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*(Dedicated to Professor G. C. Sharma on His $85^{th}$ Birth Anniversary Celebrations)*

# FUZZY SEMI-SEPARATION AXIOMS AND FUZZY SEMI-CONNECTEDNESS IN FUZYY BICLOSURE SPACES

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**Abstract**

The purpose of this paper is to introduce the notion of fuzzy semi-separation axioms and fuzzy semi-connectedness in fuzzy biclosure spaces. Further we investigate their characterizations and find relations with other already existing definitions. We generalize the results of semi-connectedness in fuzzy setting. Here we follow the definition of closure operator given by Birkhoff [5].
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## 1 Introduction

The notion of fuzzy semi-open sets as well as fuzzy semi-closed sets plays a very significant role in fuzzy topology. The concept of semi-open sets were introduced by Levine [8]. Azad [1] introduced the concept of fuzzy semi-open sets and fuzzy semi-separation axioms in 1981.

We introduce and study the concept of fuzzy semi-separation axioms in different way. Here we introduce fuzzy semi-connectedness in fuzzy biclosure spaces and also study their basic properties.

## 2 Preliminaries

The concept of fuzzy set was introduced by Zadeh (1965) in his classical paper [18]. A fuzzy set '$A$' in a non-empty set $X$ is a mapping from $X$ to $[0, 1]$.

A fuzzy point $x_r$ is a fuzzy set in $X$ taking value $r \in (0, 1)$ at $x$ and zero otherwise. A fuzzy point $x_r$ is said to belong to a fuzzy set $A$ i.e. $x_r \in A$ iff $r \leq A(x)$ [13]. A fuzzy singleton $x_r$ is a fuzzy set in $X$ taking value $r \in (0, 1]$ at $x$ and 0 elsewhere. A non-empty set $X$ together with two fuzzy topologies $\tau_1,\ \tau_2$ is called fuzzy bitopological space. It is denoted by $(X, \tau_1, \tau_2)$.

A fuzzy point $x_r$ is said to be quasi-coincident with $A$ denoted by $x_r qA$ iff $r + A(x) > 1$. A fuzzy set $A$ is said to be quasi-coincident with another fuzzy set $B$ denoted by $AqB$ iff $\exists\ x \in X$ such that $A(x) + B(x) > 1$ similarly we say that $A\bar{q}B$ iff $A \subseteq coB$ i.e. $A(x) + B(x) \leq 1$. Obviously, if $A$ and $B$ are quasi-coincident at $x$ both $A(x)$ and $B(x)$ are not zero and here $A$ and $B$ intersect at $x$. Here we follow the Lowen's definition [9] of fuzzy topology as a family $\tau$ of fuzzy sets of a non-empty $X$ is said to form a fuzzy topology on $X$ if it is preserved under Arbitrary union, finite intersection and contains all constant fuzzy sets. The members of $\tau$ are called fuzzy open sets and their complements are called fuzzy closed sets. All the definitions, results and terminology used here is taken from Ming and Ming [13]. In this paper, we introduce the concept of semi-separation axioms and semi-connectedness in fuzzy biclosure spaces. We use abbreviation fbcs for fuzzy biclosure space.

The concept of closure operator was given by Čech [7] and Birkhoff [5] separately. A lot of works has been done on Čech closure operator. We are using here Birkhoff closure operator. Now we mention the definitions of closure operator as:

**Definition 2.1** ([7]). *(Čech closure operator)*

*An operator $C : 2^X \to 2^X$ is called closure operator if it satisfies the following axioms:*
1. $C(\phi) = \phi$,
2. $A \subseteq C(A),\ \forall A \in 2^X$,
3. $C(A \cup B) = C(A) \cup C(B),\ \forall A, B \in 2^X$.

*Here $C$ is called a closure operator and $(X, C)$ is known as Čech closure space.*

**Definition 2.2** ([5])**.** *(Birkhoff closure operator)*
*An operator $C : 2^X \to 2^X$ is called closure operator on a non-empty set $X$ if it satisfies the following axioms:*
1. $C(\phi) = \phi$,
2. $A \subseteq C(A),\ \forall A \in 2^X$,
3. $A \subseteq B \Rightarrow C(A) \subseteq C(B), \forall A, B \in 2^X$,
4. $C(C(A)) = C(A),\ \forall A \in 2^X$,

*Here $C$ is called Birkhoff closure operator and the pair $(X, C)$ is called a closure space.*

**Definition 2.3** ([10])**.** *A fuzzy closure operator on a set $X$ is a function $c : I^X \to I^X$ satisfying the following three axioms:*
1. $c(\phi) = \phi$,
2. $c(A) \subset A,\ \forall A \in I^X$,
3. $c(A \cup B) = c(A) \cup c(B) <\ \forall A, B \in I^X$.

*The pair $(X, c)$ is a fuzzy closure space (in short fcs).*

**Definition 2.4** ([16])**.** *A fuzzy closure operator c on a set $X$ is a function $c : I^X \to I^X$ satisfying following the axioms:*
1. $c(\underline{\alpha}) = \alpha; \alpha \in [0, 1]$ ,
2. $A \subseteq c(A),\ \forall A \in I^X$,
3. $A \subseteq B \Rightarrow c(A) \subseteq c(B),\ \forall A, B \in I^X$,
4. $c(c(A)) = c(A),\ \forall A \in I^X$,

*Here $(X, c)$ is known as fuzzy closure space.*

**Definition 2.5** ([17])**.** *A function $c_i : I^X \to I^X (i = 1, 2)$ is called a fuzzy biclosure operator on $X$ if the following postulates are satisfied:*
1. $c_i(\underline{\alpha}) = \underline{\alpha}, \alpha \in [0, 1]$ ,
2. $A \subseteq c_i(A), \forall A \in I^X$,
3. $A \subseteq B \Rightarrow c_i(A) \subseteq c_i(B), \forall A, B \in I^X$,
4. $c_i(c_i(A)) = c_i(A),\ \forall A \in I^X$.

*Then $(X, c_1, c_2)$ is called a fuzzy biclosure spaces.*

**Definition 2.6.** *Let $(X, c_1, c_2)$ be a fuzzy biclosure space. If the closure operator satisfies the condition $c_i(A \cup B) = c_i(A) \cup c_i(B)$ then it is called additive property.*

*The concept of semi-open sets was introduced by Levine [8] as "A set $R$ in a topological space $X$ will be termed semi-open (simply written as s.o.) iff there exist an open set $P$ such that $P \subset R \subset c(P)$ where $c$ denote the closure operator in $X$ [8]." Also we know that a fuzzy set $R$ is said to be fuzzy semi-open iff $R \subseteq cl$ (int $R$ ). The complement of fuzzy semi-open set is called fuzzy semi-closed set. So, a fuzzy set $A$ is said to be fuzzy semi-closed iff int$(cl(R) \subseteq R$. Let $\{R_\alpha\}\ \alpha \in \Lambda$ be a collection of semi-open set in $X$ then $\cup_{\alpha \in \Lambda} R_\alpha$ is semiopen.*

**Definition 2.7.** *Let $(X, c_1, c_2)$ be a fuzzy biclosure space and the semi closure of a fuzzy set $A$ in $X$ is defined as:*

$c_i$ *- scl*$(A) = \cap\{B : B$ *is fuzzy closed set and* $B \supset A\}$

*Similarly the semi interior of a fuzzy set $A$ in $X$ is defined as:*

$c_i$ *- sint*$(A) = \cap\{B : B$ *is fuzzy open set and* $B \subset A\}$.

## 3 Fuzzy semi-separation axioms in fuzzy biclosure space

In this section, we define the concepts of fuzzy semi-separation axioms in fuzzy biclosure spaces:

**Definition 3.1.** *A fuzzy biclosure space $(X, c_1, c_2)$ is said to be*
1. Fuzzy Pairwise Semi $T_0$ if $\forall x, y \in X, x \neq y\ \exists$ a fuzzy semi-open set $U$ such that $U(x) \neq U(y)$,
2. Fuzzy Pairwise Semi $T_1$ if $\forall x, y \in X, x \neq y\ \exists$ fuzzy semi-open sets $U, V$ in $X$ such that $U(x) = 1, U(y) = 0$ and $V(x) = 0, V(y) = 1$,

3. Fuzzy Pairwise Weakly semi $T_1$ if $\exists$ a $c_1$-fuzzy semi-open set or a $c_2$-fuzzy semi-open set $U$ such that $U(x) = 1, U(y) = 0$,
4. Fuzzy Pairwise Semi $T_2$ if for every pair of distinct fuzzy points $x_r, y_s$ in $X$ their exists fuzzy semi-open sets $U$ and $V$ such that $x_r \in U, y_s \in V$ and $U \cap V = \emptyset$,
5. Fuzzy Pairwise Semi-Regular if for each fuzzy point $x_r$ and each fuzzy closed set $F$ such that $x_r \bar{q} F$ $\exists$ fuzzy semi-open sets $U$ and $V$ such that $x_r \subseteq U, F \subseteq V$ and $U \bar{q} V$,
6. Fuzzy Pairwise Semi-Normal if for every pair of fuzzy closed set $F_1$ and $F_2$ such that $F_1 \bar{q} F_2$, $\exists$ fuzzy semi-open sets $U, V$ such that $F_1 \subseteq U$ and $F_2 \subseteq V$ and $U \bar{q} V$.

*Clearly fuzzy semi $T_2 \Rightarrow$ fuzzy semi $T_1 \Rightarrow$ fuzzy semi $T_0$ but not conversely.*

**Theorem 3.1.** *A fbcs $(X, c_1, c_2)$ is fuzzy pairwise semi $T_0$ if either $(X, c_1)$ or $(X, c_2)$ is semi $T_0$.*

*Proof.* It is given that $(X, c_1)$ or $(X, c_2)$ is fuzzy semi $T_0$. If $(X, c_1)$ is semi $T_0$ then we have $x, y \in X, x \neq y$ $\exists$ *a* $c_1$-fuzzy semi-open set $U$ such that $U(x) \neq U(y)$. Now if $(X, c_2)$ is fuzzy semi $T_0$ then $\exists$ *a* $c_2$-fuzzy semi-open set $V$ such that $V(x) \neq V(y)$. Thus for $x, y \in X, x \neq y$ $\exists$ a fuzzy semi-open set $U$ in $c_1$ or $c_2$ such that $U(x) \neq U(y)$. Hence $(X, c_1, c_2)$ is fuzzy pairwise semi $T_0$. $\square$

**Theorem 3.2.** *A fbcs $(X, c_1, c_2)$ is fuzzy pairwise semi $T$. iff $(X, c_1)$ and $(X, c_2)$ are fuzzy semi $T_1$.*

*Proof.* First let the fbcs $(X, c_1, c_2)$ is fuzzy pairwise semi $T_1$. then for $x, y \in X, x \neq y$ $\exists$ $U_1 \in c_1$ and $V_1 \in c_1$ such that $U_1(x) = 1, V_1(y) = 0$ and $U_1(x) = 0, V_1(y) = 1$. If we take $x, y \in X$ then $U_2, V_2 \in X$ such that $U_2(x) = 0, U_2(y) = 1$ and $V_2(x) = 1, V_2(y) = 0$ .Therefore $x, y \in X, x \neq y$ we have $U_1, U_2 \in c_2$ such that $U_1(x) = 1, U_1(y) = 0$ and $U_2(x) = 0, U_2(y) = 1$ implies that $(X, c_1)$ is fuzzy semi $T_1$. Similarly $(X, c_2)$ is fuzzy semi $T_1$.

Conversely suppose that $(X, c_1)$ and $(X, c_2)$ are fuzzy semi $T_1$ then $(X, c_1)$ is fuzzy semi $T_1$ for $x, y \in X, x \neq y$ $\exists$ a $c_1$-fuzzy semi-open set $U$ such that $U(x) = 1, U(y) = 0$ and since $(X, c_2)$ is fuzzy semi $T_1$ for $x, y \in X, x \neq y$ $\exists$ a $c_2$-fuzzy semi-open set $V$ such that $V(x) = 0, V(y) = 1$. Then for $x, y \in X, x \neq y$ $\exists$ a $c_1$-fuzzy semi-open set $U$ and a $c_2$-fuzzy semi-open set $V$ such that $U(x) = 1, V(y) = 0$ and $V(x) = 0, U(y) = 1$ implies that $(X, c_1, c_2)$ is fuzzy pairwise semi $T_1$. $\square$

**Theorem 3.3.** *A fbcs $(X, c_1, c_2)$ is fuzzy weakly pairwise semi $T_1$ iff $c_1$-$scl\{x\} \cap c_2$-$scl\{x\} = \{x\}$ for every $x \in X$.*

*Proof.* Let $(X, c_1, c_2)$ be a fuzzy pairwise semi $T_1$. Let $x \in X$ and choose any $y \neq x$, then $\exists$ a $c_1$-fuzzy semi-open set or a $c_2$-fuzzy semi-open set $U$ such that $U(y) = 1, U(x) = 0$.

First let us consider $U$ as a $c_1$-fuzzy semi-open set then clearly $coU$ is $c_1$-fuzzy semi-closed set such that $coU(x) = 1,\ coU(y) = 0$. Thus $c_1$ - $scl\{x\} \supseteq \{x\}$ for $i = 1, 2$ where $\{x\}$ is semi-closed.

Hence $\{c_1\text{-}scl\{x\}\}(x) = 1$ for $i = 1, 2$ which implies that $(c_1\text{-}scl\{x\} \cap c_2\text{-}scl\{x\})(x) = 1$. let $c_1\text{-}scl\{x\} = \cap \left\{F \in I^X : F \supseteq \{x\}\right\}$ and $F$ is $c_1$-fuzzy semi-closed set. Let $\left\{F \in I^X : F \supseteq \{x\}\right\}$ be defined by $\mathcal{F}$. Then $coU \in \mathcal{F}$ and since $coU(y) = 0$, we have $(c_1\text{-}scl\{x\})(y) = 0$ which implies that $(c_1\text{-}scl\{x\} \cap c_2\text{-}scl\{x\})(y) = 0$. Let $(X, c_1, c_2)$ be a fuzzy pairwise semi $T_1$. Hence $(X, c_1, c_2)$ is fuzzy weakly pairwise semi $T_1$.

Conversely let $x, y \in X$ $x \neq y$ then $(c_1\text{-}scl\{x\} \cap c_2\text{-}scl\{x\})(x) = 1$ and $(c_1\text{-}scl\{x\} \cap c_2\text{-}scl\{x\})(y) = 0$.

Taking complement of both the sides we get

$[(X - c_1\text{-}scl\{x\}) \cup (X - c_2\text{-}scl\{x\})](x) = 0, [(X - c_1\text{-}scl\{x\}) \cup (X - c_2\text{-}scl\{x\})](y) = 1$ also $[(X - c_1 - scl\{x\})(y) = 1$ or $(X - c_2\text{-}scl\{x\})(y) = 1$.

Let us suppose that $(X - c_1\text{-}scl\{x\})(y) = 1$, hence $X - c_1\text{-}scl\{x\}$ and $X - c_2\text{-}scl\{x\}$ are fuzzy open sets such that $(X - c_1\text{-}scl\{x\})(x) = 0$ and $(X - c_2\text{-}scl\{x\})(x) = 0$ and

$(X - c_1\text{-}scl\{x\})(y) = 1$ or $(X - c_2\text{-}scl\{x\})(y) = 1$. Thus we have fuzzy semi-open set in $c_1$ viz. $X - c_1\text{-}scl\{x\}$ such that $(X - c_1\text{-}scl\{x\})(x) = 0$ and $(X - c_1\text{-}scl\{x\})(y) = 1$ which implies that $(X, c_1, c_2)$ is fuzzy weakly pairwise semi $T_1$. $\square$

**Theorem 3.4.** *A fuzzy biclosure space $(X, c_1, c_2)$ is fuzzy pairwise semi $T_2$ iff the diagonal set $\Delta_X$ is fuzzy semi-closed in $(X \times X, c_1 \times c_2)$.*

*Proof.* Let $(X, c_1, c_2)$ be fuzzy pairwise semi $T_2$. We have to show that $\Delta_X$ is fuzzy semiclosed in $X \times X$. In other side, we have to show that $X \times X - \Delta_X$ is fuzzy open. Let $x_r, y_r$ be any two distinct fuzzy points in $X$ because $(X, c_1, c_2)$ is fuzzy pairwise semi $T_2$, $\exists$ *a* $c_1$-fuzzy semi-open set $U$ and a $c_2$-fuzzy semi-open set $V$

such that $x_r \in U, y_r \in V$ and $U \cap V = \emptyset$. Consider the basic fuzzy semi-open set $U \times V$ in $(X \times X, c_1 \times c_2)$ then $(x_r, y_r) \in U \times V \subseteq X \times X - \Delta_X$ which implies that $X \times X - \Delta_X$ is fuzzy semi-open in $c_1 \times c_2$ i.e. $\Delta_X$ is fuzzy semi-closed.

Conversely, let $\Delta_X$ be fuzzy semi-closed in $(X \times X, c_1 \times c_2)$ i.e. $X \times X - \Delta_X$ be fuzzy open in $X \times X$. We show that $(X, c_1, c_2)$ is fuzzy pairwise semi $T_2$. Let $x_r, y_r \in X, x \neq y$.

Let $r \leq s$ and consider $(x, y)_S$ then $(x, y)_S$ is a fuzzy point in. $X \times X - \Delta_X$ therefore $\exists$ a basic fuzzy semi-open set $U \times V$ in $c_1 \times c_2$ such that $(x, y)_S \in U \times V \subseteq X \times X - \Delta_X$.

Here $U \in c_1$ and $V \in c_2$ where $U = \bigcup_{i \in A_1} U_i$ and $V = \bigcup_{j \in A_2} V_j$ where $U_i \in c_1$ and $V_j \in c_2$ thus $(x, y)_s \in \cup_i U_i \times \cup_j V_j \subseteq X \times X - \Delta_X$

This implies that $\exists\ i,\ j$ such that $(x, y)_s \epsilon U_i \times V_j \subseteq X \times X - \Delta_X$. Thus we may say that $x_r \in U_i$ and $y_s \in V_j$ and $U_i \cap V_j = \emptyset$ which implies that $(X, c_1, c_2)$ is fuzzy pairwise semi $T_2$. □

**Theorem 3.5.** *The fbcs $(X, c_1, c_2)$ is fuzzy pairwise semi-regular iff for each $c_i$-fuzzy open set $F$ $\exists$ a $c_i$-fuzzy semi-open set $U$ such that $x_r \subseteq U \subseteq c_1\text{-}sclU\ \bar{q}F$.*

*Proof.* Let $(X, c_1, c_2)$ be fuzzy pairwise semi-regular. Then for every $c_i$-fuzzy open set $F$ and each fuzzy point $x_r$ such that $x_r \subseteq F$, $\exists$ *a* $c_1$-fuzzy semi-open set $U$ and $c_2$-fuzzy semi-open set $V$ such that $x_r \subseteq U, coF \subseteq V$ and $U \subseteq coV$ then $x_r \subseteq U \subseteq coV \subseteq F$. Since $coV$ is a $c_i$-fuzzy semi-closed set such that $U\bar{q}V$ then $x_r \subseteq U$ and $sclU\bar{q}F$. This can also be written as $x_r \subseteq U \subseteq c_1\text{-}sclU\bar{q}F$. Conversely, let $x_r$ be a fuzzy point and $F$ be a $c_1$-fuzzy semi-closed set such that $x_r\bar{q}F$. Thus $\exists$ a $c_i$-fuzzy semi-open set $U$ such that $x_r \subseteq U \subseteq c_i\text{-}sclU\bar{q}F$. Consider fuzzy sets $U_1$ and $V_1$ where $U_1 = U$ and $V_1 = 1 - c_i\text{-}scl\ U$ clearly $U_1$ is $c_1$-fuzzy semi-open set and $V_1$ is $c_j$-fuzzy semi-open set such that $x_r \subseteq U_1, F \subseteq V_1$ and $U_1\bar{q}V_1$ since for any $z \in X, U_1(z) + V_1(z) = U(z) + 1 - c_i - sclU(z)$ which is obviously $\leq 1$. □

**Theorem 3.6.** *A fbcs $(X, c_1, c_2)$ is fuzzy pairwise semi-normal iff for any $c_i$-fuzzy semi-closed set $A$ and a $c_j$-fuzzy semi-open set $B$ such that $A \subseteq B$ $\exists$ a $c_i$-fuzzy semi-open set $U$ such that $A \subseteq U$ and $c_i\text{-}sclU \subseteq B$*

*Proof.* First let the fbcs $(X, c_1, c_2)$ be fuzzy pairwise semi-normal then for any $c_i$-fuzzy semi-closed set $A$ and a $c_j$-fuzzy semi-open set $B$ such that $A \subseteq B$ $\exists$ *a* $c_j$-fuzzy semi-open sets $U$ and a $c_i$-fuzzy semi-open set $V$ such that $A \subseteq U$ and co$B \subseteq V$ and $U\bar{q}V$ thus $A \subseteq U$, $U\bar{q}V$ and $coV \subseteq B$ or $V \subseteq$ coB. We can also write $A \subseteq U \subseteq c_1\text{-}sclU \subseteq B$.

Conversely, let $A$ be any $c_i$-fuzzy semi-closed set and $B$ be any $c_j$-fuzzy semi-closed set such that $A\bar{q}B$ then $\exists$ a $c_j$-fuzzy semi-open set $U$ such that $A \subseteq U \subseteq c_1\text{-}sclU \subseteq B$.

Consider the fuzzy sets $U_1$ and $V_1$ such that $U_1 = U$ and $V_1 = 1 - c_1\text{-}sclU$. Obviously, $U_1$ is a $c_j$-fuzzy semi-open set and $V_1$ is a $c_j$-fuzzy semi-open set such that $A \subseteq U_1, B \subseteq V_1$ and $U_1 \subseteq coV_1$ i.e. $U\bar{q}V_1$ as for any $x \in X, U_1(x) + V_1(x) = U(x) + 1 - c_1\text{-}scl(x) \leq 1$. □

## 4 Fuzzy pairwise semi-continuous maps in fuzzy biclosure space

The concepts of fuzzy biclosed maps, pairwise fuzzy bicontinuous maps and generalized fuzzy continuous maps in fbcs were introduced by Navalakhe [14, 15]. Pairwise continuity between fuzzy closure spaces was introduced by Azad [1]. Further using fuzzy semi-open sets Azad [1] introduced and studied fuzzy pairwise s-continuous mapping between fuzzy closure spaces. We introduce and study two more definitions using fuzzy semi-open sets and compare all the definitions with each other. Let $(X, c_1, c_2)$ and $(Y, c_1, c_2)$ be any two fuzzy biclosure spaces then

**Definition 4.1** ([1])**.** *A map $f : X \to Y$ is said to be fuzzy pairwise continuous if $f^{-1}(V)$ is fuzzy open in $X$ whenever $V$ is fuzzy open set in $Y$.*

**Definition 4.2** ([1])**.** *A map $f : X \to Y$ is said to be fuzzy pairwise s-continuous if $f^{-1}(V)$ is semi-open in $X$ whenever $V$ is semi-open set in $Y$.*

**Definition 4.3.** *A map $f : X \to Y$ is said to be fuzzy pairwise semi-continuous if $f^{-1}(V)$ is open in $X$ whenever $V$ is semi-open set in $Y$.*

**Definition 4.4.** *A map $f : X \to Y$ is said to be fuzzy pairwise $s^*$-continuous if $f^{-1}(V)$ is semi-open in $X$ whenever $V$ is open set in $Y$.*

Comparing them with each other we have

$$\text{Fuzzy pairwise-continuous}$$

$$\swarrow \; \Downarrow \; \searrow$$

$$\text{Fuzzy semi-continuous} \Rightarrow \text{Fuzzy s-continuous} \Leftarrow \text{Fuzzy s}^*\text{-continuous.}$$

## 5 Fuzzy semi-connectedness in fuzzy biclosure spaces

In this section, we introduce and study the concept of q-separated sets. This concept is earlier introduced by Ming-Ming in 1980.

**Definition 5.1** ([13])**.** *The fuzzy sets $A_1$ and $A_2$ in fts $(X,T)$ are said to be separated iff $\exists\, U_i \in T(i=1,2)$ : $U_i \supset A_i$ and $U_1 \cap A_2 = \emptyset = U_2 \cap A_1$.*

**Definition 5.2** ([13])**.** *Two fuzzy sets $A_1$ and $A_2$ in $fts$ $(X,T)$ are said to be Q-separated iff $\exists$ $T$ closed sets $H_i : H_i \supset A_i$ and $H_1 \cap A_2 = \emptyset = H_2 \cap A_1$.*

*It is obvious that $A_1$ and $A_2$ are Q-Separated iff $\bar{A}_1 \cap A_2 = \emptyset = \bar{A}_2 \cap A_1$*

**Definition 5.3** ([13])**.** *A fuzzy set $Y$ in $(X,c)$ is called disconnected iff there exist two non-void sets $A$ and $B$ in the subspace $Y_0$ (i.e supp $Y$) such that $A$ and $B$ are $Q$-separated and $Y = A \cup B$. A fuzzy set is called connected iff it is not disconnected.*

**Definition 5.4.** *The fuzzy sets $A$ and $B$ are said to be q-separated iff $cl(A)\bar{q}B$ and $A\bar{q}cl(B)$*

**Definition 5.5.** *The fuzzy sets $A$ and $B$ in a fbcs $(X,c_1,c_2)$ are said to be $c_i$ - q-separated (or simply q-separated) iff $c_i(A)\bar{q}B$ and $c_i(B)\bar{q}A$.*

*We introduce here the definition of semi-connectedness using $c_i - q$-separated sets in a fuzzy biclosure space:*

**Definition 5.6.** *A fuzzy set in a fuzzy closure space is said to be fuzzy semi-connected iff there doesn't exist two non-empty semi-open sets $A$ and $B$ in the subspace $Y_0$ (supp $Y$) such that $Y = A \cup B$ and $A, B$ are q-separated.*

**Theorem 5.1.** *The s-continuous onto image of a fuzzy semi-connected biclosure space which has additive property also is fuzzy semi-connected biclosure space with additive property.*

*Proof.* Let $X$ and $Y$ be two fbcs and $X$ be fuzzy semi-connected. Suppose $Y$ is not fuzzy semi-connected then $Y$ is fuzzy semi disconnected. Then $\exists$ two non-empty fuzzy sets $A$ and $B$ in the subspace $Y_0$ (supp $Y$) such that $A$ and $B$ are $q$-separated and $Y_0 = A \cup B$ (supp $Y$) or $c_i(A)\bar{q}B$ and $A\bar{q}c_i(B)$. Now $f$ is semi continuous $f^{-1}(A)$ and $f^{-1}(B)$ are subsets of $X$ therefore $X = f^{-1}(A) \cup f^{-1}(B)$ where $c_i\left(f^{-1}(A)\right)\bar{q}f^{-1}(B)$ and $f^{-1}(A)\bar{q}c_i\left(f^{-1}(B)\right)$.

We know from the additive property that $c_i(A \cup B) = c_i(A) \cup c_i(B)$. Then $f^{-1}(Y_0) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ which is a contradiction since $f^{-1}(A), f^{-1}(B)$ are q-separated. It means our assumption is wrong. Thus $Y$ is fuzzy semi-conneced biclosure space with additive property. □

**Theorem 5.2.** *Let $(X,c_1,c_2)$ be semi disconnected let $c_i \subset c_i^*(i=1,2)$ then $(X,c_1^*,c_2^*)$ is semi disconnected.*

*Proof.* Let $(X,c_1,c_2)$ be semi disconnected fbcs then $\exists$ two non-empty fuzzy semi-open sets $A$ and $B$ in the subspace $Y_0$ (supp $Y$) such that $Y = A \bigcup B$ where $A$ and $B$ are $c_i$-$q$-separated. Since $c_i \subset c_i^*$ then $A$ and $B$ are $c_i^*$-q-separated also. Thus we have two non void sets $A$ and $B$ in the space $Y_0$(supp$Y$) such that $Y = A \cup B$ where $A$ and $B$ are $c_i^*$-q-separated also. Hence $(X,c_1^*,c_2^*)$ is semi-connected. □

**Theorem 5.3.** *Let $(X,c_1,c_2)$ be a fbcs. Let $c_i^* \subset c_i$ then $(X,c_1^*,c_2^*)$ is also fuzzy semi-connected.*

*Proof.* Let $(X,c_1,c_2)$ be a semi-connected fbcs. Then it cannot be written as union of two non-empty $q$-separated sets. Let $c_i^* \subset c_i$ and suppose that $(X,c_1^*,c_2^*)$ is disconnected. Since $(X,c_1^*,c_2^*)$ is disconnected $\exists$ two non-empty semi-open set $A$ and $B$ in the subspace $Y_0$ (supp $Y$) such that $Y = A \cup B$ where $A$ and $B$ are $c_i$-$q$-separated also. Then $(X,c_1,c_2)$ is also disconnected. Since $\exists$ two non-empty semi-open sets $A$ and $B$ such that $Y_0 = A \cup B$ (supp $Y$) where $A$ and $B$ are $c_i$-$q$-separated sets which is a contradiction. Hence $(X,c_1^*,c_2^*)$ is semi-connected. □

## 6 Conclusion

In this paper, we introduce and study fuzzy semi separation axioms in fuzzy biclosure space. Though this is weaker than separation axioms already introduced and studied in fuzzy setting by various researchers but it doesn't affect its importance. It is used in Boolean algebra, convex set etc in pure mathematics. Here we introduce various types of continuities using semi-open sets and compare them with each other. Here we introduce semi-separated sets and semi connectedness in fuzzy biclosure space. Further scope of this research is in the field of medical sciences using fuzzy semi-open sets.

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