ANALYSIS OF VARIOUS METHODS TO OBTAIN SERIES SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS OF SECOND ORDER WITH VARIABLE COEFFICIENTS AND MODIFICATION IN FROBENIUS METHOD

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Abstract
This research paper presents a comprehensive comparison of various methods employed for obtaining series solutions to second-order ordinary differential equations featuring variable coefficients with only complementary functions. The primary focus lies on evaluating the efficacy and applicability of these methods in handling the complexities inherent in solving such differential equations in series. Through rigorous analysis and various examples, the strengths and limitations of each approach are delineated, shedding light on their comparative performance in terms of computational efficiency.

Furthermore, this paper proposes modification to the traditional Frobenius method, tailored to address specific challenges encountered when dealing with variable coefficients.

Keywords and Phrases: Series solution, Power series method, General method, Frobenious method, Ordinary point, Singular point.

1 Introduction
Second order ordinary differential equations with variable coefficients are very useful in various scientific disciplines, including Physics, Engineering, and Mathematics. Solutions of these differential equations play a crucial role in understanding the behavior of various dynamic systems. However, finding exact analytical solutions for such equations often poses significant challenges due to the complexity of their coefficients and boundary conditions.

In quest of analytical solutions, series methods offer a powerful approach by representing the solution as an infinite series expansion. Various methods are listed for this purpose. Power series method for solving such differential equations is described by Ramanna [5], General method for this purpose is presented by Gaur and Koul [3], Frobenius method for solving the mentioned differential equations is explained by Bali and Goyal [1].

Among these methods, the Frobenius method stands out as a fundamental technique for solving second order ordinary differential equations with variable coefficients. However, its procedure needs special attention while handling recurrence relation developed at a junction point in the solution.

Series solution of a second order differential equation with exponential or cosine function as variable coefficients has been discussed by Shivakumar and Zhang [6]. Use of power series method for providing the series solution has been discussed by Michael, Hycienth and Paul [4].

In recent past, different views and approaches for finding the series solution of various differential equations have been considered some attention. Torabi and Rohani [7] described the series solution of a differential equation having real and complex roots of concerned indicial equation by Frobenius method. Esuabana, Ekpenyong and Okon [2] provided series solution of some selected problems to verify efficiency of Frobenius method.

Earlier researchers [2] tried to find out the solution of specific problems using Frobenius method. They discussed this technique at a singular point for second order ordinary differential equations with variable coefficients.

Now, we are discussing the series solution of second order ordinary differential equations with variable coefficients using Frobenius method with its priorities over other methods. We have presented a good number
of examples in this paper to validate our claim that how Frobenius method is more efficient as compared to other methods available in the literature so far.

In this research paper, we embark on a comprehensive comparison of power series method, general method and Frobenius method for finding series solutions of second order ordinary differential equations with variable coefficients with only complementary functions. Our primary focus lies on comparing the efficacy of procedures of these methods. By systematically analyzing and contrasting these methods, our aim is to provide insights into their strengths and limitations. Then we suggest an important modification in Frobenius method regarding the recurrence relation, which develops a clear understanding during solving any second order differential equation by this method.

2 Various Methods

In order to compare power series method, general method and Frobenius method for finding series solutions of second order ordinary differential equations with variable coefficients with only complementary functions i.e.

\[ P_0 \frac{d^2 y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = 0; \quad P_0, P_1, P_2 \ \text{are functions of} \ x, \tag{2.1} \]

we discuss algorithm of these methods one by one.

2.1 Power series method

For finding series solution of (2.1) through power series method in powers of \((x - x_0)\), we assume

\[ y = \sum_{n=0}^{\infty} c_n (x - x_0)^n. \tag{2.2} \]

Then by substituting value of \(y\) and its successive derivatives in equation (2.1) and by collecting coefficients of like powers of \((x - x_0)\), it reduces to

\[ k_0 + k_1 (x - x_0) + k_2 (x - x_0)^2 + \ldots = 0; \quad k_0, k_1, k_2, \ldots \ \text{are functions of certain coefficients} \ c_n. \tag{2.3} \]

As (2.2) is solution of (2.1), \(k_0, k_1, k_2, \ldots\) all must be zero i.e. \(k_0 = 0, k_1 = 0, k_2 = 0, \ldots\), solving which we can find values of unknown coefficients \(c_n\) s in terms of the other known coefficients. Generally it is governed by a recurrence relation between \(c'_n\) s. Substituting values of \(c'_n\) s in equation (2.2), we obtain the required series solution.

2.2 General method

For finding series solution of (2.1) through general method in powers of \(x\), we check the status of point \(x = 0\) that whether it is an ordinary point or a singular point. For differential equation (2.1), \(x = 0\) is called an ordinary point if \((P_0)_{x=0} \neq 0\) and \(x = 0\) is called a singular point if \((P_0)_{x=0} = 0\). Further if \(\lim_{x \to 0} \frac{f_1}{P_0}\) and \(\lim_{x \to 0} \frac{x^2 f_2}{P_0}\) are obtained as finite values along with \((P_0)_{x=0} = 0\), then \(x = 0\) is called a regular singular point, otherwise it is called an irregular singular point.

In order to find the required series solution, we use the substitution \(y = x^m\) to convert the equation (2.1) in the following form:

\[ f_1 (m) x^m' + f_2 (m) x^{m''} = 0; \quad m'' > m'. \tag{2.4} \]

Now, by assuming \(m'' - m' = s\), series solution of (2.1) is assumed as:

\[ y = \sum_{r=0}^{\infty} A_r x^{m+rs}; A_0 \neq 0. \tag{2.5} \]

Then by substituting value of \(y\) and its successive derivatives in equation (2.1), we get an identity i.e. coefficient of each power of \(x\) vanishes separately. Hence equating to zero the coefficient of \(x^{m-\lambda}\) (taking \(\lambda\) as a whole number), a quadratic equation is obtained known as indicial equation. While equating to zero the coefficient of \(x^{m-\lambda+rs}\), a recurrence relation is obtained for finding values of \(A_1, A_2, \ldots\). Substituting these values in equation (2.5), an infinite or finite series is obtained, which provides the required series solution as per the types of roots given by indicial equation in the form

\[ y = Ay_1 + By_2, \tag{2.6} \]

where \(y_1\) and \(y_2\) are two independent infinite or finite series obtained as:

2.2.1 \(y_1 = (y)_{m=m_1}\) and \(y_2 = (y)_{m=m_2}\), where \(m_1\) and \(m_2\) are any two real and different roots of indicial equation with \(x = 0\) as an ordinary point.
2.2.3 \( y_1 = (y)_{m=m_1} \) and \( y_2 = (y)_{m=m_2} \), where \( m_1 \) and \( m_2 \) are two real and different roots of indicial equation with non-integer difference and \( x = 0 \) as a regular singular point.

2.2.4 \( y_1 = (y)_{m=m_1} \) and \( y_2 = \left( \frac{\partial y}{\partial m} \right)_{m=m_1} \), where \( m_1 \) and \( m_2 \) are two real and different roots of indicial equation with integer difference such that \( m_1 < m_2 \) and \( x = 0 \) as a regular singular point.

2.3 Frobenius method

For finding series solution of (2.1) through Frobenius method in powers of \( x \), we check the status of point \( x = 0 \) that whether it is an ordinary point or a singular point as per the process mentioned above in general method.

2.3.1 Series solution with \( x = 0 \) as an ordinary point

For finding series solution of (2.1) with \( x = 0 \) as an ordinary point, we assume

\[
y = \sum_{r=0}^{\infty} a_r x^r; a_0, a_1 \neq 0.
\]

(2.7)

Then by substituting value of \( y \) and its successive derivatives in equation (2.1) and by comparing coefficients of various powers of \( x \) in the identity thus generated, values of \( a_2, a_3, \ldots \) are obtained in terms of \( a_0 \) and \( a_1 \). Substituting these values in (2.7), series solution of (2.1) is obtained with \( a_0 \) and \( a_1 \) as arbitrary constants.

2.3.2 Series solution with \( x = 0 \) as a regular singular point

For finding series solution of (2.1) with \( x = 0 \) as a regular singular point, we assume

\[
y = \sum_{r=0}^{\infty} a_r x^{r+m}; a_0 \neq 0.
\]

(2.8)

Then by substituting value of \( y \) and its successive derivatives in equation (2.1) and by comparing coefficient of lowest power of \( x \) in the identity thus generated, a quadratic equation is obtained known as indicial equation. While equating to zero the coefficient of \( x^{m+n} \), a recurrence relation is obtained for finding values of \( a_1, a_2, \ldots \). Substituting these values in equation (2.8), an infinite or finite series is obtained, which provides the required series solution as per the types of roots given by indicial equation in the form

\[
y = \alpha y_1 + \beta y_2,
\]

(2.9)

where \( y_1 \) and \( y_2 \) are two independent infinite or finite series obtained by the similar process as mentioned above in 2.2.2, 2.2.3 and 2.2.4 points of the general method.

3 Examples

Example 3.1. Use power series method to find the series solution of differential equation

\[
(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0 \text{ in powers of } x.
\]

(3.1)

Solution. In order to obtain the series solution of (3.1), we assume

\[
y = \sum_{m=0}^{\infty} c_m x^m.
\]

(3.2)

Then by substituting the value of \( y \) and its successive derivatives, equation (3.1) reduces to

\[
\sum_{m=0}^{\infty} c_m m(m-1)x^{m-2} - \sum_{m=0}^{\infty} c_m m(m-1)x^m - \sum_{m=0}^{\infty} 2c_m m x^m + \sum_{m=0}^{\infty} n(n+1)c_m x^m = 0.
\]

After rearranging first term in left hand side, we get

\[
\sum_{m=0}^{\infty} [c_{m+2}(m+2)(m+1) - c_m m(m-1) - 2c_m m + n(n+1)c_m] x^m = 0.
\]

(3.3)

Equating to zero the coefficient of \( x^m \) in equation (3.3), we get a recurrence relation

\[
c_{m+2} = \frac{(m+n+1)(m-n)}{(m+2)(m+1)} c_m.
\]

(3.4)
Substituting \( m = 0, 1, 2, \ldots \) in equation (3.4), we get
\[
c_2 = \frac{-(n + 1)n}{2.1}c_0, c_3 = \frac{-(n + 2)(n - 1)}{3.2}c_0, c_4 = \frac{(n + 3)(n + 1)n(n - 2)}{4.3.2.1}c_0, c_5 = \frac{(n + 4)(n + 2)(n - 1)(n - 3)}{5.4.3.2}c_0, \ldots
\]

Substituting values of \( c_2, c_3, c_4, c_5, \ldots \) in equation (3.2), the required series solution is obtained as:
\[
y = c_0 \left[ 1 - \frac{(n + 1)n}{2.1}x^2 + \frac{(n + 3)(n + 1)n(n - 2)}{4.3.2.1}x^4 + \ldots \right] + c_1 \left[ x - \frac{(n + 2)(n - 1)}{3.2}x^3 + \frac{(n + 4)(n + 2)(n - 1)(n - 3)}{5.4.3.2}x^5 + \ldots \right],
\]
where \( c_0, c_1 \) are arbitrary constants as not covered in calculation of coefficients.

**Example 3.2.** Use of power series method to find the series solution of differential equation
\[
(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0 \text{ in powers of } x.
\]  

**Solution.** In order to obtain the series solution of (3.5), we assume
\[
y = \sum_{m=0}^{\infty} c_m x^m.
\]

Then by substituting the value of \( y \) and its successive derivatives, equation (3.5) reduces to
\[
\sum_{m=2}^{\infty} c_m m(m - 1) x^{m-2} - \sum_{m=0}^{\infty} c_m (m^2 + m - 2) x^m = 0.
\]

After rearranging first term in left hand side, we get
\[
\sum_{m=0}^{\infty} [c_{m+2} (m + 1) - c_m (m - 1)] x^m = 0. \tag{3.7}
\]

Equating to zero the coefficient of \( x^m \) in equation (3.7), we get a recurrence relation
\[
c_{m+2} = \frac{(m - 1)}{(m + 1)} c_m. \tag{3.8}
\]

Substituting \( m = 0, 1, 2, \ldots \) in equation (3.8), we get \( c_2 = -c_0, c_3 = 0, c_4 = 0, c_5 = 0, \ldots \), which shows that \( c_0 \) is arbitrary constant as not covered in calculation of coefficients and \( c_1 \) is arbitrary constant as per the result \( c_3 = 0, c_1 = 0 \).

Substituting values of \( c_2, c_3, c_4, c_5, \ldots \) in equation (3.6), the required series solution is obtained as:
\[
y = c_0 \left[ 1 - x^2 - \frac{1}{3} x^4 + \ldots \right] + c_1 x.
\]

**Example 3.3.** Use of general method to find series solution of
\[
(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + n (n + 1) y = 0. \tag{3.9}
\]

**Solution** As per the definition mentioned above, \( x = 0 \) is an ordinary point of given differential equation.

In order to obtain the series solution of (3.9), we assume \( y = x^m \) and substitute its successive derivatives in equation (3.9) to reduce it in the form \( m(m - 1)x^{m-2} + (n + m + 1)(n - m)x^m = 0 \), which provides \( s = m - (m - 2) = 2 \). Now, we assume
\[
y = \sum_{r=0}^{\infty} A_r x^{m+2r}; A_0 \neq 0. \tag{3.10}
\]

Then by substituting above value of \( y \) and its successive derivatives, equation (3.9) reduces to
\[
\sum_{r=0}^{\infty} A_r (m + 2r)(m + 2r - 1)x^{m+2r-2} - \sum_{r=0}^{\infty} A_r (m + 2r)(m + 2r - 1)x^{m+2r}
\]

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we get a recurrence relation

\[ -\sum_{r=0}^{\infty} 2A_r (m + 2r) x^{m+2r} + \sum_{r=0}^{\infty} n (n+1) A_r x^{m+2r} = 0. \]  

(3.11)

After breaking first term and rearranging other terms in left hand side, we get

\[ \sum_{r=1}^{\infty} [A_r (m + 2r) (m + 2r - 1) - A_{r-1} \{(m + 2r - 2) (m + 2r - 1) - n (n+1)\}] x^{m+2r-2} + A_0 m (m-1) x^{m-2} = 0. \]  

(3.12)

Equating to zero the coefficient of \(x^{m-2}\) in equation (3.11), we get an indicial equation as \(m (m-1) = 0\), which provides \(m = 0, 1\) as two real roots. Equating to zero the coefficient of \(x^{m+2r-2}\) in equation (3.11), we get a recurrence relation

\[ A_r = \frac{(m+2r-2) (m+2r-1) - n (n+1)}{(m+2r) (m+2r-1)} A_{r-1}. \]

Substituting \(r = 1, 2, \ldots\) in equation (3.12), we get

\[ A_1 = \frac{(m+n+1) (m-n)}{(m+2) (m+1)} A_0, A_2 = \frac{[(m+2) (m+3) - n (n+1)] (m+n+1) (m-n)}{(m+4) (m+3) (m+2) (m+1)} A_0, \]

Substituting values of \(A_1, A_2, \ldots\) in equation (3.10), we get

\[ y = A_0 x^m \left[ 1 + \frac{(m+n+1) (m-n)}{(m+2) (m+1)} x^2 + \frac{\{(m+2) (m+3) - n (n+1)\} (m+n+1) (m-n)}{(m+4) (m+3) (m+2) (m+1)} x^4 + \ldots \right], \]

which provides series \(y_1\) and \(y_2\) as:

\[ y_1 = (y)_{m=0} = A_0 \left[ 1 - \frac{(n+1)}{2.1} x + \frac{(n+3) (n+1) (n-2)}{4.3.2.1} x^4 + \ldots \right]. \]

\[ y_2 = (y)_{m=1} = A_0 x \left[ 1 - \frac{(n+2) (n-1)}{3.2} x + \frac{(n+4) (n+2) (n-1) (n-3)}{5.4.3.2} x^4 + \ldots \right]. \]

Thus, the required series solution is given by \(y = Ay_1 + By_2\), where \(A\) and \(B\) are arbitrary constants.

**Example 3.4.** Use general method to find series solution of

\[ (x + x^2 + x^3) \frac{d^2 y}{dx^2} + 3x^2 \frac{dy}{dx} - 2y = 0. \]  

(3.13)

**Solution.** As per the definition mentioned above, \(x = 0\) is a regular singular point of given differential equation. In order to obtain the series solution of (3.13), we assume \(y = x^m\) and use its successive derivatives in equation (3.13) to reduce it in to \((m-1) x^{m-1} + (m^2 - m - 2) x^m + (m+2) m x^{m+1} = 0\), which cannot provide value of \(s\), as there are three different powers of \(x\) i.e. \(m - 1, m, m + 1\) and for finding value of \(s\), only two different powers of \(x\) are required as per the theory of general method.

Thus, the given differential equation cannot be solved by general method.

**Example 3.5.** Use Frobenius method to find series solution of

\[ (1 - x^3) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + n (n+1) y = 0. \]  

(3.14)

**Solution** As per the definition mentioned above, \(x = 0\) is an ordinary point of given differential equation. In order to obtain the series solution of (3.14), we assume

\[ y = \sum_{r=0}^{\infty} a_r x^r; a_0, a_1 \neq 0. \]  

(3.15)

Then by substituting the value of \(y\) and its successive derivatives, equation (3.14) reduces to

\[ \sum_{r=2}^{\infty} a_r (r-1) x^{r-2} - \sum_{r=2}^{\infty} a_r (r-1) x^r - \sum_{r=1}^{\infty} 2a_r x^r + \sum_{r=0}^{\infty} n (n+1) a_r x^r = 0. \]  

(3.16)

Equating to zero the coefficient of \(x^m\) in equation (3.16), we get a recurrence relation

\[ a_{m+2} = \frac{(m+n+1) (m-n)}{(m+2) (m+1)} a_m. \]  

(3.17)
Substituting \( m = 0, 1, 2, \ldots \) in equation (3.17), we get
\[
a_2 = -\frac{(n + 1)}{2.1} a_0, a_3 = -\frac{(n + 2)}{3.2} a_1, a_4 = \frac{(n + 3)}{4.3.2.1} a_0, a_5 = \frac{(n + 4)}{5.4.3.2} a_1, \ldots \text{[ since } a_0, a_1 \neq 0]\]
Substituting values of \( a_2, a_3, a_4, a_5, \ldots \) in equation (3.15), the required series solution is obtained as:
\[
y = a_0 \left[1 - \frac{(n + 1)}{2.1} x^2 + \frac{(n + 3)}{4.3.2.1} x^4 + \ldots\right] + a_1 \left[x - \frac{(n + 2)}{3.2} x^3 + \frac{(n + 4)}{5.4.3.2} x^5 + \ldots\right].
\]

Example 3.6. Use Frobenius method to find series solution of
\[
x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + x^2 y = 0.
\]

**Solution.** As per the definition mentioned above, \( x = 0 \) is a regular singular point of given differential equation. In order to obtain the series solution of (3.18), we assume
\[
y = \sum_{r=0}^{\infty} a_r x^{m+r}; a_0 \neq 0.
\]
Then by substituting the value of \( y \) and its successive derivatives, equation (3.18) reduces to
\[
\sum_{r=0}^{\infty} a_r (m + r) (m + r - 1) x^{m+r-1} + \sum_{r=0}^{\infty} a_r (m + r) x^{m+r-1} + \sum_{r=0}^{\infty} a_r x^{m+r+2} = 0.
\]
Equating to zero the coefficient of \( x^{m-1} \) in equation (3.20), we get an indicial equation as \( m^2 = 0 \), which provides \( m = 0, 0 \) as two real and same roots. Equating to zero the coefficient of \( x^{m+n} \) in equation (3.20), we get a recurrence relation
\[
a_{n+1} = -\frac{1}{(m + n + 1)^2} a_{n-2},
\]
which is capable of providing values of \( a_3, a_4, a_5, \ldots \). So, for finding values of \( a_1 \) and \( a_2 \), coefficients of \( x^m \) and \( x^{m+1} \) in equation (3.20) are equated with zero. Thus, we obtained \( a_1 = 0 \) and \( a_2 = 0 \).
Substituting \( n = 2, 3, 4, \ldots \) in equation (3.21), we get
\[
a_3 = -\frac{1}{(m + 3)^2} a_0, a_4 = 0, a_5 = 0, a_6 = \frac{1}{(m + 6)^2 (m + 3)^2} a_0, \ldots \text{[ since } a_0 \neq 0]\]
Substituting values of \( a_1, a_2, a_3, a_4, \ldots \) in equation (3.19), we get
\[
y = a_0 x^m \left[1 - \frac{1}{(m + 3)^2} x^3 + \frac{1}{(m + 6)^2 (m + 3)^2} x^6 - \ldots\right],
\]
which provides series \( y_1 \) and \( y_2 \) as:
\[
y_1 = (y)_{m=0} = a_0 \left[1 - \frac{1}{3^2} x^3 + \frac{1}{6^2.3^2} x^6 - \ldots\right]
\]
and
\[
y_2 = \left(\frac{\partial y}{\partial m}\right)_{m=0} = y_1 \log x + 2a_0 \left[\frac{1}{3} x^3 - \frac{1}{6^2.3^2} \left(\frac{1}{3} + \frac{1}{6}\right) x^6 - \ldots\right].
\]
Thus, the required series solution is given by \( y = \alpha y_1 + \beta y_2 \), where \( \alpha \) and \( \beta \) are arbitrary constants.

4 Conclusion
As seen in above examples and methodology discussed in previous section, the strengths and limitations as well as computational efficiency of three methods for finding series solutions of second order ordinary differential equations with variable coefficients with only complementary functions i.e. power series method, general method and Frobenius method is based over the following characteristics:
• Ordinary point / singular point
• Indicial equation
• Computation of coefficients involved in the series solution
Thus, the strengths and limitations of all three methods are written as:
4.1 Power series method

**Strength** We do not need the status of point \( x = 0 \) as ordinary or singular point. Indicial equation is not involved in the solution.

**Limitation** For final phase of solution, two independent infinite or finite series are given with two arbitrary constants. These arbitrary constants are not defined initially. In fact, the relations between the coefficients involved decide the values of arbitrary constants (explained in example 3.1 and example 3.2).

4.2 General method

**Strength** Common procedure is used for both the cases that whether \( x = 0 \) is an ordinary point or a singular point.

**Limitation** For finding value of \( s \), only two powers of \( x \) are required. If three or more powers of \( x \) are involved in a problem, then it cannot be solved by general method (explained in example 3.4).

4.3 Frobenius method

**Strength** Certain procedure is used separately for both the cases i.e. \( x = 0 \) is given as an ordinary point or a singular point. Arbitrary constants are defined in initial phase of solution.

**Limitation** For finding values of various coefficients involved in the solution, recurrence relation is used without knowing the fact that will it provide all the required values or not (explained in example 3.6).

To overcome the above mentioned limitation of Frobenius method, following modification is suggested:

4.4 Modification in Frobenius method

If \( x = 0 \) is an ordinary point, then compute the difference between values of highest and lowest subscripts of coefficients in the recurrence relation. If the difference is 2, then recurrence relation will provide values of all coefficients, otherwise it is required to equate the coefficients of \( x^0 , x^1 , x^2 , \ldots \) with zero to find the values of remaining coefficients. In example 3.5, the difference between the highest and lowest subscripts of coefficients is equal to \( (m+2) - m \) i.e. 2. So, all coefficients are evaluated using the recurrence relation.

If \( x = 0 \) is a regular singular point, then compute the difference between values of highest and lowest subscripts of coefficients in the recurrence relation. If the difference is 1, then recurrence relation will provide values of all coefficients, otherwise it is required to equate the coefficients of powers of \( x \) higher than the power of \( x \) used in finding indicial equation with zero to find the values of remaining coefficients. In example 3.6, the difference between the highest and lowest subscripts of coefficients is equal to \( (n+1) - (n-2) \) i.e. 3. So, two coefficients \( a_1 \) and \( a_2 \) are evaluated by equating to zero the coefficients of \( x^m \) and \( x^{m+1} \). All remaining coefficients are evaluated using the recurrence relation.

By incorporating above modification in Frobenius method, it can easily be said that the limitation of the method is compensated. Thus, it is concluded that the Frobenius method is a well-defined method with clear procedure and approach in comparison of power series as well as general method.

References


