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(Dedicated to Professor V. P. Saxena on His 80^{th} Birth Anniversary Celebrations)

HIGHER DIMENSIONAL TOPOLOGICAL DEFECT SOLUTIONS WITH MASSIVE SCALAR FIELD IN GENERAL RELATIVITY

V. G. Mete[1] and V.S. Deshmukh[2]
[1]Department of Mathematics, R.D.I.K. & K.D. College, Badnera– Amravati (M.S.), India- 444701
[2]Department of Mathematics, P.R.M.I.T.& R. , Badnera– Amravati (M.S.), India- 444701
Email: vmete5622@gmail.com,vsdeshmukh456@gmail.com

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Abstract

In this paper, we investigate a five dimensional Locally Rotationally Symmetric *(LRS)* Bianchi type-V string cosmological model with massive scalar field in general relativity. In order to obtain an exact solution of the field equations, we used the following conditions: (i) the shear scalar is proportional to the expansion scalar, resulting in a relationship between metric potentials, and (ii) the average scale factor is proportional to the massive scalar field, resulting in a power law relationship. In addition, the physical and kinematical parameters are discussed in detail.

2020 Mathematical Sciences Classification: 83-3, 83-08, 83C15, 83E15, 83E30, 83F05.
Keywords and Phrases: Five dimensional LRS Bianchi -V model, cosmic string model, massive scalar field.

1 Introduction

General relativity *(GR)* a is geometric theory that describes gravitational phenomena. It is also useful in constructing mathematical models in cosmology which deals with the large scale structure of the universe. A phase transition in the early universe took place when the temperature dropped and symmetry of the universe broken spontaneously leading to topologically stable defects called vacuum domain walls, strings, and monopoles [7]. As cosmic strings are key parts of the description of the universe in the early stages of its evolution and give rise to density perturbations that lead to the formation of galaxies [7, 22]. It has attracted considerable interest among cosmologists to study cosmic strings within the framework of general relativity. A scalar meson field can be classified into two types, namely, zero mass scalar field and massive scalar field. Massive scalar fields describe short-range interactions while zero mass scalar fields describe long-range interactions. Cosmological models with massive scalar fields have been discussed by several authors in general and in modified theories of gravitation. Mete et al. [9] explored Bianchi type-V magnetized cosmological model with wet dark fluid in *GR*. Reddy [16] discussed the Bianchi type-V dark energy model with a scalar meson field in *GR*. Using a modified holographic Ricci dark energy model with an attractive massive scalar field, Naidu [11] studied Bianchi type-II modified holographic Ricci dark energy model. Naidu et al. [12] developed an anisotropic and spatially homogeneous Bianchi type-V dark energy cosmological model in the presence of an attractive massive scalar field in *GR*. A study by Aditya et al. [1] examined Kaluza-Klein dark energy models in Lyra manifolds with massive scalar fields. A spatially homogeneous and anisotropic Kantowski-Sachs cosmological model with anisotropic dark energy *(DE)* fluid and massive scalar fluid is presented by Raju et al. [17].

Rao et al. [18] constructed LRS Bianchi type-II cosmological models based on a mixture of a cosmic string cloud and anisotropic dark energy fluid as the source of gravitation. Aditya et al. [2] explored a spatially homogeneous and anisotropic Bianchi type- VI_0 cosmological model with dark energy fluid. An attractive massive scalar field with Bianchi type- I cosmological model with perfect fluid and attractive scalar fields in Lyra manifold has been discussed by Naidu et al. [13].Aditya et al. [3] investigated the solution of Einstein field equations using some physically relevant conditions in order to obtain an exact plane-symmetric dark energy cosmological model in the presence of an attractive massive scalar field. Poonia et al. [15] examined a Bianchi type-VI inflationary cosmological model with massive string source in general relativity. Recently Keerti Acharya et al.[4] discussed some Bianchi type-III string cosmological models for perfect fluid distribution with an alternate approach.

An Extra dimension is a concept in cosmology aimed at unifying gravity with other forces through higher-dimensional space-time. We are living in a four-dimensional stage of the universe, which may have been preceded by a multi-dimensional stage. Higher-dimensional cosmological models play an important role in studying the evolution of the universe in its early stages after the big bang due to their ability to study the early stages of the universe in a more detailed way. Kaluza Klein minimally interacting dark energy model in the presence of massive scalar field has been investigated by Naidu et al. [14]. Mohanty et al. [10] constructed a five dimensional string cosmological models in Lyra manifold when a massive string is the source of the gravitational field with $\rho = (1+\omega)\lambda$ (Takabayasi string). Very recently, a spatially homogeneous and anisotropic Bianchi type-V cosmological model coupled with a massive scalar meson field in presence of cosmic string has been studied by Raju et al. [19].

In light of the above discussion, we have investigated the higher dimensional LRS Bianchi type-V string cosmological model with massive scalar field. Some physical and kinematical properties of the model are discussed in detail.

2 Metric and field equations

Here we consider the space-time represented by five dimensional LRS Bianchi type-V metric in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x}\left(dy^2 + dz^2\right) - C^2 d\psi^2, \tag{2.1}$$

where metric coefficients A, B, C are the functions of time t.

The Einstein field equation is

$$R_{ij} - \frac{1}{2} g_{ij} R = -\left(T_{ij}^{(s)} + T_{ij}^{(\varphi)}\right), \tag{2.2}$$

where R_{ij} is the Ricci tensor, R is Ricci scalar and $T_{ij}^{(s)}$ is the energy momentum tensor corresponding to massive string defined as

$$T_{ij}^{(s)} = \rho u_i u_j - \lambda x_i x_j, \tag{2.3}$$

where ρ is the energy density, λ is the string tension density, u^i is five velocity and x_i is string direction.

$T_{ij}^{((\phi)}$ is the energy momentum tensor for attractive massive scalar field defined as,

$$T_{ij}^{(\phi)} = \varphi_{,i}\varphi_{,j} - \frac{1}{2}\left(\varphi_{,k}\varphi^{,k} - M^2\varphi^2\right), \tag{2.4}$$

where M is mass of scalar field φ which satisfies Klein-Gordan equation

$$g^{ij}\varphi_{,ij} + M^2\varphi = 0 \tag{2.5}$$

and a comma (,) and a semicolon (;) denote ordinary and covariant differentiation respectively and $\varphi = \varphi(t)$.

In a co-moving coordinates system, the velocity vector u^i and direction of string x^i satisfy the conditions

$$u^i u_i = -x^i x_i = 1. \tag{2.6}$$

$$u^i x_i = 0. \tag{2.7}$$

$$\rho = \rho_p + \lambda, \tag{2.8}$$

where ρ_p is the rest energy density of the particles attached to the string (λ) which may be negative or positive [8].

Using commoving coordinates the field equations (2.2), for the metric (2.1) with the help of equations (2.3) to (2.8) can be written as

$$2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} = \rho + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2}, \tag{2.9}$$

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = -\frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2}, \tag{2.10}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2}, \tag{2.11}$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} = \lambda - \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2}, \tag{2.12}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0. \tag{2.13}$$

From equation (2.13) we get

$$A = lB, \tag{2.14}$$

where l is a constant of integration. Thus, without loss of generality we take $l = 1$.

$$A = lB. \tag{2.15}$$

Using equation (2.15), equations (2.9) to (2.12) reduce to

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{C}}{AC} - \frac{3}{A^2} = \rho + \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2}, \tag{2.16}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2}, \tag{2.17}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} - \frac{3}{A^2} = \lambda - \frac{\dot{\varphi}^2}{2} + \frac{M^2\varphi^2}{2}, \tag{2.18}$$

where overhead dot(.) represents differentiation with respect to the cosmic time t. The conservation law for matter energy tensor gives us

$$\dot{\rho} + \rho\left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) - \lambda\frac{\dot{C}}{C} = 0 \tag{2.19}$$

and the Klein- Gordon equation for the metric (2.1) takes the form

$$\ddot{\varphi} + \dot{\varphi}\left(3\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) + M^2\varphi = 0. \tag{2.20}$$

Now, we define the following physical parameters that are useful for solving the above field equations.

The spatial volume V and the scale factor $R(t)$ are given by

$$V = AB^2C = R^4. \tag{2.21}$$

The expansion scalar (θ), the Hubble parameter (H) and the deceleration parameter (q), for the metric (2.1) are

$$\theta = 4H = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{2.22}$$

$$H = \frac{1}{4}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{2.23}$$

$$q = -\frac{R\ddot{R}}{R^2}. \tag{2.24}$$

The shear scalar $\left(\sigma^2\right)$ and the anisotropy parameter (Δ) are respectively given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 4H^2\right). \tag{2.25}$$

$$\Delta = \frac{1}{4}\left(\sum_{i=1}^{4} \frac{H_i - H}{H}\right)^2, \tag{2.26}$$

where H_i denotes the directional Hubble parameters in x, y, z and ψ directions.

3 Cosmological solutions of the field equations

Now the system of equations (2.16)-(2.18) are three independent equations in five unknowns A, C, ρ, λ and φ. However, equation (2.19) being conservation equation. To find a deterministic solution we use the following physically relevant conditions:

(i) First, using well known fact that the shear scalar is proportional to scalar expansion we assume a relation between the metric potentials as follows [6].

$$A = C^n, \tag{3.1}$$

where $n \neq 1$ is a positive constant.
(ii) Also in order to solve the highly non-linear field equations we use the following mathematical condition which several researchers have studied from different aspects of the scalar field φ. [5,20,21],

$$(3n+1)\frac{\dot{C}}{C} = -\frac{\dot{\varphi}}{\varphi}, \tag{3.2}$$

which simplifies the mathematical complexity of the field equations.

From equation (2.20), (3.1) and (3.2), we obtain

$$\varphi = \exp\left(\varphi_0 t - \frac{M^2t^2}{2} + \varphi_1\right), \tag{3.3}$$

where φ_0 and φ_1 are constants of integration.

Now using equation (3.1), (3.2) and (3.3), we obtain

$$A = B = \exp n\left(\frac{\frac{M^2t^2}{2} - \varphi_0 t - \varphi_1}{3n+1}\right), \tag{3.4}$$

$$C = \exp\left(\frac{\frac{M^2t^2}{2} - \varphi_0 t - \varphi_1}{3n+1}\right). \tag{3.5}$$

Using equations (3.4) to (3.5) in equation (2.1), we obtain five dimensional LRS Bianchi typeV model in the presence of string source and with massive scalar field given by equation (3.3).

$$ds^2 = dt^2 - \exp 2n\left(\frac{\frac{M^2t^2}{2} - \varphi_0 t - \varphi_1}{3n+1}\right)dx^2 - \left[\exp 2n\left(\frac{\frac{M^2t^2}{2} - \varphi_0 t - \varphi_1}{3n+1}\right)\right]\left(e^{2x}dy^2 + e^{2x}dz^2\right) - \exp 2\left(\frac{\frac{M^2t^2}{2} - \varphi_0 t - \varphi_1}{3n+1}\right)d\psi^2. \tag{3.6}$$

4 Cosmological Parameters

In this section, we obtained the following kinematical and physical parameters for the model (3.6) which are important in discussion of cosmology.

The average Hubble parameter is given as

$$H = \frac{1}{4}\left(M^2t - \varphi_0\right). \tag{4.1}$$

The expansion scalar is

$$\theta = \left(M^2t - \varphi_0\right). \tag{4.2}$$

The deceleration parameter for our model is given as

$$q = -\left[1 + \frac{4M^2}{(M^2t - \varphi_0)}\right]. \tag{4.3}$$

The spatial volume is

$$V = \exp\left(\frac{M^2t^2}{2} - \varphi_0 t - \varphi_1\right). \tag{4.4}$$

The shear scalar of the model is

$$\sigma^2 = \frac{3}{8}\left(\frac{n-1}{3n+1}\right)^2\left(M^2t - \varphi_0\right)^2. \tag{4.5}$$

The average anisotropy parameter is

$$\Delta = 3\left(\frac{n-1}{3n+1}\right)^2. \tag{4.6}$$

From equation (2.16) the energy density (ρ) for the model (3.6) is given by

$$\rho = \frac{3n(n+1)\left(M^2t - \varphi_0\right)^2}{(3n+1)^2} - 3e^{\frac{n\left(2\varphi_0 t - M^2t^2 + 2\varphi_1\right)}{(3n+1)}} - \left[\frac{\left(\varphi_0 - M^2t\right)^2 + M^2}{2}\right]e^{\left(2\varphi_0 t - M^2t^2 + 2\varphi_1\right)}. \tag{4.7}$$

From equation (2.18) the string density (λ) for the model (3.6) is obtained as

$$\lambda = \frac{3nM^2}{3n+1} + \frac{6n^2\left(M^2t - \varphi_0\right)^2}{(3n+1)^2} - 3e^{\frac{n\left(2\varphi_0 t - M^2t^2 + 2\varphi_1\right)}{(3n+1)}} + \left[\frac{\left(\varphi_0 - M^2t\right)^2 - M^2}{2}\right]e^{\left(2\varphi_0 - M^2t^2 + 2\varphi_1\right)}.$$

From equations (2.8), (4.7) and (4.8) the particle density (ρ_p) for the model (3.6) is given by

$$\rho_p = \frac{3n(1-n)\left(M^2t - \varphi_0\right)^2}{(3n+1)^2} - \frac{3nM^2}{3n+1} - \left(\varphi_0 - M^2t\right)^2 e^{\left(2\varphi_0 t - M^2t^2 + 2\varphi_1\right)}. \tag{4.8}$$

5 Physical discussion of the model

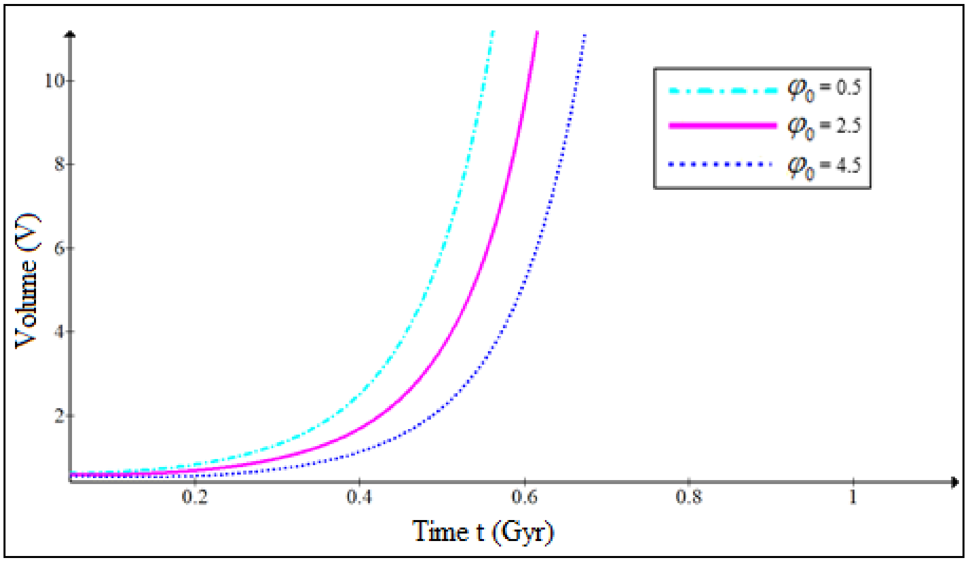

***Figure* 5.1:** Plot of volume versus time for $\varphi_1 = 0.5, n = 0.9$ and $M = 4.5$

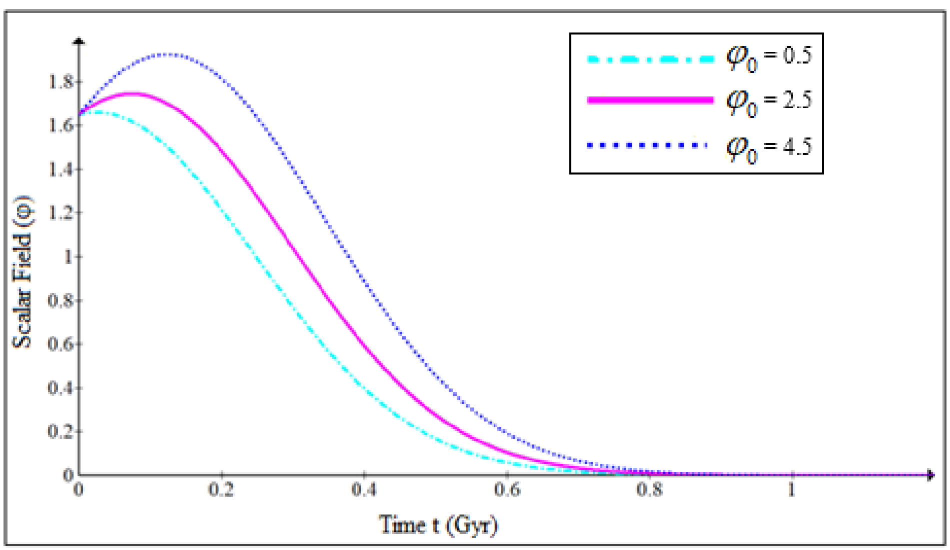

***Figure* 5.2:** Plot of scalar field versus time for $\varphi_1 = 0.5, n = 0.9$ and $M = 4.5$

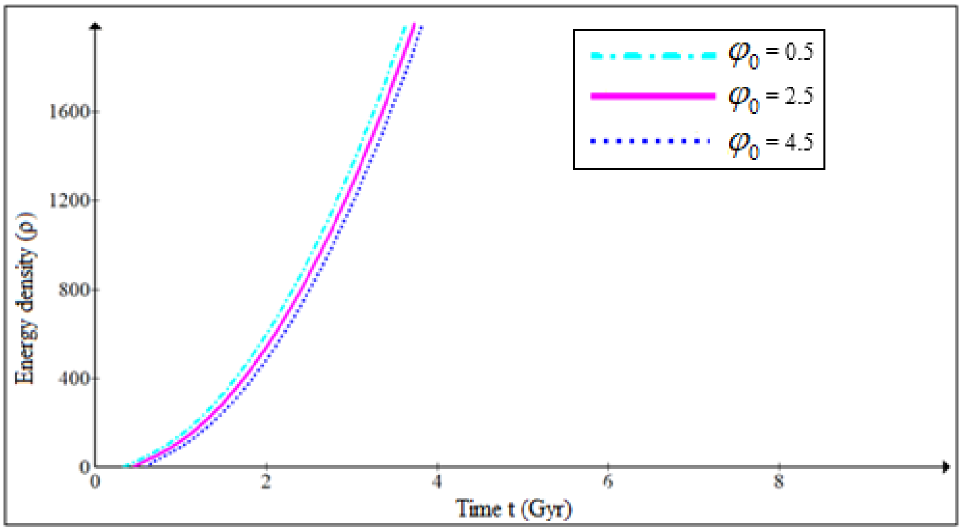

***Figure* 5.3:** Plot of energy density versus time for $\varphi_1 = 0.5, n = 0.9$ and $M = 4.5$

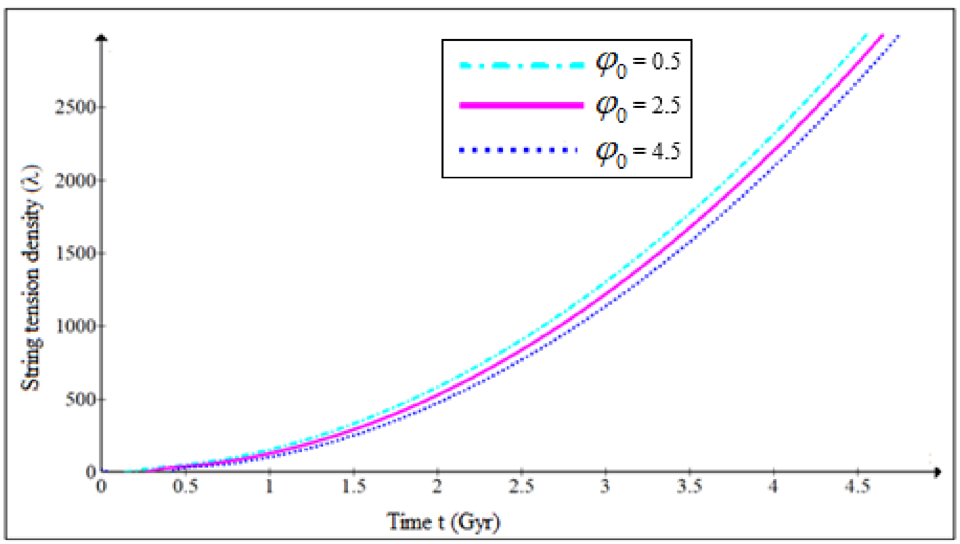

***Figure* 5.4:** Plot of string tension density versus time for $\varphi_1 = 0.5, n = 0.9$ and $M = 4.5$

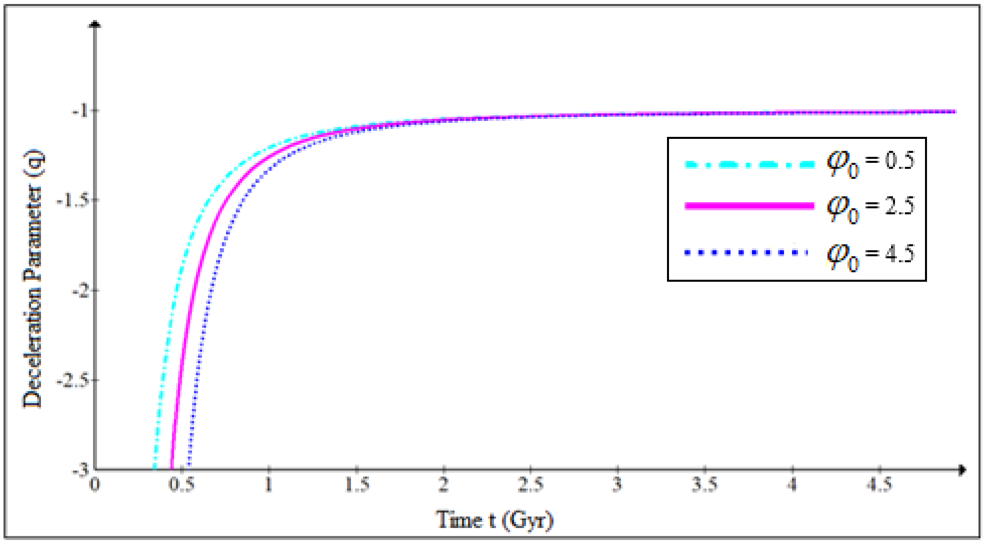

***Figure* 5.5:** Plot of deceleration parameter versus time for $\varphi_1 = 0.5$ and $M = 4.5$

Fig.5.1 depicts the behavior of the volume versus cosmic timet. For our model, it has been found that the spatial volume increases exponentially with time from a finite volume and attains infinite value as time $t \to \infty$.

Fig. 5.2 describes the behavior of scalar field versus time. It can be seen that scalar field is positive and decreasing function of cosmic timet. The behavior of scalar field of our model is quite similar to the scalar field shown in the model constructed by [18, 19].

The behavior of energy density versus cosmic time for various values of φ_0 is depicted in Fig.5.3. It is observed that the energy density is always positive throughout the evolution and is increasing function of cosmic time t. The realistic energy conditions, $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied in our model.

Fig.5.4 exhibits the behavior of string tension density versus cosmic timet . The string tension density λ is positive throughout the evolution of the model and increases with cosmic time t. This behavior of string tension density is quite different from the behavior of string tension density obtained in string cosmological model by Raju et al. [19].

In Fig.5.5 we depicted the behavior of deceleration parameter versus cosmic timet . One of the important physical quantity is deceleration parameter which shows whether the universe is accelerating or decelerating. For our model, we observe that initially since q is less than -1 , hence we obtain a universe with super exponential expansion and finally it approaches to $q = -1$, hence we obtain a universe with exponential expansion.

6 Conclusion

In this paper, we have discussed the dynamical aspects of the five-dimensional LRS Bianchi type-V string cosmological model with massive scalar field. It is noteworthy that the results obtained for our model resemble with the result obtained by Raju et al.[19] except with the behavior of string tension density (λ). Here we can observe that for the specific case $n = 1$, the anisotropy parameter and the shear scalar vanish and therefore the universe is isotropic and shear free for the model. In addition, because the average anisotropic parameter is constants, the universe is anisotropic throughout the evolution, except when $n = 1$.

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