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(Dedicated to Professor G. C. Sharma on His 85^{th} Birth Anniversary Celebrations)

META-GAME THEORETIC ANALYSIS OF SOME STANDARD GAME THEORETIC PROBLEMS

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Abstract

Meta game theory is a non-quantitative reconstruction of mathematical game theory. This paper attempts to adapt meta-game theory for conflict analysis. A conflict is a situation where parties with opposing goals affect one another. A simple approach for performing meta-game analysis is adapted in this paper and illustrated on various games standard in game theory literature. The approach presented yields the desired results, although the computation required is much lesser than the standard Game theory analysis. Even a person without detailed knowledge about meta-game analysis or game theory can implement this method.
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1 Introduction

Game playing can be used to couple direct competition with the intellectual activity. The chances of a person's winning more games improve if a person has better thinking abilities and learning skills. The opportunity to test and refine his/her intellectual skills are provided by playing the game [19]. A scientific method that reconstructs classical Game- Theory on a non-quantitative basis is Meta Game theory. Its application to actual conflicts is called Meta-Game analysis or the analysis of options [7]. The Meta- Game theory has been applied to various problems, including the fall of France, an international water allocation conflict, and the Garrison Diversion Unit *(GDU)* irrigation project in North Dakota, U.S.A. [7], the Vietnam war and arms control, and the Arab-Israeli conflict using the method of Meta- Game analysis. The technique has also been applied to environmental management [12], and analysing political conflicts, particularly water resources problems [11]. The conflict analysis uses Meta-Game theory to make non-quantitative predictions instead of long mathematical calculations like Game-Theory. Conflict analysis can be employed to perform decision-making on problems that are hard to deal with quantitatively. Further, conflict analysis avoids the assumptions taken in Game Theory studies [25].

Various standard Game-Theoretical problems find applications in day-to-day life. The GameTheoretic analysis of many such problems is described in the following text.

For the Game of Chicken, Cooper et al. [3] establish that the Pareto optimal outcome is only sometimes selected in the games. If the opponent plays a dominated strategy, the equilibrium selection disturbs [3]. Similarly, other methods are discussed by Fox et al. [8], Carbon et al. [4], and Mehta et al. [16].

For the Prisoners' dilemma, Holler et al. [13] consider the real-world cases of a state choosing a dominant strategy. Using the method of general Meta-Games, Howard [14] showed that cooperation is an equilibrium of the full Meta-Game. Zhang et al. [26] study the effect of memory on the evolution of Prisoners' dilemma. The authors construct different kinds of two-layer networks. Miettinen et al. [18] experimentally investigate behaviour and beliefs in a sequential prisoners' dilemma. Proto et al. [20] used a repeated Prisoners' dilemma. The role of attention and memory is used to show that social interactions are likely to be mediated by cognitive skills in heterogeneous groups.

The Stag-Hunt was invented by philosopher Jean-Jacques Rousseau [32] in his discourses on inequality. Boudreau et al. [1] study a three-party game of conflict. They study the potential alliance formation introducing the concept of Stag-Hunt alliance. They further use this concept as a novel solution to the alliance formation puzzle in contests. Girtz et al. [9] establish that risk-averse individuals tend to make riskier choices less often than risk-loving individuals do in a series of economic games. Guarin et al. [10] study whether the co-worker's gender affects coordination on the mutually beneficial outcome in a socially risky environment. Marsh et al. [1] conduct the first ever study to influence human cooperation by means of positive imagery. Riedl [21] perform an experiment with human subjects. They study how free neighborhood choice affects coordination. Luo et al. [15] study divide-and-conquer strategy with the Stag-Hunt Game to model a terrorism and counter terrorism environment. The authors establish the universal characteristics of cooperative dynamics in different scenarios for the N-person Stag-Hunt Game. Dong et al. [6] propose a memory bases Stag-Hunt Game. The work is concerned with the study of evolutionary games with memory effect. Capraro et al. [5] study two sets of experimental data ($N = 523$) to investigate Stag-Hunt cooperation. The authors find that the cooperation is dependent on efficiency rather than moral Preferences. Belloc et al. [2] establish that intuition and deliberation play a significant role in strategic situations that entail social coordination.

A simple approach for performing Meta-Game analysis is adapted in this paper and illustrated on various games popular in Game-Theory literature. The approach presented yields the desired results, although the computation required is much lesser than the standard Game-Theory analysis. This method can be implemented even by a person without detailed knowledge about Meta-Game analysis or Game-Theory.

2 Methodology

The information in the problem and the payoff matrices are transformed into binary to construct Preference vectors. The Preference vectors contain the possible outcomes of the problem in descending order of the players' Preferences. The equally preferred outcomes are denoted by placing a bridge on the top. The Preference vectors can be cross-checked logically. The Preference vectors are transformed into decimal form, called decimalized Preference vectors, by multiplying the entry in the upper row by 2^0 and in the lower row by 2^1. It is checked whether a player can improve his/her position while keeping the other player's binary values fixed. This is termed as *UI*. If an outcome has no *UI*, *r* is written on top of the column and u otherwise. Stability analysis is conducted for individual players and amongst the players to obtain an outcome from which no player wants to deviate. The analysis is done for various problems which were earlier solved using Game-Theory analysis only. The results obtained using the (extension of) Meta-game theoretic analysis is consistent with those obtained using GameTheoretic analysis. The significant advantage of using (extension of) Meta-game theoretic analysis over Game-Theoretic analysis is less rigorous calculations.

***Table* 2.1:** The algorithm to perform stability analysis [7]

1. Model the conflict
   (a) for a particular point in time,
   (b) as a game with players and options
   (c) create a meaningful ordering of options.
2. Construct the tableau for the conflict
   (a) Order outcomes by Preferences for each player
   (b) and list UI under each outcome.
3. Perform the stability analysis
   (a) and mark as rational (r) all outcomes with no UI
   (b) for each successive outcome, determine if it is "reasonable" for a player to improve. If it is reasonable, mark the outcome as unstable (u), or if not reasonable to improve, mark it as stable (s),
   (c) if an outcome is unstable for two or more players, check for "stability by simultaneity,"
   (d) If an outcome is stable for all players, it is an equilibrium. All other outcomes are not equilibria.
4. Return to step 1) if necessary.

3 Stability Analysis

A stable outcome is one from which no player wants to deviate. The stability can be for a player or all the players. Write 'r' (rational) above every outcome that does not have a UI, as the outcomes with no UI are stable. (a) Suppose player A has a UI and player B does not have a UI from A 's UI. In that case, that outcome is unstable for A. (b) If player A has a UI and player B has a UI from A 's UI then (i) if A prefers that outcome to B's outcome, then it is unstable. (ii) It is stable if A does not prefer that outcome to B's.

4 Game of Chicken

In the Game of Chicken, two drivers must pass a single road. If they keep driving straight, they will collide, so at least one must swerve to avoid a head-on collision. If a person swerves and the other does not (i.e., goes straight), the one who swerves is called a chicken (loser). The relative payoffs in the different scenarios that can emerge in this interaction are depicted in Table 4.1.

***Table* 4.1:** The payoff matrix

		Player B	
		Swerve	Straight
Player A	Swerve	0,0	$-1,+1$
	Straight	$+1,-1$	$-1000,-1000$

Preference vector for Player A and Player B

Let Swerve correspond to 1 and straight correspond to 0 .

For player A, in Table 4.2 (a) the first Preference vector is $(1,0)$, which means player A goes straight, and player B swerves, (b) the second Preference vector is $(0,0)$, which means both player A and player B swerve (c) the third Preference vector is $(0,1)$ which means player A swerves and player B goes straight (d) the fourth Preference vector is $(1,1)$ which means both Player A goes straight and Player B goes straight.

***Table* 4.2:** The Preference vector for player A

Outcomes					
Player A	1	0	0	1	$2^0=1$
Player B	0	0	1	1	$2^1=2$
	1	0	2	3	.

In Table 4.3, for player B (a) the first Preference vector is $(0,1)$, which means player A swerves and player B goes straight, (b) the second Preference vector is $(0,0)$, which means both players A and player B swerve (c) the third Preference vector is $(1,0)$ which means player A goes straight and player B swerves (d) the fourth Preference vector is $(1,1)$ which means both players A and B go straight.

***Table* 4.3:** The Preference vector for player B

Outcomes					
Player A	0	0	1	1	$2^0=1$
Player B	1	0	0	1	$2^1=2$
	2	0	1	3	

Decimalized Preference vector for Player A and Player B

In Tables 4.5 and 4.6, decimalized Preference vectors are obtained by multiplying the first row of the Preference vectors of A and B by 2^0 and the second row by 2^1.

***Table* 4.4:** Decimalized Preference vector for player A

Player A	1	0	2	3

***Table* 4.5:** Decimalized Preference vector for player B

Player B	2	0	1	3

Stability Analysis for Player A and Player B
The outcomes of player B are kept fixed in Table 4.3 and then checked for UI in the outcomes for player A. Player A has UI from column two to column one and column four to three (Table 4.6).

***Table* 4.6:** UI for player A

Player A	1	0	2	3
		1		2

The outcomes of player A are kept fixed in Table 4.3. The authors check for UI in the outcomes for player B. Player B has UI from column two to column one and four to three (Table 4.7).

***Table* 4.7:** UI for player B

Player B	2	0	1	3
		2		1

Player A, (Table 4.8) has UI from column two to column one, and player B (Table 4.9) has no UI from column three, which means player A can improve from column two to column one. Thus, this is an unstable outcome. Similarly, player A, (Table 4.8) has UI from column four to column three, and player B (Table 4.9) has no UI from column one, which means player A can improve from column four to column three. Thus, this too is an unstable outcome.

***Table* 4.8:** Stability analysis for player

	E		E	
	r	u	r	u
Player A	1	0	2	3
		1		2

For player B, from column two to column one (Table 4.9) and player A (Table 4.8) has no UI from column three, which means player B can improve from column two to column one. Thus, this is an unstable outcome. Player B, (Table 4.9) has UI from column four to column three, and player A (Table 4.8) has no UI from column one, which means player B can improve from column four to column three. Thus, this too is an unstable outcome.

***Table* 4.9:** Stability analysis vector for player B

	E		E	
	r	u	r	u
Player B	2	0	1	3
		2		1

Player A has a UI from column two to column one, and player B has no UI from column three. Hence, column two is unstable for player A. Player B has UI from column two to column one, and player A has no UI from column three. Hence, column two is unstable for B (Table 4.10).

Solutions
There are two stable equilibrium solutions Straight-Straight and Swerve-Swerve. When analyzed using Game-Theoretic techniques, the Nash equilibria obtained for the game are identical [23].

5 Prisoners' dilemma

There are two accused of a crime, and they are not allowed to communicate. The options left to the two accused i.e., A and B are (i) If both defect, a two-year prison sentence is awarded to both (ii) If A defects and B cooperates, A is released, and B gets a three-year prison sentence (iii) If A cooperates and B defects,

A will get three years in prison and B will be released (iv) If A and B both cooperate, they both get one-year prison sentence (Table 5.1).

***Table* 5.1:** The payoff matrix

		Player B	
		Cooperate	Defect
Player A	Cooperate	$-1, -1$	$-3, 0$
	Defect	$0, -3$	$-2, -2$

Preference vector for Player A and Player B

Let Cooperate correspond to 1 and Defect correspond to 0 .

In Table 5.2, for player A (a) the first Preference vector is $(0, 1)$, which means player A defects and player B cooperates, (b) the second Preference vector is $(1, 1)$, which means player A cooperates and player B cooperates (c) the third Preference vector is $(0, 0)$ which means player A defects and player B defects (d) the fourth Preference vector is $(1, 0)$ which means player A cooperates and player B defects.

***Table* 5.2:** The Preference vector for player A

Outcomes					
Player A	0	1	0	1	$2^0 = 1$
Player B	1	1	0	0	$2^1 = 2$
	2	3	0	1	

In Table 5.3, for player A (a) the first Preference vector is $(1, 0)$, which means player A cooperates and player B defects, (b) the second Preference vector is $(1, 1)$, which means player A cooperates and player B cooperates (c) the third Preference vector is $(0, 0)$ which means player A defects and player B defects (d) the fourth Preference vector is $(0, 1)$ which means player A defects and player B cooperates.

***Table* 5.3:** The Preference vector for player B

Outcomes					
Player A	1	1	0	0	$2^0 = 1$
Player B	0	1	0	1	$2^1 = 2$
	1	3	0	2	

Decimalized Preference vector for Player $\boldsymbol{A}$ and Player B

In Tables 5.4 and 5.5, the authors obtain decimalized Preference vectors by multiplying the first row by 2^0 and the second row by 2^1.

***Table* 5.4:** Decimalized Preference vector for player A

Player A	2	3	0	1

***Table* 5.5:** Decimalized Preference vector for player B

Player B	1	3	0	2

Stability Analysis for Player A and Player B

The outcomes of player B are kept fixed in Table 5.2, then the authors check for UI in the outcomes for player A. Player A has UI from two to one and four to three (Table 5.6).

***Table* 5.6:** UI for player A

Player A	2	3	0	1
		2		0

The outcomes of player A are kept fixed in Table 5.3. The authors check for UI in the outcomes for player B. Player B has UI from column two to column one and column four to column three (Table 5.7).

***Table* 5.7:** UI for player B

Player B	1	3	0	2
		1		0

In Table 5.8, Player A has UI from column two to one, and player B has UI from column four to three as column two is more preferred over column three. Therefore, it is a stable outcome. Player A has UI from column four to three, and player B has no UI from column three, so this is an unstable outcome.

***Table* 5.8:** Stability analysis for player A

	E		E	
	r	s	r	u
Player A	2	3	0	1
		2		0

Player B has UI from column two to column one, player A has UI from column four to column three, and player B prefers column two to column three. Therefore, it is a stable outcome. Player B has UI from column four to three, and player A has no UI from column three. Thus, it is an unstable outcome (Table 5.9).

***Table* 5.9:** Stability analysis for player B

	E		E	
	r	s	r	u
Player B	1	3	0	2
		1		0

Solution

The solution to the Prisoners' dilemma is (i) column one and one, which means both players defect, (ii) column three and three, which also means both players defect, (iii) column two and two, which means both the players cooperate.

When analyzed using Game-Theoretic techniques, the same Nash equilibria are obtained [24].

6 Stag-Hunt Game

Imagine two hunters, they can hunt a stag or a hare. They can independently hunt a hare. For hunting a stag, they need each other's help. The (Stag, Stag) is the pareto optimal outcome. But in experimental games, people choose (Hare, Hare).

***Table* 6.1:** The payoff matrix

Player A	Player B		
		Stag	Hare
	Stag	10,10	1,8
	Hare	8,1	5,5

Preference Vector for Player A and Player B
Let Stag correspond to 1 and Hare correspond to 0.

In Table 6.2, for player A (a) the first Preference vector is $(1, 1)$ which means both player A and player B want to hunt a stag, (b) the second Preference vector is $(0, 1)$ which means player A wants to hunt a Hare and player B wants to hunt a Stag (c) the third Preference vector is $(0, 0)$ which means both player A and player B want to hunt a hare (d) the fourth Preference vector is $(1, 0)$ which means player A wants to hunt a stag and player B wants to hunt a hare.

***Table* 6.2:** The Preference vector for player A

Outcomes					
Player A	1	0	0	1	$2^0 = 1$
Player B	1	1	0	0	$2^1 = 2$
	3	2	0	1	

In Table 6.3, for player A (a) the first Preference vector is $(1, 1)$ which means player A and player B wants to hunt a stag, (b) the second Preference vector is $(1, 0)$ which means player A wants to hunt a stag and player B wants to hunt a hare (c) the third Preference vector is $(0, 0)$ which means player A and player B want to hunt a hare (d) the fourth Preference vector is $(0, 1)$ which means player A wants to hunt a hare and player B wants to hunt a stag.

***Table* 6.3:** The Preference vector for player B

Outcomes					
Player A	1	1	0	0	$2^0 = 1$
Player B	1	0	0	1	$2^1 = 2$
	3	1	0	2	

Decimalized Preference vector for Player *A* and Player B
In Table 6.4 and 6.5, the authors obtain decimalized Preference vectors by multiplying first row by 2^0 and second row by 2^1.

***Table* 6.4:** Decimalized Preference vector for player A

Player A	3	2	0	1

***Table* 6.5:** Decimalized Preference vector for player B

Player B	3	1	0	2

Stability Analysis for Player A and Player B
The outcomes of player B are kept fixed in Table 6.2, then the authors check for UI in the outcomes for player A. Player A has UI from column two to column one and from column four to column three.

***Table* 6.6:** UI for player A

Player A	3	2	0	1
		3		0

The outcomes of player A are kept fixed in Table 6.3, then the authors check for UI in the outcomes for player B. Player B has UI from column two to column one and from four to three.

***Table* 6.7:** UI for player B

Player B	3	1	0	2
		3		0

The player A has UI from column two to column one and player B has no UI from column one, so it is an unstable outcome. The player A has UI from column four to column three and player B has no UI from column three, so it is an unstable outcome.

***Table* 6.8:** Stability Analysis for player A

	E		E	
		u		u
Player A	3	2	0	1
		3		0

The player B has UI from column two to column one to 3 and player A has no UI from column one, so it is an unstable outcome. The player B has UI from column four to column three and player A has no UI from column three, so it is an unstable outcome.

***Table* 6.9:** Stability Analysis for player B

	E		E	
		u		u
Player B	3	1	0	2
		3		0

Solutions

There are two stable equilibrium values, Stag- Stag and Hare- Hare.

When analyzed using Game-Theoretic techniques, the game yields exactly same values as the Nash equilibrium values [22].

7 Conclusion

The authors attempted to do a novel analysis of standard Game-Theoretic problems using the MetaGame analysis techniques. The Meta-Game analysis has many advantages. It includes all the information about the conflict, is easy to do by hand, and can be used for hyper games and very complex conflicts. The results obtained show the stability analysis for various standard Game Theoretic problems. These solutions obtained in the three cases viz., Game of Chicken, Prisoners' dilemma, and Stag-Hunt Game are consistent with the Nash equilibrium values obtained from the Game-Theoretic analysis [22-24]. Thus, Meta-game theory can replace Game-Theory in these situations as it includes no tedious mathematical calculations.

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