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ACQUIRING HALF COMPANION SEQUENCES OF DIOPHANTINE TRIPLES INCLUSIVE OF GNOMONIC NUMBERS

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Abstract

This paper aims at accomplishing half companion sequence of Diophantine triples featuring Gnomonic numbers such that the product of any two terms of the half companion sequence of the Diophantine triple and combined by an integer or a polynomial with integer coefficients results in a perfect square.
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1 Introduction

The investigation of integers and arithmetic functions is the main of the discipline of pure mathematics known as number theory. There are a tonne of straightforward problems regarding integers that are either impossible to answer or are exceedingly challenging to answer because they only require the fundamental operations of addition and multiplication [1,5,10,12,13].

A polynomial equation with two or more unknowns that only takes into account or looks for integer solutions is known as a Diophantine equation in number theory. The word "Diophantine" refers to the Greek mathematician Diophantus of Alexandria, who developed the use of symbols in variable-based mathematics and investigated the corresponding problems in the third century.

The problem of the occurrence of Dio triples and quadruples with the property $D(n)$ for any integer n as well as for any linear polynomial in n was studied by a number of mathematicians. In this particular situation, one can seek a thorough review of many problems involving Diophantine triples and special Diophantine triples [2,3,4,6-9,14,16-21]. Half companion diophantine triple sequences were studied in [11, 15]. These findings inspired us to search for Diophantine triple sequences using gnomonic number words. A half companion sequence of Diophantine triples for gnomonic numbers with the property $D($*Square of the difference of the ranks*$)$ is what this study seeks to build.

Basic Definition

A set of three different integers or polynomials with integer coefficients (s_1, s_2, s_3) is said to be a diophantine triple with property $D(n)$ if $s_i * s_j + n$ is a perfect square for all $1 \leq i \leq j \leq 3$, where n may be non-zero integer or polynomial with integer coefficients.

2 Proposed Strategy

Notation

Gno_n: Gnomonic number of rank n.

Half Companion Sequence of Diophantine triples inclusive of Gnomonic numbers

Choose $s_1 = Gno_n = 2n - 1$,

$s_2 = Gno_{n+k} = 2(n + k) - 1$,

the Gnomonic numbers of rank n and $n + k$ respectively, where $k = ... - 2, -1, 0, 1, 2, 3....$

so that $s_1 s_2 = 4n^2 + 4nk - 4n - 2k + 1$.

Property

Rank of $s_1 = n$ and rank of $s_2 = n + k$.

Square of the difference of the ranks $= (n+k-n)^2 = k^2$.
Property $= D($*Square of the difference of the ranks*$) = D(k^2)$.
As a result, according to the Diophantine triples,
$s_1 s_2 +$ (*Square of the difference of the ranks*)

$$= 4n^2 + 4nk - 4n - 2k + 1 + k^2$$
$$= (2n+k-1)^2$$

is a perfect square say λ^2.

If 's_3' represents a non-zero integer or a polynomial with integer coefficients then

$$s_1 s_3 + k^2 = \nu_1^2, \tag{2.1}$$
$$s_2 s_3 + k^2 = \omega_1^2. \tag{2.2}$$

Solving (2.1) and (2.2), we have

$$(s_2 - s_1)k^2 = s_2\nu_1^2 - s_1\omega_1^2.$$

Using the linear transformations

$$\nu_1 = u_1 + s_1 v_1 \text{ and } \omega_1 = u_1 + s_2 v_1, \tag{2.3}$$

we obtain

$$u_1^2 = s_1 s_2 v_1^2 + k^2. \tag{2.4}$$

Therefore, $\nu_1 = 4n + k - 2$, with the initial solution $u_1 = 2n + k - 1, v_1 = 1$.

On substitution of the values of s_1 and ν_1 in equation (2.1), we get

$$s_3 = 8n + 4k - 4.$$

Hence, the triple (s_1, s_2, s_3) with property $D(k^2)$ is a Diophantine triple.

Now let 's_4' be a non-zero integer or a polynomial with integer coefficients such that

$$s_2 s_4 + k^2 = \nu_2^2, \tag{2.5}$$
$$s_3 s_4 + k^2 = \omega_2^2. \tag{2.6}$$

Using the linear transformations,

$$\nu_2 = u_2 + s_2 v_2, \omega_2 = u_2 + s_3 v_2 \tag{2.7}$$

and using some algebra from (2.5), (2.6) and (2.7), we have

$$u_2 = 4n + 3k - 2, v_2 = 1.$$

Thus,

$$\nu_2 = 6n + 5k - 3.$$

Upon changing the value of ν_2 in (2.5),

$$s_4 = 18n + 12k - 9.$$

Hence, the triple (s_2, s_3, s_4) is a Diophantine triple with property $D(k^2)$.

Assuming that 's_5' be non-zero integer or a polynomial with integer coefficients,

$$s_3 s_5 + k^2 = \nu_3^2, \tag{2.8}$$
$$s_4 s_5 + k^2 = \omega_3^2. \tag{2.9}$$

Introducing the linear transformations,

$$\nu_3 = u_3 + s_3 v_3, \omega_3 = u_3 + s_4 v_3. \tag{2.10}$$

Using some algebra from (2.8), (2.9) and (2.10), we have

$$u_3 = 12n + 7k - 6, v_3 = 1,$$

so that,

$$\nu_3 = 20n + 11k - 10.$$

Substituting ν_3 in (2.8),

$$s_5 = 50n + 30k - 25.$$

The triple (s_3, s_4, s_5) possesses the property $D(k^2)$ of being a Diophantine triple as a result.

As an outcome, the triples (s_1, s_2, s_3), (s_2, s_3, s_4), (s_3, s_4, s_5), ... form a half companion sequence of Diophantine triples with property $D(k^2)$.

A few numerical instances that satisfy the property are displayed in the table below.

n	k	(s_1, s_2, s_3)	(s_2, s_3, s_4)	(s_3, s_4, s_5)	Property
1	0	(1,1,4)	(1,4,9)	(4,9,25)	0
2	1	(3,5,16)	(5,16,39)	(16,39,105)	1
3	-1	(5,3,16)	(3,16,33)	(16,33,95)	1
4	2	(7,11,36)	(11,36,87)	(36,87,235)	4
5	3	(9,15,48)	(15,48,117)	(48,117,315)	9

3 Conclusion

We have given a few examples in this paper of how to construct a sequence of Diophantine triples for Gnomonic numbers of various ranks with reasonable properties. To finish up one may look for other sequence of triples or quadruples for different numbers with their associated properties.

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