business performance, Lee *et al.* [15] suggested trade credit. A supply chain scheme with a capital-constrained producer and a well-financed contractor dealing with uncertain demand was described by Simin and Bo [22]. The manufacturer may seek green credit financing (GCF) from banks in this scenario. For a supply chain linked to emissions, Cao and Yu [3] offered trade credit financing and synchronization. The EOQ model with trade credit investments for non-diminishing demand was explained by Teng *et al.* [26]. The inventory ordering practices of belatedly failed commodities under trade credits were examined by Musa and Saini [18]. An EOQ model of fading products with inventory-linked demand under varying situations was suggested by Tripathi and Mishra [28]. An EOQ model for commodities with a quadratic demand under shortages was studied by Khanra *et al.* [12] over a fixed time horizon. The outcome of economic strategy certainty on trade credit was presented by Mello and Toscano [16]. An ideal ordering strategy for sellers that consider buyer credit under two-stage trade credit financing was proposed by Zhang *et al.* [34]. The influence of business strategy on companies' trade credit practices was suggested by Cao *et al.* [4]. The connection between the establishment of bankruptcy courts and trade credit financing was first presented by Wang *et al.* [31]. The impact of shared executives and administrators with important providers on a firm's ability to obtain trade credit was established by Ding *et al.* [9]. The EOQ model was created by Jaggi *et al.* [11] using shortages under allowable delay. In the proposed EOQ model, Katariya and Shukla [13] investigated green technology investment and selling price linked to linear demand. The incomplete trade credit financing in a supply chain EOQ model for deteriorating items with shortages was developed by Annadurai and Kumar [1]. A two-stage payment delay is part of the EOQ model developed by Banu and Mondal [2] for deteriorating goods over an infinite time horizon. For non-instantaneous deteriorating items, Tripathy and Sharma [29] examined an EOQ model where a progressive trade credit approach is crucial.

The paper is structured as follows. Section 2 specifies notations and assumptions before moving on to mathematical formulation. Section 4 discusses solution methods. Section 5 goes into detail about sensitivity and numerical examples. Section 6 elaborates on the model expansion, and the conclusion follows.

2 Notations and Assumptions

2.1 Notations

$q(t)$: level of inventory at time t
$D(t)$: demand rate
s : unit selling cost
c : unit purchase cost
K : ordering cost
I_e : interest earned /dollar/ year
M : vendors trade-credit phase offered by dealer
R : retailers recorder point
θ : deterioration rate $0 < \theta \leq 1$
$\Pi_i(T, R)$: retailers average profit, $i = 1, ..., 3$
Q : retailers order quantity
I_p : interest charged /dollar/ year
N : purchasers trade-credit period presented by trader ($N < M$)
ξ : space accessibility
h : carrying cost/unit/year
T : inventory stock cycle length
HC : carrying cost
SR : sales revenue
IEN_i : interest earned, $i = 1, ..., 3$
IPD_i : interest paid, $i = 1, ..., 3$
T^* : optimal T

2.2 Assumptions

(i) Demand rate is measured as the retailers instantaneous stock level and exponential time sensitive, i.e. $D(t) = \alpha q(t) + ae^{bt},\ a > 0,\ 0 < b < 1,\ \alpha > 0$.
(ii) The model is used for one item, unremitting-review and endless horizon scheme.
(iii) Deterioration rate is constant.
(iv) Shortage is not allowed.
(v) Lead time is zero.
(vi) The selling price and all other cost are unvarying.

3 Mathematical Formulation

The presumptive inventory system begins at time $t = 0$ of the replenishment cycle; products of Q units are held in reserve, and the joint effects of stock level and exponential time-dependent demand and decline cause the commodities to be used up gradually during $[0, T]$. This is an illustration of the inventory echelon over time:

$$\frac{dq(t)}{dt} + (\alpha + \theta)q(t) = -ae^{bt}, \quad 0 \leq t \leq T, \tag{3.1}$$

with the margin condition

$$q(T) = R. \tag{3.2}$$

Solution of (3.1) under the condition (3.2) is:

$$q(t) = \frac{ae^{bT}}{(\alpha + b + \theta)} \left\{ e^{(\alpha+\theta)(T-t)} - 1 \right\} + Re^{(\alpha+\theta)(T-t)}. \tag{3.3}$$

The last phrase, $Re^{(\alpha+\theta)(T-t)}$, denotes the augmentation inventory on hand due to sustaining optimistic inventory at the finish of order phase.

The order quantity is:

$$Q = q(0) - R = \frac{a}{b + \theta + \alpha} \left\{ e^{(\alpha+\theta+b)T} - 1 \right\} + R \left\{ e^{(\alpha+\theta)T} - 1 \right\}. \tag{3.4}$$

The profit function per cycle is given below.

(i) Sales Revenue

For each unit that is requested, the vendor is paid 's' a fixed selling price per unit. Thus,

$$\begin{aligned} SR &= s \int_0^T D(t)\, dt \\ &= s \left[\frac{a}{b} \left(e^{bT} - 1 \right) + \left\{ \frac{a\alpha}{b + \theta + \alpha} e^{(b+\theta+\alpha)T} + \mathrm{Re} \left(e^{(b+\theta)T} \right) \right.\right. \\ &\quad \left.\left. \times \left(\frac{1 - e^{-(\alpha+\theta)T}}{\alpha + \theta} - \frac{a\alpha}{b + \theta + \alpha} \cdot \frac{e^{bT} - 1}{b} \right) \right\} \right]. \end{aligned} \tag{3.5}$$

(ii) Interest Earned

For the goods purchased up until time N, the purchaser returns them to the vendor. The seller deposits this money into an account and earns interest at a rate of I_e. As the goods are sold, the buyer reimburses the dealer for the unpaid goods. Until the trader is reimbursed at time M, interest accumulates. Depending on T, three scenarios are examined with N and M.

Case I: If $T \leq N$. Since every item is sold to the buyer within M, the dealer is eligible to receive interest on the sales returns from N to M.

$$\begin{aligned} IEN_1 &= sI_\varepsilon(M - N) \int_0^T \left\{ \alpha q(t) + ae^{bt} \right\} dt \\ &= sI_\varepsilon(M - N) \left[\frac{a}{b} \left(e^{bT} - 1 \right) + e^{(b+\theta)T} \left\{ \frac{a\alpha e^{\alpha T}}{b + \theta + \alpha} + R \left(\frac{1 - e^{-(\alpha+\theta)T}}{\alpha + \theta} - \frac{a\alpha}{b + \theta + \alpha} \cdot \frac{e^{bT} - 1}{b} \right) \right\} \right]. \end{aligned} \tag{3.6}$$

Case II: If $N \leq T \leq M$. For the objects procured up to N, the buyer shells out the dealer; for the outstanding items, they pay the dealer from N to T.

$$IEN_2 = sI_\varepsilon\left[\int_N^T\left(\int_N^t\left[ae^{bu}+\alpha q(u)\right]du\right)dt+(M-T)\int_0^T\left[ae^{bu}+\alpha q(u)\right]du\right]$$

$$= sI_\varepsilon\left[\frac{a}{b}\left(\frac{e^{bT}-e^{bN}}{b}\right)(T-N)+\frac{a\alpha e^{(b+\theta+\alpha)T}}{b+\theta+\alpha}\left(\frac{e^{-(\alpha+\theta)(T-N)}}{\alpha+\theta}+(T-N)\right)\right.$$

$$-\frac{a\alpha e^{bT}(T^2-N^2)}{2(b+\theta+\alpha)}-\mathrm{Re}\left(e^{(b+\theta)T}\right)\left(\frac{e^{-(\alpha+\theta)(T-N)}}{\alpha+\theta}+(T-N)\right)$$

$$\left.+(M-T)\left\{\frac{a}{b}\left(e^{bT}-1\right)+e^{(b+\theta)T}\left[\frac{a\alpha e^{\alpha T}}{b+\theta+\alpha}+R\left(\frac{1-e^{-(\alpha+\theta)T}}{\alpha+\theta}-\frac{a\alpha}{b+\theta+\alpha}\cdot\frac{e^{bT}-1}{b}\right)\right]\right\}\right]. \tag{3.7}$$

Case III: If $T \geq M$. Until the retailer reimburses the broker from N to M, the client reimburses the seller for the commodities purchased up to N and the lasting items as they are purchased. As the commodities are purchased, the seller immediately pays the dealer after the client pays the seller from M to T.

$$IEN_3 = sI_\varepsilon\left[\int_N^M\left(\int_0^t\left[ae^{bu}+\alpha q(u)\right]du\right)dt\right]$$

$$= sI_\varepsilon\left[\frac{a}{b}(M-N)\left(e^{bT}-1\right)+\frac{a\alpha e^{(b+\theta+\alpha)T}}{b+\theta+\alpha}\left(\frac{e^{-(\alpha+\theta)(M-N)}}{\alpha+\theta}+(M-N)\right)\right.$$

$$\left.-\frac{a\alpha e^{bT}}{2(b+\theta+\alpha)}(M^2-N^2)+\mathrm{Re}\left(e^{(\alpha+\theta)T}\right)\left(\frac{e^{-(\alpha+\theta)(M-N)}}{\alpha+\theta}-(M-N)\right)\right]. \tag{3.8}$$

(iii) Purchasing Cost

The purchase cost is obtained by multiplying unit purchase cost c and order quantity Q.

$$PC = cQ = c\left[\frac{a}{b+\theta+\alpha}\left\{e^{(\alpha+\theta+b)T}-1\right\}+R\left\{e^{(\alpha+\theta)T}-1\right\}\right]. \tag{3.9}$$

(iv) Holding Cost

$$HC = \int_0^T hq(t)dt = \int_0^T\left[\frac{he^{(\alpha+\theta)t}}{\alpha+\theta}+\frac{ae^{bT}}{b+\theta+\alpha}+R\right]\left(1-e^{-(\alpha+\theta)t}\right)dt+\frac{ha}{(b+\theta+\alpha)}\cdot\frac{1-e^{bT}}{b}. \tag{3.10}$$

(v) Ordering Cost

A fixed cost K is sustained for every order located:

$$OC = K. \tag{3.11}$$

(vi) Interest Paid

For the commodities that the vendor has sold to the client, the seller pays the contractor at instant M; the remaining commodities are paid to the provider as they are sold, plus interest I_p. For instances I and II, there is no interest paid.

$$IPD_3 = \frac{cI_phe^{(\alpha+\theta)T}}{\alpha+\theta}\left(\frac{ae^{bT}}{b+\theta+\alpha}+R\right)\left(1-e^{-(\alpha+\theta)(T-M)}\right)+\frac{cI_pha}{(b+\theta+\alpha)}\cdot\frac{1-e^{b(T-M)}}{b}. \tag{3.12}$$

The average profit per unit time is as follows:

$$\Pi_i(T,R) = \frac{SR+IEN_i-OC-PC-HC-IPD_i}{T}, \quad \text{for } i=1,...,3. \tag{3.13}$$

Again,

$$\Pi_1(T,R) = \frac{SR+IEN_1-PC-HC-OC-IAPD_1}{T}$$

$$= \frac{s\left\{1+I_\varepsilon(M-N)\right\}}{T}\left[\frac{a}{b}\left(e^{bT}-1\right)+e^{(b+\theta)T}\left\{\frac{a\alpha e^{\alpha T}}{b+\theta+\alpha}+R\left(\frac{1-e^{-(\alpha+\theta)T}}{\alpha+\theta}\right)\right\}\right]$$

$$
\begin{aligned}
&- \frac{\{1+I_\varepsilon(M-N)\}\, s\alpha + h}{T(b+\theta+\alpha)} \cdot \frac{a\left(e^{bT}-1\right)}{b} \\
&- \frac{ae^{(b+\theta+\alpha)T}}{T(\alpha+\theta)(b+\theta+\alpha)} \left\{c(\alpha+\theta) + h\left(1-e^{-(\alpha+\theta)T}\right)\right\} \\
&- \frac{R}{T(\alpha+\theta)} \left\{c(\alpha+\theta) + h\left(1-e^{-(\alpha+\theta)T}\right)\right\} e^{(\alpha+\theta)T} \\
&- \frac{c}{T}\left(R - \frac{a}{b+\theta+\alpha}\right) - \frac{K}{T}. \qquad (3.14)
\end{aligned}
$$

$$
\begin{aligned}
\Pi_2(T,R) &= \frac{(SR + IEN_2 - PC - HC - OC - IPD_2)}{T} \\
&= \frac{s\{1+I_e(M-T)\}}{T}\left[\frac{a}{b}(e^{bT}-1) + e^{(b+\theta)T}\left\{\frac{a\alpha e^{\alpha T}}{b+\theta+\alpha} + R\left(\frac{1-e^{-(\alpha+\theta)T}}{\alpha+\theta}\right)\right\}\right] \\
&- \frac{\{1+(M-T)I_e\}\, s\alpha - h}{T} \cdot \frac{a(e^{bT}-1)}{b(b+\theta+\alpha)} + \frac{sI_e}{T}\left[\frac{e^{bT}-e^{bN}}{b}(T-N)\right] \\
&+ \frac{a\alpha e^{(b+\theta+\alpha)T}}{b+\theta+\alpha}\left(\frac{e^{-(\alpha+\theta)(T-N)}}{\theta+\alpha} + (T-N)\right) - \frac{a\alpha e^{bT}(T^2-N^2)}{2(\alpha+b+\theta)} \\
&- \mathrm{Re}(e^{(\alpha+\theta)T})\left(\frac{e^{-(\alpha+\theta)(T-N)}}{\theta+\alpha} + (T-N)\right) \cdot \frac{a\left\{c(\alpha+\theta) + h\left(1-e^{-(\alpha+\theta)T}\right)\right\} e^{(b+\theta+\alpha)T}}{T(\alpha+\theta)(b+\theta+\alpha)} \\
&- \frac{R\left\{c(\alpha+\theta) + h(1-e^{-(\alpha+\theta)T})\right\} e^{(\alpha+\theta)T}}{T(\alpha+\theta)} - \frac{c}{T}\left(\frac{a}{b+\theta+\alpha} + R\right) - \frac{K}{T}. \qquad (3.15)
\end{aligned}
$$

$$
\begin{aligned}
\Pi_3(T,R) =& \frac{(SR + IEN_3 - PC - HC - OC - IPD_3)}{T} \\
=& \frac{s}{T}\left[\frac{a}{b}\left(e^{bT}-1\right) + \mathrm{e}^{(b+\theta)T}\left\{\frac{a\alpha \mathrm{e}^{\alpha T}}{b+\theta+\alpha} + R\right\}\frac{\left(1-e^{-(\alpha+\theta)T}\right)}{\alpha+\theta} - \frac{a\alpha}{b+\theta+\alpha}\frac{\left(e^{bT}-1\right)}{b}\right] + \\
& \frac{sI_e}{T}\left[\frac{a}{b}(M-N)\left(e^{bT}-1\right) + \frac{a\alpha e^{(b+\alpha+\theta)T}}{(b+\theta+\alpha)}\left\{\frac{e^{-(\theta+\alpha)(M-N)}}{(\theta+\alpha)} + (M-N)\right\} - \frac{a\alpha e^{bT}\left(M^2-N^2\right)}{2(b+\alpha+\theta)}\right. \\
& \left.+ \frac{\mathrm{Re}^{(\alpha+\theta)T}}{(\alpha+\theta)}\left\{\frac{e^{-(\alpha+\theta)(M-N)}}{(\alpha+\theta)} + (M-N)\right\}\right] - \frac{c}{T}\left[\frac{a}{(b+\theta+\alpha)}\left\{e^{(b+\theta+\alpha)T}-1\right\} + R\left\{e^{(\alpha+\theta)T}-1\right\}\right] \\
& - \frac{h\mathrm{e}^{(\alpha+\theta)T}}{(\alpha+\theta)T}\left\{\frac{a}{(b+\theta+\alpha)}e^{bT} + R\right\}\left(1-e^{-(\alpha+\theta)T}\right) + \frac{ha}{T(b+\theta+\alpha)}\frac{\left(1-e^{bT}\right)}{b} - \frac{K}{T} \\
& - \frac{cI_p\mathrm{e}^{(\alpha+\theta)T}}{(\alpha+\theta)T}\left\{\frac{a}{(b+\theta+\alpha)}e^{bT} + R\right\}\left\{1-e^{-(\alpha+\theta)(T-M)}\right\} - \frac{cI_pha}{(b+\theta+\alpha)T}\frac{\left\{1-e^{b(T-M)}\right\}}{b}. \qquad (3.16)
\end{aligned}
$$

4 Solution Methodology

Let us examine objective function (3.13) without considering the constraint on the highest inventory echelon by classifying the situations when the planned model outperforms the *MZZ* model. The additional benefit obtained by adding the positive recorder point is as follows:

$$\Delta\Pi_i(T,R) = \Pi_i(T,R) - \Pi_i(T,0), \quad \text{for } i = 1,\dots,3, \qquad (4.1)$$

where $\Pi_i(T,R)$ is taken from (3.13) and $\Pi_i(T,R)$ is taken $R=0$ in (3.14). We find the incremental profit obtained by incorporating the positive recorder point:

$$\Delta\Pi_1(T,R) = \frac{Re^{(b+\theta)T}}{T(\alpha+\theta)}\left[s\{1+I_e(M-N)\}\left(1-e^{-(\alpha+\theta)T}\right) - c(\alpha+\theta)\left(1-e^{-(\alpha+\theta)T}\right) - h\right], \qquad (4.2)$$

$$\Delta\Pi_2(T,R) = \frac{R}{T(\alpha+\theta)}\left[s\{1+I_\varepsilon(M-T)\}\left(1-e^{-(\alpha+\theta)T}\right)e^{(b+\theta)T}\right.$$

$$
- sI_{\varepsilon}\left\{\frac{e^{(b+\theta+\alpha)(T-N)}}{b+\theta+\alpha}-\frac{e^{(b-\alpha)(T-N)}}{b-\alpha}\right\}
$$

$$
\left. - c(\alpha+\theta)\left(1-e^{-(\alpha+\theta)T}\right) - he^{(\alpha+\theta)T}\right], \tag{4.3}
$$

$$
\Delta\Pi_3(T,R) = \frac{R}{T(\alpha+\theta)}\left[se^{(b+\theta)T}\left(1-e^{-(\alpha+\theta)T}\right)+e^{b(M-N)}\left\{\frac{e^{\theta(M-N)}}{b+\theta}-\frac{e^{-\alpha(M-N)}}{b-\alpha}\right\}\right.
$$

$$
+ c(\alpha+\theta)\left(1-e^{-(\alpha+\theta)T}\right) - he^{(\alpha+\theta)T}\left(1-e^{-(\alpha+\theta)T}\right)
$$

$$
\left. - cI_p he^{(\alpha+\theta)T}\left\{1-e^{-(\alpha+\theta)(T-N)}\right\}\right]. \tag{4.4}
$$

Allowing a nonzero rearrange point will not result in any profit if the profit is negative. Conversely, if the additional profits are beneficial, then additional productivity is made available by the planned model from (4.2)-(4.4). Additionally, it is evident that, for a predetermined order cycle, the incremental turnover is a linear function of the rearrangement point.

We take into account both interest earned and sales revenue. Constraints can therefore be written as $Q + R \leq \xi$. When an optimistic reorder point is in order, the reorganize point has no boundaries. The reorder point will increase under this assumption until the inventory level at the start of the order cycle equals the constraint.

Thus,

$$
Q + R = q(0) = \xi \tag{4.5}
$$

or

$$
\frac{a}{(b+\theta+\alpha)}\left(e^{(b+\theta+\alpha)T}-1\right) + R\left(e^{(\alpha+\theta)T}-1\right) + R - \xi = 0. \tag{4.6}
$$

Solving (4.6), we get:

$$
T = \frac{2\xi}{\{a+R(\alpha+\theta)\}}\left[1-\sqrt{\frac{2\xi\left\{(\alpha+\theta+b)+R(\alpha+\theta)^2\right\}}{\{a+R(\alpha+\theta)\}^2}}\right]. \tag{4.7}
$$

If $2\xi\left\{(\alpha+\theta+b)+R(\alpha+\theta)^2\right\} < \{a+R(\alpha+\theta)\}^2$, then T will be positive.

5 Numerical Examples and Sensitivity Analysis

Example 5.1: *Case I ($T \leq N$).* On substituting these parameters (mentioned in Table 5.1) in (4.7), we get $T^* = 0.324074$ yrs, accordingly $Q^* = 220.209$ units, and $\Pi_1(T,0) = \$5159.99$.

Example 5.2: *Case II ($N \leq T \leq M$).* On replacing these parameters (cited in Table 5.1) in (4.7), we get $T^* = 0.381944$ yrs, accordingly $Q^* = 211.017$ units, and $\Pi_2(T,0) = \$7124.08$.

Example 5.3: *Case III ($T \geq M$).* On replacing these parameters (quoted in Table 5.1) in (4.7), we get $T^* = 0.428241$ yrs, accordingly $Q^* = 200.74$ units, and $\Pi_3(T,0) = \$15912.7$.

Table 5.1: *Varying the parameters keeping the other values similar as used in Examples* 5.1–5.3.

Parameter	**Ex. 5.1**	**Ex. 5.2**	**Ex. 5.3**
a (units)	12	12	12
b (units)	5	5	5
θ	0.10	0.05	0.05
e	2.72	2.72	2.72
ξ	10	10	10
α	0.70	0.50	0.30
I_e (\$/unit/yr)	0.13	0.08	0.13
M (yrs)	0.5	0.5	0.4
N (yrs)	0.4	0.3	0.3
K (in \$)	50	50	50
c (/unit)	5	6	5
h (\$/unit/yr)	3	3	3
s (/unit)	20	30	30
I_p (\$/unit/yr)	0.15	0.20	0.15
MZZ solution			
T^* (yrs)	0.324074	0.381944	0.428241
Q^* (units)	220.209	211.017	200.74
$\Pi(T^*, 0)$ (in \$)	5159.99	7124.08	15912.70

Table 5.2: *Impact of* α *on* ***MZZ****'s solution of Ex.* 5.1

α	T^*	Q^*	SR	IEN	PC	HC
0.1	0.462963	191.997	1641.82	5.88763	959.983	1797.31
0.2	0.439815	197.902	2178.85	5.32130	989.510	1388.69
0.3	0.416667	203.485	2554.13	4.78239	1017.42	1102.62
0.4	0.393519	208.639	2854.88	4.27294	1043.19	955.039
0.5	0.370370	213.242	3108.50	3.79438	1066.21	851.192
0.6	0.347222	217.153	3325.06	3.34766	1085.77	771.813

Table 5.3: *Impact of* α *on cost and profit measures*

α	OC	IPD	$\Pi(T^*, 0)$
0.1	50	0.00	4897.99
0.2	50	0.00	4931.63
0.3	50	0.00	4970.01
0.4	50	0.00	5012.57
0.5	50	0.00	5058.77
0.6	50	0.00	5108.09

Table 5.4: *Outcomes of greatest inventory restrictions on optimal result to Ex.* 5.2

ξ	**Solution to MZZ model**			**Solution to planned model**				Increased profit (%)
	T^*	Q^*	$\Pi(T^*, 0)$	T^*	$R^* = \xi - Q$	Q^*	$\Pi(T^*, R^*)$	
100	0.428089	296.0	8820.58	0.336490	58.1325	41.868	133895	1417.98
200	0.645689	447.8	8718.09	0.295052	73.7230	126.28	175059	1907.99
300	0.652800	452.7	8742.37	0.192269	140.511	159.49	357569	3990.06
400	0.494422	310.9	8757.07	0.107956	288.856	111.14	771122	8705.68
500	0.035555	22.10	9389.37	0.067661	489.733	10.267	132761	14039.50

From Table 5.3 it can be simply seen that as ξ increases, T^* diminishes, R^* increases, while Q^* and $\Pi(T^*, R^*)$ increase. The increase in profit also increases.

Model Extension

Here we present some extension model related to specified planed model.

(i) Interest Earned on Profit Margin

Our consideration in this paper of Min *et al.* [17] (*MZZ* model) is: given interest is not gained from the time imbursement is made to the vendor till the end process of the order cycle, like as Case-III ($T \geq M$). In this case, profit margin $(s-c)$ is not added in the interest carrying account.

The possible interest gain on the earnings boundary from M to time T, the average profit function (3.13) for Case III is given below:

$$\begin{aligned} IRN_{3,4} &= (s-c)I_\varepsilon \left\{ \int_M^T \left(\int_M^t \left[ae^{bu} + \alpha q(u) \right] du \right) dt \right\} \\ &= a(s-c)I_\varepsilon \left[\frac{1}{b}\left(e^{bT} - e^{bM}\right) - (T-M) \right. \\ &\quad + \frac{\alpha}{(\alpha+\theta)(b+\theta+\alpha)} \left(e^{(b+\theta+\alpha)(M-N)} \cdot \frac{1}{b+\theta+\alpha} + e^{-b(M-N)} \right) \\ &\quad + \frac{R}{\alpha+\theta} \left(\frac{e^{(b+\theta)(T-M)}}{b+\theta} - \frac{e^{(b-\alpha)(T-M)}}{b-\alpha} \right) \\ &\quad \left. - \frac{\alpha a}{b(b+\theta+\alpha)} \left(e^{b(T-M)} - (T-M) \right) \right]. \end{aligned} \tag{5.1}$$

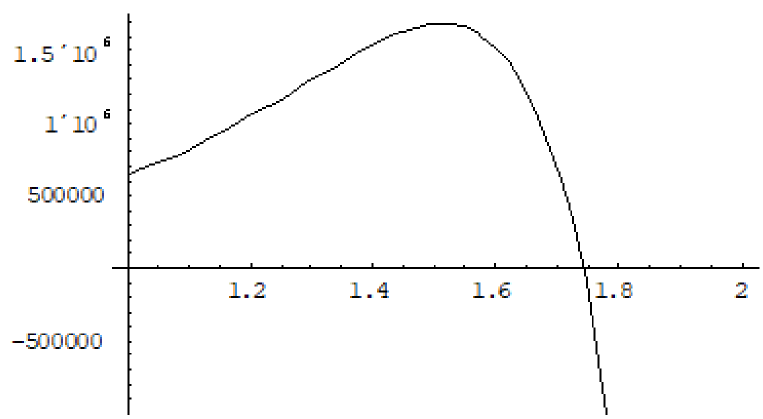

***Figure* 5.1:** *Average profit curve for Ex.* 5.3.

(ii) Return of deteriorating items

If we consider ψ be the value of decayed item in \$/unit, the value of ψ will be positive then it behaves as a revenue flow. This ψ function inserts into the proposed model, then average profit function from (3.13) for all cases:

$$\begin{aligned} DETER &= \psi \left\{ Q - \int_0^T \left[ae^{bt} + \alpha q(t) \right] dt \right\} \\ &= \psi \left[\frac{ae^{(b+\theta)T}}{b+\theta+\alpha} \left\{ 1 + \alpha \cdot \frac{1-e^{-(\alpha+\theta)T}}{\alpha+\theta} \right\} + R \left\{ e^{(\alpha+\theta)T} + e^{(b+\theta)T} \cdot \frac{1-e^{-(\alpha+\theta)T}}{\alpha+\theta} \right\} \right. \\ &\quad \left. - \frac{a}{b+\theta+\alpha} \left\{ \left(1 + \frac{e^{bT}-1}{b} \right) - \frac{a}{b}(e^{bT}-1) \right\} \right]. \end{aligned} \tag{5.2}$$

6 Conclusion and Future Research Directions

The EOQ model is presented in this paper for a retailer with exponential time-linked demand and stock. Boundary conditions for every cycle are provided T. There has been discussion of three different examples. The best results for determining the Q^*, T^*, and Π^* of each of the three scenarios are obtained. We also contrast the outcome with that of the earlier model.

Changes in a number of important characteristics have a significant impact on the variations. The managerial insights of this paper are how to on the interaction of credit provisions and changing demand patterns in determining inventory levels, working capital and profit optimization. Numerical examples and sensitivity analysis are validating the proposed model, verifying that combining time-dependent and stock-level demand functions with financial credit policies support better and more stable inventory management.

The extension of this paper may include variable decay and demand. Time-dependent carrying costs are also included in the expanded study. It is possible to simplify the model by using fuzzy demand and fuzzy decay.

Declaration

All authors declare that they have no conflict of interest.

References

[1] K. Annadurai and R. Uthaya Kumar, Analysis of partial trade credit financing in supply chain by EOQ-based model for decaying items with shortage, *The International Journal of Advanced Manufacturing Technology*, **61** (2012), 1139-1159.

[2] A. Banu and S.K. Mondal, Analyzing an inventory model with two level trade credit period including the effect of customers credit of the demand function using q-fuzzy number, *International Journal of Operational Research*, **20** (2020), 1559-1587.

[3] E. Cao and M. Yu, Trade credit financing and coordination for an emission-dependent supply chain, *Computer and Industrial Engineering*, **119**(119) (2018), 50-62.

[4] Z. Cao, S.X. Chen and E. Lee, Does business strategy influence interfirm financing? Evidence from trade credit, *Journal of Business Research*, **141** (2022), 495-511.

[5] S.C. Chen and J.T. Teng, Retailers optimal ordering policy for deteriorating items with maximum lifetime under suppliers trade credit financing, *Applied Mathematical Modelling*, **38**(15-16) (2014), 4049-4061.

[6] S.C. Chen, L.E. Cardenas-Barron and J.T. Teng, Retailers economic order quantity when the supplier offers conditionally permissible delay in payments linked to order quantity, *International Journal of Production Economics*, **155** (2014), 284-291.

[7] K.J. Chung and J.J. Liao, The simplified solution algorithm for an integrated supplier-buyer inventory model with two part trade credit in a supply chain system, *European Journal of Operational Research*, **213**(1) (2011), 56-165.

[8] K.J. Chung, J.J. Liao, S.D. Lin, S.T. Chung and H.M. Srivastava, Manufacturers optimal pricing and lot-sizing policies under trade credit financing, *Mathematical Science in Applied Sciences*, **43**(6) (2020), 1-18.

[9] F. Ding, Q. Liu, H. Shi, W. Wang and S. Wu, Firms access to informal financing: The role of shared manager in trade credit access, *Journal of Corporate Finance*, **79** (2023), Article ID 103388.

[10] Y.F. Huang, Optimal retailers replenishment decisions in the *EPQ* model under levels of trade credit policy, *European Journal of Operational Research*, **176**(3) (2007), 1577-1591.

[11] C.K. Jaggi, V.S.S. Yadavalli, M. Verma and A. Sharma, An EOQ model with allowable shortage under trade credit in different scenario, *Applied Mathematics and Computations*, **252** (2015), 541-551.

[12] S. Khanra, B. Mandal and B. Sarkar, An inventory model with time dependent demand and shortages under trade credit policy, *Economic Modelling*, **35** (2013), 349-355.

[13] D.K. Katariya and K.T. Shukla, An EOQ model for deteriorating product with green technology investment and trade credit financing, *International Journal of Production Economics*, **18**(3) (2023), 300-320.

[14] M. Khouja and A. Mehrez, Optimal inventory policy under different supplier credits, *Journal of Manufacturing Systems*, **15**(5) (1996), 334-339.

[15] H.H. Lee, J. Zhou and J. Wang, Trade credit financing under competition and its impact on firm performance in supply chain, *Manufacturing and Service Operations Management*, **20**(1) (2017), 1-17.

[16] R.D. Mello and F. Toscano, Economic policy uncertainty and short term financing: The case of trade credit, *Journal of Corporate Finance*, **64** (2020), Article ID 1016686.
[17] Y.W. Min, J. Zhou and Zhao, An inventory model for deteriorating items under stock-dependent demand and two-level trade credit, *Applied Mathematical Modelling*, **34**(11) (2010), 3273-3285.
[18] A. Musa and B. Sani, Inventory ordering policies of delayed deteriorating items under permissible delay in payments, *International Journal of Production Economics*, **136**(1) (2012), 75-83.
[19] D.W. Pentico and M.J. Drake, A survey of deterministic models for the EOQ and *EPQ* with partial backordering, *European Journal of Operational Research*, **214**(2) (2011), 179-198.
[20] J. Scillia, M.C.D. L. Rosa and J.F. Acosta, An inventory model for deteriorating items with shortages and time varying demand, *International Journal of Production Economic*, **155**(2014), 155-162.
[21] D. Seifert, R.W. Seifert and M.P. Sieke, A review of trade credit literature: Opportunities for Research in Operation, *European Journal of Operational Research*, **231**(2) (2013), 245-256.
[22] A.B.L. Simin, L. Bo and D.S.X. Chen, Green credit financing versus trade credit financing in a supply chain with carbon emission limits, *European Journal of Operational Research*, 292(1) (2021), 125-142.
[23] H.N. Soni, Optimal replenishment policies for deteriorating items with stock-sensitive demand under two-levels trade credit and limited capacity, *Applied Mathematical Modelling*, **38**(8) (2013), 5887-5895.
[24] J.T. Teng, C.T. Chang and S.K. Goyal, Optimal pricing and ordering policy under permissible delay in payments, *International Journal of Production Economics*, **97**(2) (2005), 1121-129.
[25] J.T. Teng, L.Y. Ouyang and L.H. Chen, Optimal manufacturers pricing and lot-sizing policies under trade credit financing, *International Journal of Production Economics*, **137** (2006), 102-115.
[26] J.T. Teng, J. Min and Q. Pan, Economic order quantity model with trade credit financing for non-decreasing demand, *Omega*, **40**(3) (2012), 328-335.
[27] R.P. Tripathi and M. Kaur, A linear time dependent deteriorating inventory model with linearly time dependent demand rate and inflation, *International Journal of Computing Science and Mathematics*, 9(4) (2018), 352-364.
[28] R.P. Tripathi and S. Mishra, A comprehensive study of EOQ system for spoilage commodities with stock-sensitive demand trade credit, *International Journal of Applied and Computational Mathematics*, **7**(2) (2021), 1-19.
[29] M. Tripathy, G. Sharma and A.K. Sharma, An EOQ inventory model for non-instantaneous deteriorating item with constant demand under progressive financial trade credit facility, *Opsearch*, **59** (2022), 1215-1243.
[30] T.L. Urban, An extension of inventory model incorporating financing agreements with both suppliers and customers, *Applied Mathematical Modelling*, **36**(2) (2012), 6323-6330.
[31] S. Wang, J. Wang, U.S. Maqsood, K. Wang and Q. Li, Creditor protection and trade Credit financing: Evidence from a quasi-natural experiment in China, *Finance Research Letters*, **57** (2023), Article ID 104226.
[32] H.M. Wee, S.L. Chung and P.C. Yang, A modified EOQ model with temporary sale price derived without derivatives, *The Engineering Economist*, **48**(2) (2023), 190-195.
[33] C.T. Yang, An inventory model with both stock-dependent demand rate and stock-dependent holding cost rate, *International Journal of Production Economics*, **155** (2014), 214-221.
[34] C. Zhang, Y.X. Tian, L.W. Fan and S.M. Yang, Optimal ordering policy for retailer with consideration of customer credit under two-level trade credit financing, *Operational Research*, **29**(3) (2021), 915-1942.