MODIFICATION IN $\phi$-METHOD TO FIND OBLIQUE ASYMPTOTES OF AN ALGEBRAIC CURVE

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Abstract

Asymptote is a vital phenomenon in the field of Curve Tracing and allied subjects. Asymptotes for algebraic curve can be classified into two types, viz., Parallel and Oblique Asymptotes. Our main focus in this paper is to identify the limitations of $\phi$ - method, where it fails. The modification of this limitation is described in this paper that how one can modify the existing method to our proposed method. Some numerical examples are being presented here to validate our claim.

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1 Introduction

Many earlier researchers presented the various methods for finding asymptotes of different types of curves. A method for obtaining asymptotes of some special curves has been discussed by Keckic [2]. Procedure for obtaining asymptotes for a real plane algebraic curve has been discussed by Zeng [6].

In recent past, different views and approaches for finding asymptotes have been considered some attention. Mudaly and Mpofu [3] provided learners’ views on asymptotes of a hyperbola and exponential function. Blasco and Diaz [1] described a new approach for computing the asymptotes of a parametric curve. After that Sevilla, Benedicto and Diaz [5] provided asymptotes of meromorphic functions.

In this paper, limitations and modifications of traditional $\phi$ - method for finding oblique asymptotes of an algebraic curve are discussed. Several important numerical examples are given to explain the proposed study.

2 Limitations and Modifications in $\phi$ - Method

As per [4], to find oblique asymptotes of an algebraic curve of the form

$$(a_0x^n + a_1x^{n-1}y + \ldots + a_{n-1}xy^{n-1} + a_ny^n) + (b_1x^{n-1} + b_2x^{n-2}y + \ldots + b_{n-1}xy^{n-2} + b_ny^{n-1}) + (c_2x^{n-2} + c_3x^{n-3}y + \ldots + c_{n-1}xy^{n-3} + c_ny^{n-2}) + \ldots + (k_0x + k_1y) + k = 0, \quad (2.1)$$

we put $x = 1$ and $y = m$ in all terms of degree $n - 1$ to find $\phi_n(m)$, $\phi_{n-1}(m)$, respectively. Then by using the equation $\phi_n(m) = 0$, $n$ values of $m$ are obtained and as per nature of values of $m$ (real and unique or real and few/all identical), different oblique asymptotes can be obtained.

Now, there exist many curves, for which above $\phi$ - method provides asymptotes, which actually do not exist. This can be considered as a limitation of $\phi$ - method.

In order to overcome this limitation, we modify above $\phi$ - method by imposing a necessary constraint over coefficients $a_0, a_1, \ldots, a_{n-1}, a_n$ that above $\phi$ - method is not applicable, if either $a_0 = a_1 = \ldots = a_{n-1} = 0$ or $a_1 = a_2 = \ldots = a_n = 0$. In other words, it is imposed that $n^{th}$ degree terms of above curve (2.1) of degree 'n' cannot be made by only one variable out of $x$ or $y$ for perfect existence of $\phi$ - method.

3 Examples

Example 3.1.

$$y^2 = ax + b; \quad b \geq 0. \quad (3.1)$$
For finding asymptotes of above curve (3.1), we use $\phi$ - method and get $y = \pm \sqrt{b}$ as asymptotes, while as per the graph(s) 3.1 and 3.2 of curve (3.1) shown below, there does not exist any asymptote for the same.

Example 3.2.

$y^3 = ax^2$. (3.2)

For finding asymptotes of above curve (3.2), we use $\phi$ - method and get $y = 0$ as asymptote, while as per the graph(s) 3.3 and 3.4 of curve (3.2) shown below, there does not exist any asymptote for the same.

4 Conclusion

As seen in above examples, $\phi$ - method provided asymptotes, which is invalid because of actual geometry of the concerned curves. Results of this kind are considered under limitations of $\phi$ - method indicated in this paper.

By analyzing equations of the curves mentioned in above examples, it is obvious to state that these equations are composed in a specific way that $n^{th}$degree term of curve of degree '$n$' is made by only one variable out of $x$ or $y$. In particular in example - 1, term of degree 2 in curve of degree 2 is made by only variable $y$ and in example - 2, term of degree 3 in curve of degree 3 is made by only variable $y$.

It is easy to mention that there exist many examples like above, in which the indicated limitations of $\phi$- method can be seen.

For rectifying above mentioned limitations of $\phi$ - method, a condition is imposed that $n^{th}$degree terms of any curve of degree 'n' cannot be made by only one variable out of $x$ or $y$ for existence of $\phi$ - method.

Mathematically, if $n^{th}$degree terms of any curve of degree 'n' are given by $a_0x^n + a_1x^{n-1}y + ... + a_{n-1}xy^{n-1} + a_ny^n$, then the imposed condition will be given as neither $a_0 = a_1 = ....a_{n-1} = 0$ nor $a_1 = a_2 = ....a_n = 0$ must be satisfied for existence of $\phi$ - method.
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References