STUDY OF VARYING CONCENTRATION AND SIZE OF NANOPARTICLES IN A CATHETERIZED ARTERY WITH CLOT AND STENOSIS*

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Abstract

This mathematical study comprising of a catheterized artery with clot and stenosis is conducted to highlight the usage of nanoparticles in treatment of thrombosis. Catheter coated with silver nanoparticles is inserted in the lumen of artery having clot and stenosis. The behavior of blood with nanoparticles is described using nanofluid. Navier-Stokes equation and diffusion equation for temperature as well as concentration are used to model the flow problem. Our prime intention is to study how concentration and nanoparticle size effect nanofluid flow considering the influence of various thermal features like thermal conductivity, specific heat capacity and thermal expansion. Solution has been obtained for concentration, temperature and velocity is obtained using finite difference method. The effects of radius of nanoparticle, Brownian motion parameter, stenosis depth, Grashof number and Darcy number have been examined graphically using *MATLAB*. It has been concluded that nanoparticles highly concentrate on the clot and stenosis and thus point to possible significant use of nanoparticles in antithrombotic therapy. This model can be, thus, utilized in thrombolytic therapies by proper optimization of concentration of nanoparticles as well as their geometries.

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1 Introduction

Nanoparticles have emerged as a promising technology that has revolutionized every field of science [11]. Recent years have witnessed an extensive attention of scientific researchers and clinicians in the field of nanomedicine or the use of nanoparticles in medicine. Nanoparticles provide enhanced treatment efficiency due to their convertible geometries and physiochemical properties because they mimic platelets by moving rapidly towards clots. Many nanoparticles-based drug delivery system have been used in medication and therapy of cardiovascular diseases and cancer. The application of nanoparticle in the therapeutics of thrombosis have exhibited amplified treatment efficiency [19]. In this paper we seek to understand the behavior of nanoparticles at the clot by controlling their concentration and size.

Thrombosis is the buildup of malignant clot in the blood vessels. It is a global health issue. The flow conditions of blood are affected by thrombus formation because clotted arteries have higher shear rates than healthy arteries. The thrombus or the malignant clot can be dissolved or reduced with the help of antiplatelet and anticoagulant agents like heparin, recombinant tPA *(rtPA)*, urokinase plasminogen activator *(uPA)* and streptokinase *(SK)* [19]. These agents are protein-based and have lesser bio-availability, thus, lesser therapeutic effect. Thus, it is important to develop such therapeutics that have higher bio-availability and efficiency. Here, nanoparticles have proven useful as their geometry and physio-chemical properties can be suitably controlled. Thus, nanoparticles have growing appeals in the treatment of clots.

Nanofluids are advanced fluids containing nanometer size particles suspended in a standard fluid like alcohol, water etc. Nanofluids hold an aptitude for heat transfer owing to its enhanced thermophysical properties. Thus, nanofluids are advantageous due to their better stability and better viscosity and dispersion properties.

Saleem et al. [20] analysed nanofluid in an artery with a catheter having stenosis and clot. Rathore and Srikanth [15] worked on an artery with stenosis, clot and catheter whose outer surface is layered with nanoparticles. Guan and Dou [5] outlined the recent advances in the use of nanoparticles as thrombus-targeting agents. Shah and Kumar [16] studied blood with nanoparticles in a tapered artery having a blood clot. By the above literature survey, it is clear that the effect of nanoparticle concentration present in blood along with their temperature of nanofluid has not been inspected much. Thus, in the current mathematical analysis, we have developed a model for an artery with a clot including a catheter layered with nanoparticles and probed into influence of varying concentrations of nanoparticles and temperature of nanofluid.

Primary properties of nanoparticles depend on their thermal conductivity [17]. In return viscosity and thermal conductivity of nanofluid rely on Reynolds number and Prandtl number because of convections arising in them. Saito [21] gave a model for viscosity of nanofluids containing very small spherical nanoparticles with pronounced Brownian motion, as

$$(1.1) \qquad \mu_{nf} = \mu_f \left(1 + \frac{2.5\phi}{1 - \frac{\phi}{0.87}} \right),$$

where ϕ is volume fraction of nanoparticles, μ_{nf} describes viscosity of nanofluid while μ_f is viscosity of base fluid. The interactions of nanoparticles caused by Brownian motion produces effects similar to convection at the nanoscale level. Thus, we have used this model to describe the viscosity of nanofluid.

The Navier-Stokes equation and temperature diffusion equation show that nanoparticle dispersion is elevated under strong Brownian forces. Jang and Choi [9] fabricated a model to define thermal conductivity accounting for contribution of nanoparticle Brownian motion in nanofluid, given as

$$(1.2) \qquad k_{nf} = k_f(1 - \phi) + k_p\phi + 3s\frac{r_0}{r_p}k_f Re^2 Pr\phi,$$

where ϕ is volume fraction of nanoparticles, k_{nf} describes thermal conductivity of nanofluid while k_f is thermal conductivity of base fluid and k_p is thermal conductivity of nanoparticles. Pr is Prandtl number and Re is Reynolds number. r_0 is radius of base fluid particles and r_p is radius of nanoparticles. s is an empirical constant. The vital role of Brownian motion is thus considered in our problem as we have used this model to describe the thermal conductivity of nanofluid.

Volume fraction of a solute present in a solvent is a measure of concentration of solute. The volume fraction is same as the concentration in an ideal solution i.e. where there is no reaction between the solute and solvent particles. In our case, the blood cells do not react with the nanoparticles in the nanofluid but accumulate only at the clot and stenosis. Thus, we have considered volume fraction of nanoparticles as concentration of nanoparticles in the nanofluid. The formulations have been carried out following the same.

When nanoparticles are administered in systemic circulation, they have their first encounter with blood cells. Nanoparticles are schemed specifically to deal with diseased cells to treat thrombosis. The compatibility of administered nanoparticles depends on their concentrations. Thus, to fine tune the nanoparticles before they are used in nanomedicine, it is important to understand their mathematical modelling. Hence, in this paper we have made an attempt to study blood flow in an artery with a clot in presence of a catheter coated with nanoparticles. The mathematical equations are modelled using Navier-Stokes equation, temperature and concentration diffusion equation in cylindrical co-ordinates. The concentration, temperature and velocity of nanofluid is found using finite difference method. The effects of nanoparticle concentration, temperature and velocity of nanofluid has been observed on parameters like radius of nanoparticle, Brownian motion parameter, stenosis depth, Grashof number and Darcy number. Outcomes have been discussed through graphs plotted using *MATLAB*. This study could act as a prototype in bio-medicine for the use of nanoparticles in treating thrombosis.

2 Mathematical Formulation

The incompressible, steady and laminar blood flow is assumed in an artery of length L and radius R_0 with a clot $\varepsilon'(z')$ and stenosis $R(z)$ (Fig 2.1). Silver nanoparticles are coated on the catheter of radius R_c. Cylindrical co-ordinates (r', θ', z') are taken into consideration. Equation of continuity, Navier-Stokes equation and diffusion equations for temperature and concentration are employed to frame the mathematical model.

The clot $\varepsilon'(z')$ [20] is defined as

$$(2.1) \qquad \varepsilon'(z') = \begin{cases} R_0(1 + e^{(-\pi^2(z' - 0.5)^2)}) & a' \leq z' \leq a' + b', \\ R_c & otherwise. \end{cases}$$

The geometry of the stenosis $R'(z')$[23] is given as:-

(2.2)
$$R'(z') = \begin{cases} R_0 - \delta' e^{\left(-\frac{m^2 z'^2}{L'^2} \right)} & a' \le z' \le a' + b', \\ R_0 & otherwise. \end{cases}$$

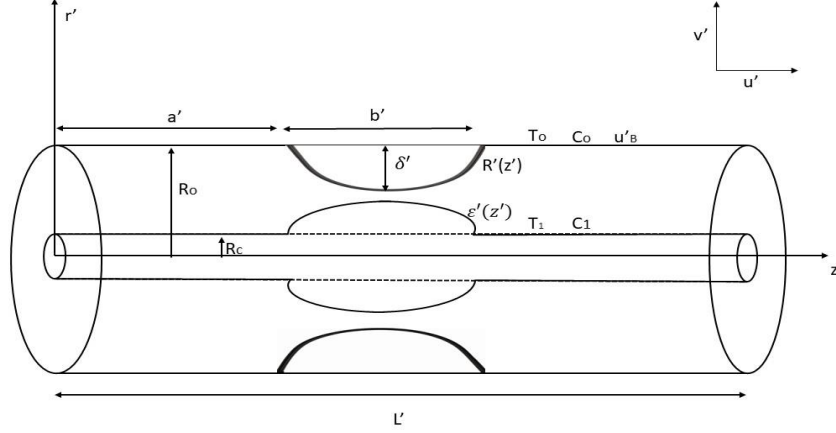where δ' is the depth of stenosis and m is a parametric constant

Figure **2.1:** Geometrical representation.

The governing equations are given as: Equation of continuity in cylindrical co-ordinates

(2.3)
$$\frac{\partial \rho_{nf}}{\partial t'} = \frac{1}{r'}\frac{\partial (r\, \rho_{nf} v')}{\partial r'} + \frac{1}{r'}\frac{\partial \rho_{nf} w'}{\partial \theta'} + \frac{\partial \rho_{nf} u'}{\partial z'} = 0,$$

Navier-Stokes equation in cylindrical co-ordinates

$$\rho_{nf}\left(\frac{\partial v'}{\partial t'} + v'\frac{\partial v'}{\partial r'} + \frac{u'}{r'}\frac{\partial v'}{\partial \theta'} - \frac{u'^2}{r'} + u'\frac{\partial v'}{\partial z'} \right)$$

(2.4)
$$= F_{r'} - \frac{\partial p'}{\partial r'} + \mu_{nf}\left(-\frac{v'}{r^2} + \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial v'}{\partial r'} \right) + \frac{1}{r'^2}\frac{\partial^2 v'}{\partial \theta'^2} + \frac{\partial^2 v'}{\partial z'^2} - \frac{2}{r'^2}\frac{\partial w'}{\partial \theta'} \right),$$

$$\rho_{nf}\left(\frac{\partial w'}{\partial t'} + v'\frac{\partial w'}{\partial r'} + \frac{u'}{r'}\frac{\partial w'}{\partial \theta'} - \frac{v'w'}{r'} + u'\frac{\partial w'}{\partial z'} \right)$$

(2.5)
$$= F_{\theta'} - \frac{\partial p'}{\partial \theta'} + \mu_{nf}\left(-\frac{w'}{r^2} + \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial w'}{\partial r'} \right) + \frac{1}{r'^2}\frac{\partial^2 w'}{\partial \theta'^2} + \frac{\partial^2 w'}{\partial z'^2} + \frac{2}{r'^2}\frac{\partial v'}{\partial \theta'} \right),$$

$$\rho_{nf}\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial r'} + \frac{u'}{r'}\frac{\partial u'}{\partial \theta'} + u'\frac{\partial u'}{\partial z'} \right)$$

(2.6)
$$= F_{z'} - \frac{\partial p'}{\partial z'} + \mu_{nf}\left(\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u}{\partial r'} \right) + \frac{1}{r'^2}\frac{\partial^2 u'}{\partial \theta'^2} + \frac{\partial^2 u'}{\partial z'^2} \right),$$

where F' in different indices stands for body forces in different co-ordinates and ρ_{nf} is density of nanofluid.

Diffusion equation for temperature T' of nanofluid in cylindrical co-ordinates

$$\left(v'\frac{\partial T'}{\partial r'} + u'\frac{\partial T'}{\partial z'} \right)$$

(2.7)
$$= \frac{k_{nf}}{\rho_{nf} c_{p_{nf}}}\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'} + \frac{\partial^2 T'}{\partial z'^2} \right) + \frac{D_B}{\rho_{nf} c_{p_{nf}}}\left(\frac{\partial c'}{\partial r'}\frac{\partial T'}{\partial r'} + \frac{\partial c'}{\partial z'}\frac{\partial T'}{\partial z'} \right),$$

where $c_{p_{nf}}$ is specific heat capacity of nanofluid, k_{nf} is thermal conductivity of nanofluid and ρ_{nf} is density of the nanofluid. D_B is Brownian diffusion coefficient. c' is concentration of nanoparticles. Temperature sensitive silver nanoparticles are coated on the catheter inserted in the lumen of artery [18]. The temperature is provided on the catheter to release nanoparticles for treating the clot.

Diffusion equation for concentration c' of nanoparticles in cylindrical co-ordinates

(2.8) $$\frac{\partial c'}{\partial t'} + u'\frac{\partial c'}{\partial z'} + v'\frac{\partial c'}{\partial r'} + w'\frac{\partial c'}{\partial \theta'} = D_B(\frac{\partial^2 c'}{\partial r'^2} + \frac{1}{r'}\frac{\partial c'}{\partial r'} + \frac{1}{r'^2}\frac{\partial^2 c'}{\partial \theta'^2} + \frac{\partial^2 c'}{\partial z'^2}),$$

where D_B is Brownian diffusion coefficient. The silver nanoparticles are highly concentrated on the surface of catheter.

The governing equations (2.3) - (2.8) are solved under the following assumptions

1. Catheter has been inserted at the center of the clot in the artery,
2. Flow is considered two dimensional,
3. Flow is steady,
4. Flow is axisymmetric,
5. The azimuthal component of fluid velocity is zero,
6. The cross-section area is very small; thus, flow is described by low Reynolds number,
7. Free convection effects are ignored,
8. Nanoparticles and blood are in thermal equilibrium,
9. No chemical reaction takes place in the blood,
10. There is no heat transfer due to radiation.

Nanofluids are highly developed colloidal fluids attained by dispersing 1-100 nm nanoparticles in standard fluid. Studies over the time have proven that nanofluids hold outstanding thermophysical properties as compared to base fluids. The parameters like volume fraction, size of base fluid particles, their thermal conductivity, hold significance in defining thermal characteristics of nanofluids like viscosity, thermal conductivity, and specific heat capacity.

The better thermal characteristics of nanofluids is because of the small sized nanoparticles dispersed in it. Viscosity is an important thermal property in this momentum because it is caused by interparticle interactions. It has been observed that viscosity of a base fluid enhances when nanoparticles are suspended in it. Viscosity is thus a governing factor of the behaviour of nanofluids which is described by the dynamics of nanoparticles in it. Brownian motion of nanoparticles controls their thermal motion which is responsible for defining the viscosity. Saito [21] gave the model for describing viscosity of nanofluids by accounting for Brownian motion of spherical nanoparticles described as:

(2.9) $$\mu_{nf} = \mu_f(1 + \frac{2.5c'}{1 - c'/0.87}),$$

where c' is concentration of nanoparticles; μ_{nf} is viscosity of nanofluid and μ_f is viscosity of blood.

Thermal conductivity is a relevant property of nanofluids as it is influenced by nanoparticle geometry, concentration and viscosity of base fluid. Thermal conductivity of nanofluids is evolved than their respective base fluids. The significant mechanism thar effects thermal conductivity of nanofluid is Brownian motion. Jang and Choi [9] gave the formula for thermal conductivity of nanofluid considering vital role of Brownian motion in thermal conduction. It has been reported by Gupta and Kumar [6] that Brownian motion enhances the thermal conductivity to 6 percent than their base fluids. Nanoparticles have a high random diffusion because of Brownian motion owing to their small dimensions. Thus, to study thermal conductivity of nanofluids, we use the formulation by Jang and Choi [9],

(2.10) $$k_{nf} = k_f(1 - c') + k_p c' + 3s\frac{r_0}{r_p}k_f Re^2 Pr c',$$

where c' is concentration of nanoparticles; k_{nf} is thermal conductivity of nanofluid, k_f is thermal conductivity of blood and k_p is thermal conductivity of nanoparticles; Pr is Prandtl number and Re is Reynolds number; r_0 is radius of blood particles (taken average), r_p is radius of nanoparticles and s is an empirical constant.

Specific heat capacity is also one of the relevant parameters for stating the thermal characteristics of nanofluids. Specific heat capacity dictates transfer of heat. It has been proved that specific heat capacity of nanofluids is lesser compared to their base fluid. Xuan et al. [25] modelled specific heat capacity for thermal equilibrium in nanoparticles and its base fluid which is given as,

(2.11) $$c_{p_{nf}} = \frac{(1 - c')\rho_f c_{p_f} + c'\rho_p c_{p_p}}{(1 - c')\rho_f + c'\rho_p},$$

where c' is concentration of nanoparticles; $c_{p_{nf}}$ is specific heat capacity of nanofluid, c_{p_f} is specific heat capacity of blood and c_{p_p} is specific heat capacity of nanoparticles; ρ_f density of blood and ρ_p is density of nanoparticles.

The thermal expansion of the nanofluid is modelled using a simple formula based on mixture rule as

$$(2.12) \qquad (\rho\gamma)_{nf} = (1 - c')\rho_f\gamma_f + c'\rho_p\gamma_p,$$

where c' is concentration of nanoparticles; $(\rho\gamma)_{nf}$ is thermal expansion of nanofluid, γ_f is specific thermal expansion of blood and γ_p is specific thermal expansion of nanoparticles; ρ_f density of blood and ρ_p is density of nanoparticles.

The modified equations using the assumptions and equations (2.9), (2.10), (2.11) and (2.12), along with their boundary conditions are given henceforth.

The equation of continuity

$$(2.13) \qquad \frac{\partial u'}{\partial z'} = 0,$$

The equation of motion in the catheterized artery with clot at the center

$$(2.14) \qquad g(\rho\gamma)_{nf}(T' - T_0) + g(\rho\gamma)_{nf}(c' - c_0) - (1/\rho_{nf})\frac{\partial p'}{\partial z'} + (\mu_{nf}/\rho_{nf})(\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u}{\partial r'}\right)) = 0,$$

No-slip at the boundary of the catheter is assumed.

$$(2.15) \qquad u' = 0 \ \ at \ \ r' = \varepsilon'(z'),$$

Using Beavers and Joseph condition [1] at the boundary of the artery, we get

$$(2.16) \qquad u' = u'_B \ \ and \ \ \frac{\partial u'}{\partial r'} = \frac{\sigma'}{\sqrt{Da}}(u'_B - u'_p) \ \ at \ \ r' = R'(z'),$$

where

$$(2.17) \qquad u'_p = -\frac{Da}{\mu_{nf}}\frac{\partial p'}{\partial z'},$$

is velocity at the permeable boundary where u'_B is slip velocity, σ' is slip parameter, Da is Darcy number

The diffusion equation for temperature of the catheterized artery with clot at the center

$$(2.18) \qquad \frac{k_{nf}}{\rho_{nf}c_{p_{nf}}}\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) + \frac{D_B}{\rho_{nf}c_{p_{nf}}}\left(\frac{\partial c'}{\partial r'}\frac{\partial T'}{\partial r'}\right) = 0.$$

Temperature T_1 is prescribed over the catheter and clot for releasing nanodrug

$$(2.19) \qquad T' = T_1 \ \ at \ \ r' = \varepsilon'(z').$$

Temperature at the boundary of the artery is T_o

$$(2.20) \qquad T' = T_1 \ \ at \ \ r' = R'(z').$$

The diffusion equation for concentration of nanoparticles in the catheterized artery with clot at the center

$$(2.21) \qquad D_B\left(\frac{\partial^2 c'}{\partial r'^2} + \frac{1}{r'}\frac{\partial c'}{\partial r'}\right) = 0.$$

Concentration c_1 of nanoparticles on the catheter and clot

$$(2.22) \qquad c' = c_1 \ \ at \ \ r' = \varepsilon'(z').$$

Concentration of nanoparticles at the boundary of the artery is c_o

$$(2.23) \qquad c' = c_0 \ \ at \ \ r' = R'(z').$$

Non-dimensional scheme
is given below as

$$(2.24) \qquad \begin{cases} r = \frac{r'}{R_0}, z = \frac{z'}{R_0}, u = \frac{u'}{u_{avg}}, P = \frac{P'}{\rho_f u_{avg}^2} Re = \frac{R_o\ u_{avg}\rho_f}{\mu_f}, Da = \frac{k_f}{R_0^2}, Pr = \frac{\mu_f}{D_B}, \\ N_b = \frac{\rho_f c_{p_f} D_B(c_1 - c_2)}{k_f}, \theta = \frac{T' - T_0}{T_1 - T_0}, c = \frac{c' - c_0}{c_1 - c_0}, Gr = \frac{g(\rho\gamma)_f R_0^2(T_1 - T_0)}{u_{avg}\mu_f}, \\ Br = \frac{g(\rho\gamma)_f R_0^2(c_1 - c_0)}{u_{avg}\mu_f}, \sigma' = \frac{\sigma}{R_0}, \delta' = \frac{\delta}{R_0}, \end{cases}$$

where u_{avg} is average reference velocity, Re is Reynolds number, Da is Darcy number, Pr is Prandtl Number, Gr is Grashof number and Br is solutary Grashof number, N_b is Brownian motion parameter.

Non-dimensional equation for clot

is given below as

(2.25)
$$\varepsilon\left(z\right) = \begin{cases} 1 + e^{-\pi^2(z-0.5)^2} & a \le z \le a+b, \\ 0.1 & otherwise. \end{cases}$$

Non-dimensional equation for stenosis

is given below as

(2.26)
$$\varepsilon\left(z\right) = \begin{cases} 1 - \frac{\delta}{R_0} e^{-m^2 z^2/L^2} & a \le z \le a+b, \\ 1 & otherwise. \end{cases}$$

Non-dimensional equations

are given below as

(2.27)
$$\frac{\partial u}{\partial z} = 0,$$

(2.28)
$$\frac{\partial P}{\partial z} = \mu_f \left(1 + \frac{2.5c}{1 - c/0.87} \right) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + \theta Gr \left((1-c) + c \frac{(\rho\gamma)_p}{(\rho\gamma)_f} \right) + c Br \left((1-c) + c \frac{(\rho\gamma)_p}{(\rho\gamma)_f} \right),$$

(2.29)
$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial \theta}{\partial r} \frac{\partial c}{\partial r} N_b \left((1-c) + c \frac{k_p}{k_f} + 3s \frac{r_0}{r_p} Re^2 Prc \right) \frac{\left(\frac{(1-c)}{c_{pf}} + c \frac{\rho_p}{\rho_f c_{pf}} \right)}{\left((1-c) + c \frac{\rho_p c_{pp}}{\rho_{fc_{pf}}} \right)} = 0,$$

(2.30)
$$\frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} = 0,$$

Non-dimensional boundary conditions

are given below as

(2.31)
$$c = 0 \ \ at \ \ r = R(z),$$

(2.32)
$$\theta = 0 \ \ at \ \ r = R(z),$$

(2.33)
$$u = u_B \ \ and \ \ \frac{\partial u}{\partial r} = \frac{\sigma}{\sqrt{Da}} \left(u_B - u_p \right) \ \ at \ \ r = R(z),$$

(2.34)
$$u = 0 \ \ at \ \ r = \varepsilon(z),$$

(2.35)
$$\theta = 1 \ \ at \ \ r = \varepsilon(z),$$

(2.36)
$$c = 1 \ \ at \ \ r = \varepsilon(z).$$

3 Solution

Mathematical solution for equations (2.25) to (2,30) employing the boundary conditions (2.31) to (2.36) is calculated numerically using *MATLAB* version 9.1R2016b.

Finite difference method

Denote c_i^k or Θ_{i+1}^k as the value of c or Θ at node r_i or z_i. In this notation, the finite difference formulation of various partial derivatives are given as

(3.1)
$$\frac{\partial c}{\partial r} \cong \frac{c_{i+1}^k - c_{i-1}^k}{2\Delta r} = c_r,$$

(3.2)
$$\frac{\partial^2 c}{\partial r^2} \cong \frac{c_{i+1}^k - 2c_i^k + c_{i-1}^k}{(\Delta r)^2} = c_{rr},$$

(3.3)
$$\frac{\partial \Theta}{\partial r} \cong \frac{\Theta_{i+1}^k - \Theta_{I+1}^k}{2\Delta r} = \Theta_r,$$

$$(3.4) \qquad \frac{\partial^2 \Theta}{\partial r^2} \cong \frac{\Theta_{i+1}^k - 2\Theta_i^k + \Theta_{i-1}^k}{(\Delta r)^2} = \Theta_{rr},$$

$$(3.5) \qquad \frac{\partial u}{\partial r} \cong \frac{u_{i+1}^k - u_{i-1}^k}{2\Delta r} = u_r,$$

$$(3.6) \qquad \frac{\partial^2 u}{\partial r^2} \cong \frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{(\Delta r)^2} = u_{rr},$$

The governing equations (2.28), (2.29) and (2.30) are as follows

$$(3.7) \qquad \frac{c_{i+1}^k - c_{i-1}^k}{2\Delta z} + \frac{1}{r}\frac{c_{i+1}^k - 2c_i^k + c_{i-1}^k}{(\Delta r)^2} = 0,$$

$$\frac{\Theta_{i+1}^k - 2\Theta_i^k + \Theta_{i-1}^k}{(\Delta r)^2}\frac{1}{r}\frac{\Theta_{i+1}^k - \Theta_{I+1}^k}{2\Delta r} + \frac{\Theta_{i+1}^k - \Theta_{I+1}^k}{2\Delta r}\frac{c_{i+1}^k - c_{i-1}^k}{2\Delta z}$$

$$(3.8) \qquad N_b\Big((1 - c_i^k) + \frac{k_p}{k_f}c_i^k 3s\frac{r_0}{r_p}Re^2 Prc_i^k\Big)\frac{(1 - c_i^k) + \frac{k_p}{k_f}c_i^k 3s\frac{r_0}{r_p}Re^2 Prc_i^k}{\big((1 - c_i^k) + \frac{k_p}{k_f}c_i^k 3s\frac{r_0}{r_p}Re^2 Prc_i^k\big)} = 0,$$

$$\frac{\Delta P}{\Delta z} = \mu_f\Big(1 + \frac{2.5c_i}{1 - c_i/0.87}\Big)\Big(\frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{(\Delta r)^2} + \frac{1}{r}\frac{u_{i+1}^k - u_{I+1}^k}{2\Delta r}\Big)$$

$$(3.9) \qquad +\theta_i^k Gr\Big((1 - c_i^k) + c_i^k\frac{(\rho\gamma)_p}{(\rho\gamma)_f}\Big) + c_i^k Br\Big((1 - c_i^k) + c_i^k\frac{(\rho\gamma)_p}{(\rho\gamma)_f}\Big),$$

$$(3.10) \qquad c_i^k = 1 \ \ at \ \ r_i = \varepsilon(z_i),$$

$$(3.11) \qquad c_i^k = 0 \ \ at \ \ r_i = R(z_i),$$

$$(3.12) \qquad \Theta_i^k = 1 \ \ at \ \ r_i = \varepsilon(z_i),$$

$$(3.13) \qquad \Theta_i^k = 0 \ \ at \ \ r_i = R(z_i),$$

$$(3.14) \qquad u_i^k = 0 \ \ at \ \ r_i = \varepsilon(z_i),$$

$$(3.15) \qquad u_i^k = u_{iB} \ \ and \ \ u_r = \frac{\sigma}{\sqrt{Da}}(u_{iB} - u_p) \ \ at \ \ r_i = R(z_i).$$

The algorithm for solving the equations is given as
1. The radial domain is represented by a mesh of $(n + 1)$ grid points $0 = r_0 < r_1 < \ldots < r_{n-1} < r_n = 1$.
2. We seek the solution for c, θ and u at the mesh points for their respective regions.
3. The difference equations (3.7) to (3.9) and boundary conditions (3.10) to (3.15) are solved using $bvp4c$ solver to obtain the values at each grid point applying Thomas algorithm for tridiagonal system of matrices

The value of concentration c in the thrombolytic and non-thrombolytic regions against radial direction r is given by Table 3.1 as $Rc = 0.1, m = 1, \delta = 0.01$

Table **3.1**

Radius	Concentration in non-thrombolytic region	Concentration in thrombolytic region
0.1	1	19.0743
0.2	0.6989	15.7190
0.3	0.5228	12.4227
0.4	0.3979	9.5837
0.5	0.3010	7.2690
0.6	0.2218	5.3592
0.7	0.1549	3.7422
0.8	0.0969	2.3142
0.9	0.4575	1.1054
1.0	0	0

The value of temperature of nanofluid θ against radial direction r is given by Table 3.2 as $Rc = 0.1, \delta = 0.01, rp = 30nm, Nb = 1.5$

Table **3.2**

Radius	Temperature of nanofluid
0.1	0
0.2	0.6225
0.3	0.4359
0.4	0.3168
0.5	0.2313
0.6	0.1657
0.7	0.1129
0.8	0.0692
0.9	0.3210
1.0	0

The value of velocity of nanofluid u against radial direction r is given by Table 3.3 as $Rc = 0.1, \delta = 0.01, rp = 30nm, Nb = 1.5, Gr = 0.2, Br = 0.1, Da = 0.1$

Table **3.3**

Radius	Velocity of nanofluid
0.1	0
0.2	2.6577
0.3	3.8602
0.4	4.3308
0.5	4.3001
0.6	3.8707
0.7	3.0970
0.8	2.0117
0.9	0.6354
1.0	-1.0176

4   Graphical results and discussions

This article gives theoretical research about the effects of treating clot in an artery using nanoparticles with respect to concentration of nanoparticles, radius of nanoparticles, Brownian motion parameter, Grashof number and Darcy number on velocity and temperature of nanofluids. Figures 4.1-4.11 show the graphs of results obtained.

Fig 4.1 shows graph of concentration of nanoparticles (c) against radial direction (r) for thrombolytic and non-thrombolytic regions. The graph shows that concentration is decreasing with the increase in radial direction. This is because the concentration of nanoparticles is highest at the catheter and clot as compared to the wall of the artery and stenosis. It can be concluded from the graph that concentration of nanoparticles is greater in thrombolytic region than in non-thrombolytic region. This result highlights the applications of nanoparticles in the treatment of clot. Khurshid et al. [12] gave identical conclusions in their experimental study.

Fig 4.2 shows graph of concentration of nanoparticles (c) against radial direction (r) for different values of catheter radius (Rc). The plots show that greater the value of catheter radius greater the concentration of nanoparticles in the artery. This is directly related to the fact that greater radius would accommodate greater number of nanoparticles on the surface of catheter. However, the radius of catheter should be optimized depending upon the severity of the clot. Comparative results have also been given by Karami et al. [13].

Fig 4.3 depicts graph of temperature of nanofluid (θ) against radial direction (r) for different values of radius of nanoparticles (r_p). Graph of temperature of decreases with increasing radial distance. This happens because nanoparticles present towards wall of catheter are at a higher temperature which causes them to migrate to walls of the artery which is at a lower temperature. The trend shows that the increase in radius of nanoparticle brings about a rise in the temperature of nanofluid. Qu et al. [14] gave this result in their experimental study. This happens because the increase in radius enhances the size of nanoparticles which causes greater interparticle collision owing to reduction in interparticle space. Hoshyar et al. [7] summarized similar results in their review on effect of nanoparticle size on their cellular interactions. They reported that larger diameter nanoparticles offer decreased cellular uptake. The optimal size of nanoparticle should be 30nm- 60 nm for effective delivery of drug.

Fig 4.4 displays graph of temperature of nanofluid (θ) against radial direction (r) for different values of Brownian motion parameter (N_b). The graph shows that temperature of nanofluid increases with increase in Brownian motion parameter. Nanoparticle motion increases with rise in Brownian motion, thus, temperature increases. Experimental validation was given by Jiang et al. [10].

Fig 4.5 shows graph of temperature of nanofluid (θ) against radial direction (r) for different values of stenosis depth (δ). Greater the stenosis depth, lesser the temperature. Xinting et al. [26] presented comparable experimental result for the effect of stenosis depth on the temperature of nanofluid.

Fig 4.6 depicts graph of velocity of nanofluid (u) against radial direction (r) for different values of radius of nanoparticles (r_p). Graph shows a parabolic variation similar to Hagen-Poiseuille flow. This is because the velocity is affected by zero acceleration because of constant pressure drop in the artery. Graph also shows that greater radius of nanoparticle lesser the velocity. Larger sized nanoparticles aggregate to increase flow resistance, hence velocity decreases. Hu et al. [8] analyzed similar result in their experimental study of effect of nanoparticle size on viscosity.

Fig 4.7 displays graph of velocity of nanofluid (u) against radial direction (r) for different values of Brownian motion parameter (N_b). It is seen that velocity decreases with increase in value of Brownian motion parameter. Brownian motion parameter is directly related to size of nanoparticles. Thus, larger the size, greater the Brownian motion parameter, lesser is the velocity. Saghir and Rahman [22] proved analogous experimental results.

Fig 4.8 shows graph of velocity of nanofluid (u) against radial direction (r) for different values of stenosis depth (δ). The results show that velocity increases with increase in stenosis depth. It follows from Bernoullis law for incompressible fluids, that reduction in cross-section area increases the velocity of fluid. This can also be supported by the fact that arteriosclerotic and thrombolytic arteries have higher blood pressure as compared to normal arteries [2].

Fig 4.9 shows graph of velocity of nanofluid (u) against radial direction (r) for different values of Grashof number (Gr). Grashof number stands for ratio of buoyancy force to viscous force. Thus, increase in its value increases the velocity of nanofluid because of the increase in temperature due to reduction in viscous forces [24].

Fig 4.10 depicts graph of velocity of nanofluid (u) against radial direction (r) for different values of solutary Grashof number (Br). Solutary Grashof number Br defines ratio between buoyant force and viscous hydrodynamic forces [18]. The trend observed is similar to Grashof number. It is because as the concentration increases the flow increases, thus increasing velocity.

Fig 4.11 displays graph of velocity of nanofluid (u) against radial direction (r) for different values of

Darcy number (Da). It is seen that increase in Darcy number increases velocity. Darcy number physically represents permeability at the arterial wall. Enhancing its value reduces flow resistance at the wall thus increasing velocity at arterial wall. Such experimental investigation was given by Boettcher et al. [3].
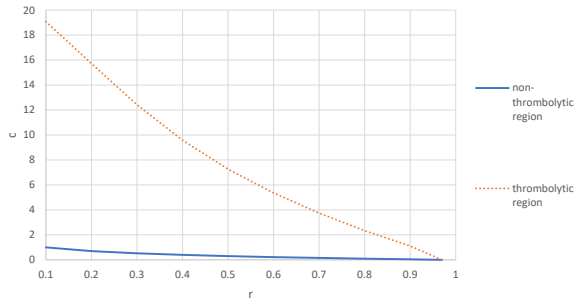

Fig 4.1 Variation of concentration of nanoparticles c against radial direction r in thrombolytic and non-thrombolytic regions Rc =0.1
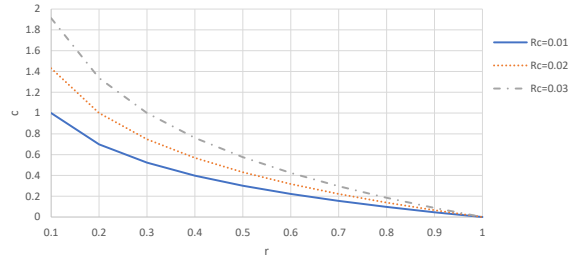

Fig 4.2 Variation of concentration of nanoparticles c against radial direction r for different values of catheter radius
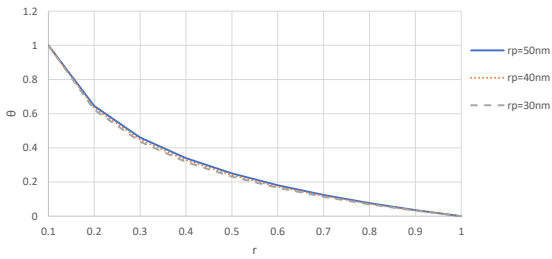
Fig 4.3 Variation of temperature of nanofluid θ against radial direction r for different values of nanoparticle radius rp Rc=0.1, Nb=1.0, δ=0.01
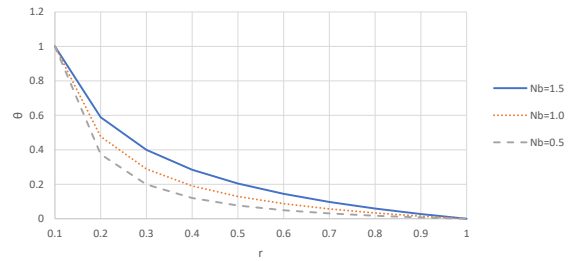
Fig 4.4 Variation of tempertaure of nanofluid θ against radial direction r for different values of Brownian motion parameter Nb Rc=0.1, rp=30nm, δ=0.01
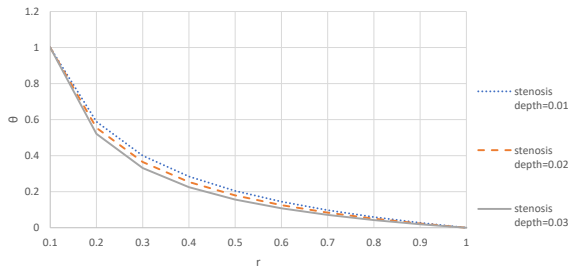

Fig 4.5 Variation of tempertaure of nanofluid θ against radial direction r for different values of stenosis depth δ Rc=0.1, Nb=1.0, rp=30 nm
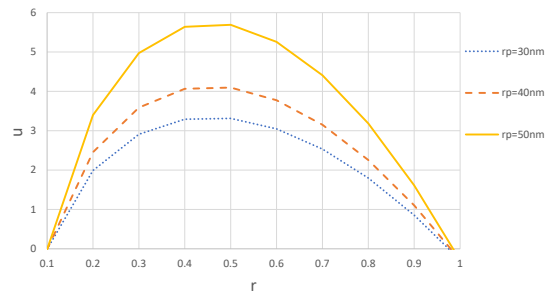

Fig 4.6 Variation of velocity of nanofluid u against radial direction r for different values of nanoparticle radius rp Nb=0.1, δ=0.01, Gr=2.0, Br=2.0, Da=0.1
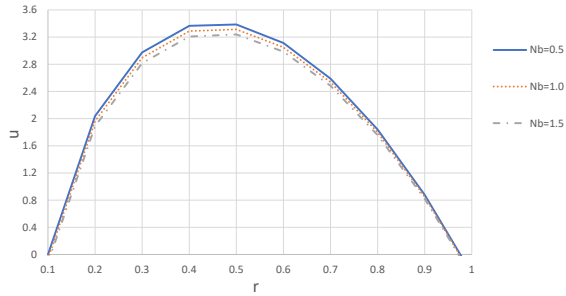
***Fig* 4.7** Variation of velocity of nanofluid *u* against radial direction *r* for
different values of Brownian motion parameter *Nb*
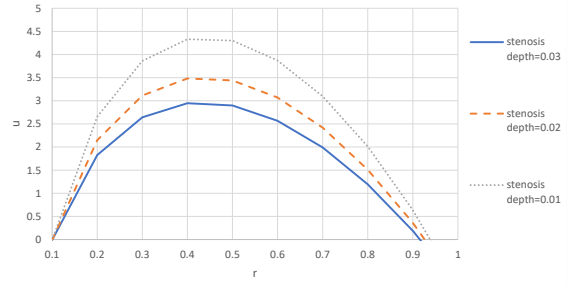*rp*=30nm, δ=0.01, *Gr*=2.0, *Br*=2.0, *Da*=0.1

***Fig* 4.8** Variation of velocity *u* against radial direction *r* for different
values of stenosis depth δ
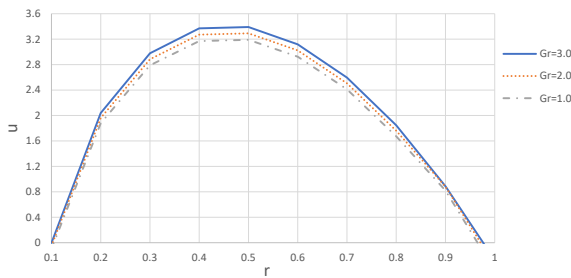*Rc*=0.1, *Nb*=1.0, *rp*=30nm, *Gr*=2.0, *Br*=3.0, *Da*=0.1

***Fig* 4.9** Variation of velocity of nanofluid *u* against radial direction *r* for
different values of Grashof number *Gr*
Nb=0.1, δ=0.01, *rp*=30 nm, *Br*=2.0, *Da*=0.1
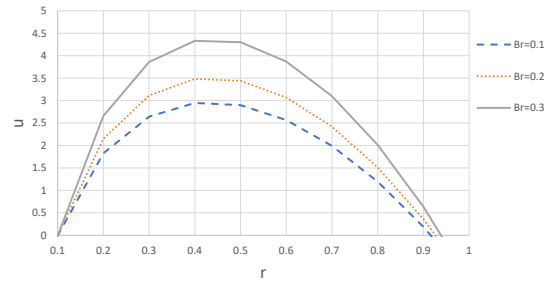

***Fig* 4.10** Variation of velocity of nanofluid *u* against radial direction *r* for
different values of solutary Grashof number *Br*
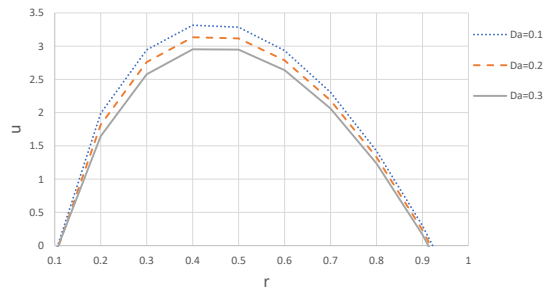*Nb*=0.1, δ=0.01, *rp*=30nm, *Gr*=2.0, *Da*=0.1

***Fig* 4.11** Variation of velocity of nanofluid *u* against radial direction *r* for
different values of Darcy number *Da*
Nb=0.1, δ=0.01, *rp*=30nm, *Gr*=2.0, *Br*=2.0

5   Conclusion

This study focuses on the influence of nanoparticle concentration, temperature and velocity of nanofluid
in a catheterized artery with clot and stenosis. The study contributes to the understanding and use of
nanoparticles as anti-thrombolytic agents. The outcomes are encapsulated as

1. The concentration of nanoparticles is higher at the clot compared to other regions.
2. The temperature of nanofluid increases with increase in nanoparticle radius, Brownian motion
   parameter and decreases with increase in stenosis depth.
3. The velocity of nanofluid decreases with increase in nanoparticle radius and Brownian motion
   parameter.
4. The velocity of nanofluid increases with increase in stenosis depth, Grashof number, solutary Grashof
   number and Darcy number.

   The above model has useful application in the treatment of cardiovascular diseases.

6 Appendix

The thermophysical properties of blood are

Table **6.1**

Physical properties	Blood
Heat Capacitance (c_p)	$3594 J/KgK$
Thermal Conductivity (k)	$0.492 W/mK$
Density (ρ)	$1060 Kg/m^3$
Thermal expansion coefficient (γ)	$0.18 X 10^{-5} K^{-1}$

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