ISSN 0304-9892 (Print) ISSN 2455-7463 (Online)
www.vijnanaparishadofindia.org/jnanabha
***Jñānābha*, Vol. 53(2) (2023), 79-85**
(Dedicated to Professor V. P. Saxena on His 80^{th} Birth Anniversary Celebrations)

BIANCHI TYPE V MAGNETIZED STRING DUST BULK VISCOUS FLUID COSMOLOGICAL MODELS WITH VARIABLE DECELERATION PARAMETER

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(Received: January 30, 2022; In format: February 11, 2023; Revised : August 16, 2023; Accepted : September 05, 2023)

DOI: https://doi.org/10.58250/jnanabha.2023.53210

Abstract

We have investigated Bianchi type V viscous fluid cosmological models with string dust universe in general relativity. Exact solution of Einstein field equations have been obtained by choosing deceleration parameter is function of cosmic time. Solutions for exponential and polynomial form are obtained. Some geometrical and physical aspects of models are also discussed.

2020 Mathematical Sciences Classification: 83C10, 83C20, 83E30.
Keywords and Phrases: Deceleration parameter, Bianchi type- V, Cosmological term Λ.

1 Introduction

Bianchi type-V cosmological models plays an important role in the investigation of origin and evaluation of universe and the study is more interesting as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some point of time. The string theory of cosmology plays a significant role in the investigation of physical situation at the very early stages of the formation of the universe. It is generally assumed that after the big bang, the universe may have undergone a series of phase transitions as its temperature was lowered down below some critical temperature as predicted by grand unified theories [5, 6, 7, 26, 28, 29, 30]. At the very early stages of evolution of the universe, during phase transition, it is believed that the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as domain walls, strings and monopoles. In these three cosmological structures, cosmic strings are the most interesting [27] because they are believed to give rise to density perturbations which lead to the formation of galaxies [30]. These cosmic strings can be closed like loops or opened like a hair which move through time and trace out a tube or a sheet, according to whether it is closed or open. The string is free to vibrate and its different vibration modes present different types of particles carrying the force of gravitation. Hence, investigation of universe is very interesting to study the gravitational effect that arises from strings using Einstein's field equations.

Bulk viscosity is useful for the study of early stages of evolution of the universe. Bulk viscosity driven inflation is primarily due to the negative bulk viscous pressure giving rise to a total negative effective pressure which may overcome the pressure due to the usual gravity of matter distribution in the universe and provides an impetus for rapid expansion of the universe. Thus many workers have been study bulk viscous string cosmological model in the context of Einstein theory or modified theories of gravity. Bulk viscous cosmological models in general relativity of material distribution have been investigated by a number of workers [1, 8, 12, 13, 14, 16, 18, 21]. Singh and Kale [22] has examined anisotropic bulk viscous cosmological models with particle creation. Rao and Sireesha [19] investigated the Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation. Banerjee and Banerjee [2] investigated stationary distribution of dust and electromagnetic fields in general relativity. Banerjee et al. [3] studied an axially symmetric Bianchi Type I string dust cosmological model in presence and absence of magnetic field. Recently, Bali and Upadhaya [4] examined LRS Bianchi Type I strings dust-magnetized cosmological models.

Motivated by the above discussion, we have constructed Bianchi type- V magnetized string dust dust bulk viscous fluid cosmological models with variable Λ and deceleration parameter. The main reason to explore the Bianchi type- V model is that the standard FLRW models are contained as special cases of the Bianchi models. The Bianchi type- V model generalizes the open $(k=-1)$ Friedmann model and represents a model in which the fluid flow is not necessarily orthogonal to the three surfaces of homogeneity. At early stage of evolution, the universe was not so smooth as it looks in present time. Therefore anisotropic cosmological models have taken considerable interest of researcher workers.

In this paper, we have study the role of variable deceleration parameter in Bianchi type- V space time with magnetized string dust bulk viscous fluid.

2 Metric and Field Equations

We consider the Bianchi type V space-time in orthogonal form represented by the line-element

$$ds^2=-dt^2+A^2dx^2+B^2e^{2x}dy^2+C^2e^{2x}dz^2. \tag{2.1}$$

where A and B are the metric potentials considered as function of cosmic time only. The energy-momentum tensor $\left(T_i^j\right)$ for cloud of string dust is given by Letelier [9] with bulk viscous fluid and electromagnetic field $\left(E_i^j\right)$ given by Lichnerowicz [10] as

$$T_i^j=\rho v_i v^j-\lambda x_i x^j-\varepsilon\theta\left(g_i^j+v_i v^j\right)+E_i^j. \tag{2.2}$$

Where ρ is the rest energy density of the cloud of strings with particles attached to them, $\rho=\rho_{\mathrm{p}}+\lambda$ with ρ_{p} being the rest energy density of particles, λ the tension density of the cloud of strings, $\theta=v^i_{;i}$ is the scalar of expansion and ε the coefficient of bulk viscosity. The vector $v^i=(0,0,0,1)$ is the four-velocity of the particles and x^i is a unit space-like vector representing the direction of string.

The vector v^i and x^i satisfy the conditions

$$v_i v^i=-x_i x^i=-1, v^i x_i=0. \tag{2.3}$$

Choosing x^i parallel to $\frac{\partial}{\partial x}$, we have

$$x^i=\left(A^{-1},0,0,0\right). \tag{2.4}$$

The electromagnetic field E_i^j is given by

$$E_i^j=\bar{\mu}\left[|h|^2\left(v_i v^j+\frac{1}{2}g_i^j\right)-h_i h^j\right], \tag{2.5}$$

where h_i is the magnetic flux vector given by

$$h_1=\frac{\sqrt{-g}}{2\bar{\mu}}\in_{ijkl}F^{kl}v^j. \tag{2.6}$$

Here $\bar{\mu}$ is the magnetic permeability and ϵ_{ijkl} the Levi-Civita tensor. We assume that current is flowing along x-axis.

Thus F_{23} is the only non-vanishing component of F_{ij}.

Maxwell's equations

$$F[ij;k]=0 \tag{2.7}$$

and

$$F_{\mathrm{ij}}=0 \tag{2.8}$$

are satisfied by

$$F_{23}=\text{ constant }=K(\text{ say }). \tag{2.9}$$

Thus

$$h_1\neq 0, h_2=0=h_3=h_4. \tag{2.10}$$

Thus equation (2.6) lead to

$$h_1=\frac{AK}{\bar{\mu}BCe^{2x}}. \tag{2.11}$$

$F_{14}=0=F_{24}=F_{34}$ due to assumption of infinite electrical conductivity (Maartens[15]).

We assume that magnetic permeability ($\bar{\mu}$) is a variable and consider

$$\bar{\mu} = e^{-4x} \text{ i.e. when } x \to \infty, \text{ then } \bar{\mu} \to 0.$$

Thus, from (2.5) and (2.11), we have

$$E_1^1 = -\frac{K^2}{2B^2C^2} = -E_2^2 = -E_3^3 = E_4^4. \tag{2.12}$$

The Einstein field equation (in gravitational units $c = 1, 8\ \Lambda\ G = 1$) with time varying cosmological term $\Lambda(t)$ are given by

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j + \Lambda g_i^j. \tag{2.13}$$

The field equation (2.13) with subsequently lead to the following system of equation

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = \lambda + \varepsilon\theta + \frac{K^2}{2B^2C^2} + \Lambda, \tag{2.14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \frac{-K^2}{2B^2C^2} + \varepsilon\theta + \Lambda, \tag{2.15}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\frac{K^2}{2B^2C^2} + \varepsilon\theta + \Lambda, \tag{2.16}$$

$$\frac{\ddot{A}\dot{B}}{AB} + \frac{\ddot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3}{A^2} = \rho + \frac{K^2}{B^2C^2} + \Lambda, \tag{2.17}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{2.18}$$

In the above and elsewhere overhead dot stands for ordinary time-derivative of the concerned quantity. We define the average scale factor R as

$$R^3 = ABC. \tag{2.19}$$

Integrating (2.18), we get

$$A^2 = LBC = BC. \tag{2.20}$$

where $L = 1$ is constant of integration.

In analogy with FRW universe, we define a generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{2.21}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{-\dot{H}}{H^2} - 1, \tag{2.22}$$

where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factor in the x, y and z directional respectively.

The anisotropy parameter

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{2.23}$$

The physical quantities of observational interest in cosmology

$$\theta = \frac{3\dot{R}}{R}. \tag{2.24}$$

The components of shear tensor $\left(\sigma_j^i\right)$

$$\sigma_1^1 = \frac{1}{3}\left(\frac{\dot{2A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \tag{2.25}$$

$$\sigma_2^2 = \frac{1}{3}\left(\frac{2\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right), \tag{2.26}$$

$$\sigma_3^3 = \frac{1}{3}\left(\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right), \tag{2.27}$$

$$\sigma_4^4 = 0. \tag{2.28}$$

Therefore,

$$\sigma^2 = \frac{1}{2}\left[\left(\sigma_1^1\right)^2 + \left(\sigma_2^2\right)^2 + \left(\sigma_3^3\right)^2 + \left(\sigma_4^4\right)^2\right]. \tag{2.29}$$

3 Solution of the Filed Equation

Observation of type Ia supernovae [20] allow to probe the expansion history of universe. In literature it is common to use a constant deceleration parameter, as it duly gives a power law for metric function or corresponding quantity. But at present the expansion of universe is accelerating and decelerating in the past. Also the transition redshift from deceleration phase to accelerating phase is about 0.5. Now for the universe which was decelerating in the past and accelerating at present time, the deceleration parameter must show signature flipping [17, 23, 24, 25]. So, in general, deceleration parameter is not constant but variable. On basis of supernovae searches, we consider the deceleration parameter to the variable i.e.

$$-\frac{R\ddot{R}}{\dot{R}^2} = q \text{ (variable)}, \tag{3.1}$$

where R is a average scale factor.

We assume

$$\varepsilon = \varepsilon_0 + \varepsilon_1 H, \tag{3.2}$$

where ε_0 and ε_1 are positive constant.

In this paper, we show how the variable deceleration parameter models with metric (2.1) behave in presence of string fluid as a source of matter.

From equation (3.1), we have

$$\frac{\ddot{R}}{R} + q\frac{\dot{R}^2}{R^2} = 0. \tag{3.3}$$

In order to solve equation (3.2), we have to assume $q = q(R)$. It is important to note here that one can assume $q = q(t) = q(R(t))$, as R is also a time dependent function. But this is possible only when one avoid singularity like big-bang or big rip because both t and R are increasing function.

Thus the general solution of equation (3.1) with assumption $q = q(R)$ is given by

$$\int e^{\int \frac{q}{R}dR} dR = t + t_0, \tag{3.4}$$

where t_0 is the constant of integration.

Without loss of generality, we chose

$$\int \frac{q}{R} dR = \log L(R). \tag{3.5}$$

Therefore,

$$\int L(R) dR = t + t_0. \tag{3.6}$$

The choice of $L(R)$ in equation (3.5), quite arbitrary but, since we are looking for a physically viable models of universe consistent with observations. We assume the following two cases.

4 Solution in Exponential Form

Let us consider $L(R) = \frac{1}{aR+b}$, where a & b are constant on integration, equation (3.6), we get

$$R = \frac{1}{a}\left(e^{aT} - b\right), \tag{4.1}$$

where $T = t + t_0$. In this case, the expression for the proper energy density (ρ), the string tension (λ), the cosmological constant (Λ) and particle density (ρ_p) are given by

$$\rho = \frac{-a^2 - 2a^2e^{-2aT} + 2a^2be^{-aT} + 3a^2\varepsilon_1}{\left(1 - be^{-aT}\right)^2} - \frac{a^6k_1^2e^{-6a}}{2\left(1 - be^{-aT}\right)} - \frac{K^2a^4e^{-4aT}}{\left(1 - be^{-aT}\right)^4} + \frac{3a^2\varepsilon_0}{\left(1 - be^{-aT}\right)}, \tag{4.2}$$

$$\lambda = \frac{-K^2a^4e^{-4aT}}{\left(1 - be^{-aT}\right)^4}, \tag{4.3}$$

$$\Lambda = \frac{-2a^2be^{-aT} + 3a^2 - a^2e^{-2aT} - 3a^2\varepsilon_1}{\left(1 - be^{-aT}\right)^2} + \frac{a^6k_1^2e^{-6aT}}{4\left(1 - be^{-aT}\right)^6} + \frac{K^2e^{-4aT}a^4}{2\left(1 - be^{-aT}\right)^4} - \frac{3a\varepsilon_0}{\left(1 - be^{-aT}\right)}, \tag{4.4}$$

$$\rho_p = \frac{-a^2 - 2a^2e^{-2aT} + 2a^2be^{-aT} + 3a^2\varepsilon_1}{\left(1 - be^{-aT}\right)^2} - \frac{a^6k_1^2e^{-6aT}}{2\left(1 - be^{-aT}\right)} + \frac{3a\varepsilon_0}{\left(1 - be^{-aT}\right)}. \tag{4.5}$$

The rate of expansion in the direction of x, y and z are given by

$$H_x = \frac{\dot{A}}{A} = \frac{a}{1 - be^{-aT}}, \tag{4.6}$$

$$H_y = \frac{\dot{B}}{B} = \frac{a}{1 - be^{-aT}} - \frac{a^3 k_1 e^{-3aT}}{2\left(1 - be^{-aT}\right)^3}, \tag{4.7}$$

$$H_z = \frac{\dot{C}}{C} = \frac{a}{1 - be^{-aT}} + \frac{a^3 k_1 e^{-3aT}}{2\left(1 - be^{-aT}\right)^3}. \tag{4.8}$$

Expansion θ, shear σ^2, deceleration parameter q, spatial volume V, bulk viscosity ε and anisotropy parameter $\bar{A}$ of the model take the form

$$\theta = \frac{3a}{1 - be^{-aT}}, \tag{4.9}$$

$$\sigma^2 = \frac{a^6 k_1^2 e^{-6aT}}{4\left(1 - be^{-aT}\right)^6}, \tag{4.10}$$

$$q = -1 + be^{-aT} \tag{4.11}$$

$$V = \frac{1}{a^3}\left(e^{aT} - b\right)^3, \tag{4.12}$$

$$\varepsilon = \varepsilon_0 + \frac{\varepsilon_1 a}{1 - be^{-aT}}, \tag{4.13}$$

$$\bar{A} = \frac{1}{6}\frac{a^4 k_1^2 e^{-6aT}}{\left(1 - be^{-aT}\right)^4}. \tag{4.14}$$

We observe that model has singularity at $T = \frac{\log b}{a} = T_0$ (say). The model starts expanding with a big bang at $T = T_0$ and the expansion in the model decreases as time T increases. Expansion in the model becomes finite at $T = \infty$. Singularity in the model is of point type. Since scale factor cannot be negative, we find $R(T)$ is positive, if $a > 0$. Therefore, in the early stage of the universe i.e. near $T = T_0$, the scale factor of the universe had been approximately constant and had increased very slowly. Sometime later, the universe has exploded suddenly and expanded to a large scale. This picture is consistent with big-bang scenario. We observe that $q = 0$ for $T = T_0$ and as $T \to \infty, q = -1$. Thus the model represents an accelerating universe at late times. The physical quantities $\rho, \sigma, \Lambda, \varepsilon$ and ρ_p all diverge at $T = T_0$. In the limit of large times i.e. $T \to \infty, \rho \to 3a^2\varepsilon_1 + 3a^2\varepsilon_0 - a^2, \sigma \to 0, \Lambda \to 3a^2 - 3a^2\varepsilon_1 - 3a\varepsilon_0, \varepsilon \to \varepsilon_0 + \varepsilon_1 a$ and $\rho_p \to 3a^2\varepsilon_1 + 3a\varepsilon_0 - a^2$.

From (4.3) it is found that tension density λ is negative. It is pointed out by Letelier[11] that λ may be positive or negative. When $\lambda < 0$, the string phase of the universe disappears i.e. we have an anisotropic fluid of particles. The mean anisotropic parameter is decreasing function of time.

For the model

$$\frac{\sigma}{\theta} = \frac{a^2 k_1 e^{-3aT}}{6\left(1 - be^{-aT}\right)^2}. \tag{4.15}$$

For the large value of $t, \frac{\sigma}{\theta} \to 0$ implying that the model approaches isotropy at late time. We observe that the pressure of shear viscosity accelerates the process of isotropization.

5 Solution in polynomial form

Let $L(R) = \frac{1}{2a\sqrt{R+b}}$, where a and b are constant.

On integration, equation (3.6), we get

$$R = a^2T^2 - b \text{ where } t + t_0 = T. \tag{5.1}$$

In this case, the expansion for the proper energy density (ρ), the string tension (λ), the cosmological constant (Λ) and particle density (ρ_p) are given by

$$\rho = \frac{-4a^2b - 2 + 12\varepsilon_1 a^4 T^2}{\left(a^2T^2 - b\right)^2} - \frac{k_1}{2\left(a^2T^2 - b\right)^6} + \frac{6\varepsilon_0 a^2 T}{a^2T^2 - b} - \frac{K^2}{\left(a^2T^2 - b\right)^4}, \tag{5.2}$$

$$\lambda = \frac{-K^2}{\left(a^2T^2 - b\right)^4}, \tag{5.3}$$

$$\Lambda = \frac{a^4T^2 - 4a^2b - 1 - 12\varepsilon_1 a^4T^2}{\left(a^2T^2-b\right)^2} + \frac{k_1}{4\left(a^2T^2-b\right)^6} - \frac{6\varepsilon_0 a^2T}{a^2T^2-b} + \frac{K^2}{2\left(a^2T^2-b\right)}, \tag{5.4}$$

$$\rho_p = \frac{-4a^2b - 2 + 12\varepsilon_1 a^4T^2}{\left(a^2T^2-b\right)^2} - \frac{k_1}{2\left(a^2T^2-b\right)^6} + \frac{6\varepsilon_0 a^2T}{\left(a^2T^2-b\right)}. \tag{5.5}$$

The rate of expansion in the direction of x, y and z are given by

$$H_x = \frac{\dot{A}}{A} = \frac{2a^2T}{a^2T^2-b}, \tag{5.6}$$

$$H_y = \frac{\dot{B}}{B} = \frac{2a^2T}{a^2T^2-b} - \frac{k_1}{2\left(a^2T^2-b\right)^3}, \tag{5.7}$$

$$H_z = \frac{\dot{C}}{C} = \frac{2a^2T}{a^2T^2-b} + \frac{k_1}{2\left(a^2T^2-b\right)^3}. \tag{5.8}$$

Expansion θ, shear σ^2, deceleration parameter q, spatial volume V, bulk viscosity ε and anisotropy parameter $\bar{A}$ of the model take the form

$$\theta = \frac{6a^2T}{a^2T^2-b}, \tag{5.9}$$

$$\sigma^2 = \frac{1}{4}\frac{k_1^2}{\left(a^2T^2-b\right)^6}, \tag{5.10}$$

$$q = -\frac{1}{2} + \frac{b}{2a^2T^2}, \tag{5.11}$$

$$V = \left(a^2T^2-b\right)^3, \tag{5.12}$$

$$\varepsilon = \varepsilon_0 + \frac{2\varepsilon_0 a^2T}{a^2T^2-b}, \tag{5.13}$$

$$\bar{A} = \frac{k_1^2}{12\left(a^2T^2-b\right)^2 a^4T^2}. \tag{5.14}$$

We observe that model has singularity at $T = \sqrt{b}/a = T_1$ (say). The model starts with a big bang at $T = T_1$ and the expansion in the model decreases as time increases. Expansion in the model stops at $T = \infty$.

In the early stage of the universe i.e. near $T = T_1$, the scale factor of the universe had been approximately constant and had increased very slowly. Sometime later, the universe had exploded suddenly and expanded to large scale. This picture is consistent with big-bang scenario. At $T = T_1$, $q = 0$ and $q \to -1/2$ as $T \to \infty$. Thus the model represents an accelerating universe at late times. The physical quantity $\rho, \sigma, \Lambda, \varepsilon$ and ρ_p all diverge at $T = T_1$. In the limit of large times i.e. $T \to \infty, \rho, \sigma, \Lambda, \rho_p$ are negligible and $\varepsilon \to \varepsilon_0$. From (5.3), it is found that tension density λ is negative. Therefore the string phase of the universe disappears i.e. we have an anisotropic fluid of particles. The mean anisotropic parameter is decreasing function of time. For the model

$$\frac{\sigma}{\theta} = \frac{k_1}{12a^2T\left(a^2T-b\right)^2}. \tag{5.15}$$

For large value of T, $\frac{\sigma}{\theta} \to 0$ implying that the model approaches isotropy at late times. In the absence of magnetic field, the model represents an isotropic universe.

6 Conclusion

In this paper, we have studied Bianchi type-V magnetized string dust bulk viscous fluid cosmological models with variable deceleration parameter q in the context of general relativity. The Einstein's field equation have been solved exactly by considering a deceleration parameter $q =$ variable which yields time dependent scale factor. We have found that cosmological term Λ being very large at initial times relaxes to genuine cosmological constant at late times. The models are found to be compatible with the results of recent observations.

Acknowledgement. We are grateful to the Editors and anonymous referee for suggestions that have led to the improvements in the manuscript.

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