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(Dedicated to Professor G. C. Sharma on His 85^{th} Birth Anniversary Celebrations)

STRUCTURE OF WEAKLY SEMI-I-OPEN SETS VIA SEMI LOCAL FUNCTIONS

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Abstract

We introduce, study and investigate the concepts of weakly semi -I_s- open sets and some properties of the set. We introduce weakly semi -I_s- open functions and weakly semi -I_s- closed functions. Also, we introduced notion of weakly semi -I_s- open sets and weakly semi -I_s- closed sets. We discussed its properties and its relationship between other sets in topological spaces as said in below introduction. We also furnish decomposition of continuity in this paper.

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1 Introduction

Topology as a well-defined mathematical discipline dates from the early twentieth century, though some isolated results can be traced back several centuries. An ideal topological space is a triplet (X,τ,I), where X is a nonempty set, τ is a topology on X and I is an ideal of subsets of X. Levine [13] introduced and investigated the concept of semi-open sets and semi-continuity in 1963. In 2006, in his paper *on weakly semi-I-open sets and another decomposition of continuity via ideals*, Hatir and Jafari [6] introduced the notions of weakly semi-I-open sets and weakly semi-I-continuous functions and obtained a decomposition of continuity. Khan and Noiri [11] introduced and investigated the concept of semilocal functions in his paper Semi-local functions in ideal topological spaces in 2010. Santhi and Rameshkumar [16] obtained several characterizations of semi-I_s-open sets and semi-I_s-continuous functions in 2013. Also, they introduce new semi-I_s-open and semi-I_s-closed functions as well. In 2014, Santhi and Rameshkumar [17] presented B_{Is}-sets, C_{Is}-sets, S_{Is}-sets, α-I_s-sets, semi-I_s-sets, and pre-I_s-sets to obtain a decomposition of continuity in ideal topological spaces using semi-local functions.

In this paper, we are introducing some properties of weakly semi-I_s-open sets and weakly semi-I_s-closed sets in ideal topological space via semilocal functions. We will study the relationship between weakly semi-I_s-open sets and weakly semi-I_s-closed sets, weakly semi-I_s-open sets and preopen set, weakly semi-I_s-open sets and α-I_s-open set, etc,.

2 Preliminaries

Let A be the subset of a topological space *(X,τ)* then *cl(A)* and *int(A)* denote closure and interior of A in *(X,τ)* respectively.

An Ideal I on a topological space *(X,τ)* is a non-empty collection of subsets of X which satisfies :

1. $A \in I$ and $B \subseteq A$ implies $B \in I$.
2. $A \in I$ and $B \in I$ implies $A \cup B \in I$.

The space *(X,τ,I)* is called an Ideal topological space or Ideal space.

Definition 2.1. *Let $P(X)$ be the power set of X. Then the operator $()^* : P(X) \rightarrow P(X)$ called a local function* [12] *of A with respect to τ and I is defined as follows : for $A \subseteq X$, $A^*(I,\tau) = \{x \in X \mid U \cap A \notin I$ for every open set U containing x $\}$. We simply write A^* instead of $A^*(I,\tau)$.*

Definition 2.2. *For $A \subseteq X$, $A_*(I,\tau)$ =$\{$ $x \in X \mid U \cap A \notin I$ for every $U \in SO(X)\}$ is called semi-local function* [11] *of A with respect to I and τ, where SO(X,x) = $\{U \in SO(X) \mid x \in U\}$. We simply write A_* instead of $A_*(I,\tau)$.*

Definition 2.3. *It is given in* [4] *that $\tau^{*s}(I)$ is a topology on X, generated by a sub basis* $\{U – E : U \in SO(X)$ *and* $E \in I\}$ *or equivalently* $\tau^{*s}(I) = \{U \subseteq X : cl^{*s}(X – U) = X – U\}$.

Definition 2.4. *The closure operator* [4] cl^{*s} *for a topology* $\tau^{*s}(I)$ *is defined as follows: for* $A \subseteq X$, $cl^{*s}(A) = A \cup A_*$ *and* int^{*s} *denotes the interior of the set A in* (X, τ^{*s}, I). *It is known that* $\tau \subseteq \tau^*(I) \subseteq \tau^{*s}(I)$.

Definition 2.5. *A subset A of* (X,τ,I) *is called semi-*-perfect* [10] *if* $A = A_*$. *A subset A of* (X,τ,I) *is called *-semi dense in-itself* [10] *if* $A \subset A_*$. *A subset A of* (X,τ,I) *is called semi-*-closed in-itself* [10] *if* $A_* \subseteq A$.

Definition 2.6. *A subset A of a space* (X,τ) *is said to be*
1. *regular closed* [15] *if* $cl(int(A)) = A$.
2. *semi-open* [13] *if* $A \subset cl(int(A))$. *The complement of semi open set is said to be semi-closed.*
3. *semi-closed* [13] *iff* $int(A) = int(cl(A))$.
4. *semi-closure* [13] *if intersections of all semi-closed sets containing A and it is denoted by* $scl(A)$.

Definition 2.7 ([11])**.** *Let* (X,τ,I) *be an ideal topological space and A, B subsets of X. Then for the semi-local function the following properties hold:*
1. *If* $A \subseteq B$ *then* $A_* \subseteq B_*$.
2. *If* $U \in \tau$ *then* $U \cap A_* \subseteq (U \cap A)_*$.
3. $A_* = scl(A_*) \subseteq scl(A)$ *and* A_* *is semi-closed in X.*
4. $(A_*)_* \subseteq A_*$.
5. $(A \cup B)_* = A_* \cup B_*$.
6. *If* $I = \{\emptyset\}$, *then* $A_* = scl(A)$.

Definition 2.8. *A subset A of a topological space X is said to be*
1. *α-open* [14] *if* $A \subseteq int(cl(int(A)))$,
2. *pre-open* [3] *if* $A \subseteq int(cl(A))$,
3. *β-open* [5] *if* $A \subseteq cl(int(cl(A)))$.

Definition 2.9. *A subset A of an ideal topological space* (X,τ,I) *is said to be*
1. *α-I-open* [8] *if* $A \subseteq int(cl^*(int(A)))$,
2. *semi-I-open* [8] *if* $A \subseteq cl^*(int(A))$,
3. *pre-I-open* [1] *if* $A \subseteq int(cl^*(A))$,
4. *almost strong I-open* [7] *if* $A \subset cl^*(int(A^*))$,
5. *almost I-open* [2] *if* $A \subset cl(int(A^*))$,
6. *β-I-open* [8] *if* $A \subset cl(int(cl^*(A)))$,
7. *strong β-I-open* [7] *if* $A \subset cl^*(int(cl^*(A)))$,
8. *weakly semi-I-open* [15] *if* $A \subset cl^*(int(cl(A)))$.

Definition 2.10. *A subset A of an ideal topological space* (X,τ,I) *is said to be*
1. *α-I_s-open* [18] *if* $A \subseteq int(cl^{*s}(int(A)))$,
2. *s-I_s-set* [18] *if* $cl^{*s}(int(A)) = int(A)$,
3. *α^*-I_s-set* [18] *if* $int(cl^{*s}(int(A))) = int(A)$.

Corollary 2.1. *A subset A of an ideal topological space* (X,τ,I) *is said to be*
1. *Every almost strong I-open set is almost I-open but not converse* [7],
2. *Every almost strong I-open set is a strong β-I-open set but not converse* [7],
3. *Every strong β-I-open set is a β-I-open set but not converse* [7],
4. *Every β-I-open set is a β-open set but not converse* [7],
5. *Every almost I-open set is a β-I-open set but not converse* [7],
6. *Every weakly semi-I-open set is a β-open set but not converse* [6],
7. *Every strong β-I-open set is a weakly semi-I-open set but not converse* [6].

Definition 2.11. *Let* (X,τ,I) *be an ideal space and M be a *-semi dense in itself* [10] *subset of X. Then* $A_* = cl(A) = cl^{*s}(A)$.

Definition 2.12. *Let* (X,τ,I) *be an ideal space and* $A \subset X$.
Then $cl^{*s}(int(cl^{*s}(int(A)))) = cl^{*s}(int(A))$.

Definition 2.13. *A subset A of an ideal space (X,τ,I) is said to be semi-I_s-open* [18] *if* $A \subseteq cl^{*s}(int(A)))$.

Definition 2.14. *A subset A of an ideal space (X,τ,I) is said to be semi-I_s-open* [16] *iff there exists* $U \in \tau$ *such that* $U \subseteq A \subseteq cl^{*s}(U)$. *A subset H of an ideal space (X,τ,I) is said to be semi-I_s-closed* [16] if its complement is semi-I_s-open.

Definition 2.15. *A subset A of an ideal space (X,τ,I) is said to be pre-I_s-open* [18] *if* $A \subseteq (int(cl^{*s}(A)))$.

Definition 2.16. *A subset F of an ideal space (X,τ,I) is said to be pre-I_s-closed* [17] *if its complement is pre-I_s-open.*

Definition 2.17. *A subset A of an ideal space (X,τ,I) is called*

1. *An A_{IS}-set* [9] *if $A = U \cap V$, where U is open and $cl^{*s}(int(V)) = V$.*
2. *A B_{1IS}-set* [9] *if $A = U \cap V$, where U is α-I_s-open and $cl^{*s}(int(V)) = X$.*
3. *A B_{2IS}-set* [9] *if $A = U \cap V$, where U is α-I_s-open and $cl^{*s}(V) = X$.*
4. *An αA_{IS}-set* [9] *if $A = U \cap V$, where U is α-I_s-open and $cl^{*s}(int(V)) = V$.*
5. *An αC_{IS}-set* [9] *if $A = U \cap V$, where U is α-I_s-open and $int(cl^{*s}(int(V))) \subset V$.*
6. *A WLC_{IS}-set* [9] *if $A = U \cap V$, where U is open and $cl^{*s}(V) = V$.*
7. *A S_{IS}-set* [18] *if $A = U \cap V$, where $U \in \tau$ and V is S-I_s-set.*

3 Weakly semi-I_s-open sets

Definition 3.1. *A subset M of an ideal space (X,τ,I) is said to be weakly semi-I_s-open if* $M \subseteq cl^{*s}(int(cl(M)))$.

Example 3.1. Consider X = {m,n,o} in an ideal space *(X,τ,I)*, where τ = {$\emptyset$, {m}, {m,o}, X} and I ={$\emptyset$, {m}}. Let the semi open set of τ be B = {$\emptyset$, X} and M = {m,n}. Then $cl^{*s}(int(cl(M))) = cl^{*s}(int(cl(\{m,n\}))) = cl^{*s}(int(X)) = cl^{*s}(X) = X \supset M$ and so M is weakly semi-I_s-open.

Example 3.2. Consider X = $\{1,2,3,4\}$ in an ideal space *(X,τ,I)*, where $\tau = \{\emptyset, \{1\}, \{3\}, \{1,3\}, X\}$ and $I = \{\emptyset, \{1\}\}$. Let the semi open set of τ be B = $\{\emptyset, \{1,2,4\}, \{1,3,4\}, X\}$, M = $\{1,3\}$, $M_* = \{3\}$. Then $cl^{*s}(int(cl(M))) = cl^{*s}(int(cl(\{1,3\}))) = cl^{*s}(int(X)) = cl^{*s}(X) = X \supset M$ and so M is weakly semi-I_s-open.

Lemma 3.1. *Every semi-I_s-open set is weakly semi-I_s-open set, but converse doesn't hold.*

Example 3.3. Consider X = {m,n,o} in an ideal space *(X,τ,I)*, where τ = {$\emptyset$, {m,n}, X} and I ={$\emptyset$, {o}}. Then M = {m}, $cl^{*s}(int(cl(M))) = cl^{*s}(int(cl(\{m\}))) = cl^{*s}(int(X)) = cl^{*s}(X) = X \supset M$ and so M is weakly semi-I_s-open, but $cl^{*s}(int(M)) = cl^{*s}(int\{m\}) = cl^{*s}(\emptyset) = \emptyset \not\supset M$ and so M is not semi-I_s-open.

Theorem 3.1. *Let (X,τ,I) be an ideal topological space. If M is weakly semi-I_s-open set then M is β-open, but not conversely.*

Proof. If M is weakly semi-I_s-open, then $M \subset cl^{*s}(int(cl(M))) = (int(cl(M)))_* \cup (int(cl(M)))$ $\subset cl(int(cl(M))) \cup int(cl(M)) = cl(int(cl(M)))$. Therefore M is β-open and converse doesn't hold. □

Example 3.4. Consider X = {m,n,o} in an ideal space *(X,τ,I)*, where τ = {$\emptyset$, {m}, {n}, {m,n}, X} and I ={$\emptyset$, {m}}. Then M = {m,o} is β-open, but not weakly semi-I_s-open.

Example 3.5. Consider X = {m,n,o,p} in an ideal space *(X,τ,I)*, where τ = {$\emptyset$, {m}, {o}, {m,o}, X} and I ={$\emptyset$, {m}}. Let the semi open set of τ be B = {$\emptyset$, {m,n,p}, {m,o,p}, X}, M = {m,o}, M_* = {c}. Then cl^{*s}(int(cl(M))) = cl^{*s}(int(cl({m,o}))) = cl^{*s}(int(X)) = cl^{*s}(X) = X $\supset M$ and so M is weakly semi-I_s-open. Also $M \neq M_*$, hence M is not semi-*-perfect. $M \not\subset M_*$, hence M is not *-semidense. $M \subseteq M_*$, hence M is semi-*-closed.

Corollary 3.1. *Let (X,τ,I) be an ideal space and M is *-semi dense in itself, then the following are equivalent :*

(a) M is β-open,

(b) M is weakly semi-I_s-open.

Theorem 3.2. *Let the ideal topological space be (X,τ,I) and M, N be the subsets of X. If M is weakly semi-I_s-open set and $N \in \tau$, then $M \cap N$ is weakly semi-I_s-open.*

Proof. Let M is weakly semi-I_s-open and $N \in \tau$. If $M \subset cl^{*s}(int(cl(M)))$, then $M \cap N \subset cl^{*s}(int(cl(M))) \cap N = ((int(cl(M)))_* \cup int(cl(M))) \cap N = (int(cl(M)))_* \cap N \cup int(cl(M)) \cap N \subset (int(cl(M)) \cap N_*) \cup int(cl(M \cap N)) = (int(cl(M \cap N)))_* \cup int(cl(M \cap N)) = cl^{*s}(int(cl(M \cap N)))$. This shows that $M \cap N$ is weakly semi-I_s-open. □

Remark 3.1. In general, the finite intersection of weakly semi-I_s-open sets need not be weakly semi-I_s-open.

Lemma 3.2. *Let the ideal topological space be (X,τ,I), where $M \subset X$ and $U \in$ semiopen set of τ. Then $cl^{*s}(M) \cap U = cl^{*s}(M \cap U)$.*

Proof. $cl^{*s}(M) \cap (U) = (M_* \cup M) \cap U = (M_* \cap U) \cup (M \cap U) \subset (M \cap U)_* \cup (M \cap U) = cl^{*s}(M \cap U)$. □

Example 3.6. Consider X $= \{1, 2, 3, 4\}$ in an ideal space (X,τ,I), where $\tau = \{\emptyset, \{1\}, \{3\}, \{1, 3\}, X\}$ and $I = \{\emptyset, \{1\}\}$. Let $M = \{1, 3\}$ and $M_* = \{3\}$. From example 3.2, M is weakly semi-I_s-open. $M \not\subset M_*$, hence M is not *-semidense.

Example 3.7. Consider X = {m,n,o} in an ideal space (X,τ,I), where τ = {$\emptyset$, {m}, {n}, {m,n}, X} and I ={$\emptyset$, {m}}. Let the semi open set of τ be B = {$\emptyset$, {m,o}, {n,o}, X} and M = {m,o}, where $cl^{*s}(int(cl(M))) = cl^{*s}(int(cl(\{m, o\}))) = cl^{*s}(int(\{m, o\})) = cl^{*s}(\{m\}) = \emptyset \not\supset M$ and so M is not weakly semi-I_s-open. since $cl(int(cl^{*s}(M))) = cl(int(cl^{*s}(\{m, o\}))) = cl(int(X)) = cl(X) = X \supset M$ and so M is $\beta - I_s$-open.

The above example shows that weakly semi-I_s-openness and β-I_s-openness are independent concepts.

Theorem 3.3. *Let an ideal space be (X,τ,I). If M is pre-open, then M is weakly semi-I_s-open.*

Proof. If M is pre-open, then $M \subset int(cl(M))$ and so $M \subset cl^{*s}(int(cl(M)))$ which implies that M is weakly semi-I_s-open. □

Example 3.8. Consider X = {m,n,o} in an ideal space (X,τ,I), where τ = {$\emptyset$, {m}, {m,o}, X} and I ={$\emptyset$, {m}}. Let the semi open set of τ be B = {$\emptyset$, X} and M = {m,n}. Then $cl^{*s}(int(cl(M))) = cl^{*s}(int(cl(\{m, n\}))) = cl^{*s}(int(X)) = cl^{*s}(X) = X \supset M$ and so M is weakly semi-I_s-open. Also, $int(cl(M)) = int(cl(\{m, n\})) = int(X) = X \supset M$ and therefore, M is pre-open.

Theorem 3.4. *Let an ideal space be (X,τ,I). If $M \subset N \subset cl^{*s}(M)$ and M is weakly semi-I_s-open, then N is weakly semi-I_s-open. In particular, if M is weakly semi-I_s-open, then $cl^{*s}(M)$ is weakly semi-I_s-open.*

Proof. If M is weakly semi-I_s-open, then $M \subset cl^{*s}(int(cl(M)))$. Since $N \subset cl^{*s}(M) \subset cl^{*s}(cl^{*s}(int(cl(M)))) = cl^{*s}(int(cl(M))) \subset cl^{*s}(int(cl(N)))$. Hence N is weakly semi-I_s-open. □

Theorem 3.5. *Let the ideal space be (X,τ,I). If M is α-I_s-open and N is weakly semi-I_s-open, then $M \cap N$ is weakly semi-I_s-open.*

Proof. Since M is α-I_s-open, $M \subset int(cl^{*s}(int(M)))$ and N is weakly semi-I_s-open, $N \subset cl^{*s}(int(cl(N)))$. Now $M \cap N \subset int(cl^{*s}(int(M))) \cap cl^{*s}(int(cl(N))) \subset cl^{*s}(int(cl^{*s}(int(M))) \cap int(cl(N))) = cl^{*s}(int(cl^{*s}(int(M) \cap int(cl(N))))) \subset cl^{*s}(int(cl^{*s}(int(M) \cap int(cl(N))))) = cl^{*s}(int(cl^{*s}(int(int(M) \cap cl(N))))) \subset cl^{*s}(int(cl^{*s}(int(cl(int(M) \cap N))))) \subset cl^{*s}(int(cl^{*s}(int(cl(M \cap N))))) = cl^{*s}(int(cl(M \cap N)))$ by Definition 2.12, which implies that $M \cap N$ is weakly semi-I_s-open. □

Theorem 3.6. *Let the ideal space be (X,τ,I) and $M \subset X$ be weakly semi-I_s-open. If M is either semiclosed or I_s-locally closed, then M is semi-I_s-open.*

Proof. Suppose M is I_s-locally closed. M is I_s-locally closed implies that $M = U \cap M_*$ for some semi open set U. M is weakly semi-I_s-open implies that $M \subset cl^{*s}(int(cl(M)))$. Now $M = U \cap M_* \subset U \cap (cl^{*s}(int(cl(M))))_* \subset U \cap cl^{*s}(cl^{*s}(int(cl(U \cap M_*)))) = U \cap cl^{*s}(int(cl(U \cap M_*))) \subset cl^{*s}(U \cap int(cl(U \cap M_*))) = cl^{*s}(int(U \cap cl(U \cap M_*))) \subset cl^{*s}(int(U \cap cl(U) \cap cl(M_*))) = cl^{*s}(int(U \cap M_*)) = cl^{*s}(int(M))$. Hence M is semi-I_s-open. Suppose M is semiclosed. Then $int(cl(M)) = int(M)$. Since M is weakly semi-I_s-open implies that $M \subset cl^{*s}(int(cl(M))) = cl^{*s}(int(M))$. Hence M is semi-I_s-open. □

Example 3.9. Consider X = {m,n,o} in an ideal space *(X,τ,I)*, where $\tau = \{\emptyset, \{m,n\}, X\}$ and $I = \{\emptyset, \{o\}\}$. Let the semi open set of τ be B = {$\emptyset$, X} and $M = \{m\}$, then $M_* =$ X and $cl^{*s}(int(cl(M))) = cl^{*s}(int(X)) = X \subset M$ and so M is weakly semi-I_s-open. Also, $cl^{*s}(int(M)) = cl^{*s}(\emptyset) = \emptyset$. Hence M is not semi-I_s-open. Moreover, M is neither I_s-locally closed nor semiclosed.

Definition 3.2. *A subset M of an ideal space (X,τ,I) is said to be weakly semi-I_s-closed if $M \subseteq int^{*s}(cl(int(M)))$.*

Theorem 3.7. *A subset M of a space (X,τ,I) is weakly semi-I_s-closed iff $int^{*s}(cl(int(M))) \subset M$. Also, if M is weakly semi-I_s-closed subset of X, then M is an α^*-I_s-set.*

Proof. Let M be weakly semi-I_s-closed set of *(X,τ,I)*. Then X - M is weakly semi-I_s-open and hence X - $M \subset cl^{*s}(int(cl(X - M))) = X - int^{*s}(cl(int(M)))$. Therefore, we have $int^{*s}(cl(int(M))) \subset M$. Conversely, let $int^{*s}(cl(int(M))) \subset M$. Then X - $M \subset cl^{*s}(int(cl(X - M)))$ and hence X - M is weakly semi-I_s-open. Therefore, M is weakly semi-I_s-closed. Also $int^{*s}(cl(int(M))) \subset M$ and so $int^{*s}(cl(int(M))) \subset int(M)$. Hence it follows that $int^{*s}(cl(int(M))) = int(M)$ which implies that M is α^*-I_s-set. □

Definition 3.3. *A subset M of an ideal space (X,τ,I) is said to be weakly S_{IS}-set (resp. C_{IS}-set* [18]*) if $M = G \cap V$ where G is open and V is weakly semi-I_s-closed (resp. α^*-I_s-set).*

Remark 3.2. Every open set is a weakly S_{IS}-set and every weakly S_{IS}-set is a C_{IS}-set.

Theorem 3.8. *Let (X,τ,I) be an ideal space. Then the following are equivalent :*
(a) M is open,
(b) M is α-I_s-open and a weakly S_{IS}-set,
(c) M is α-I_s-open and a C_{IS}-set.

Proof. If M is open, (a) implies (b) and (b) implies (c) are clear. Then (c) implies (a) follows from the preposition 4.16 of [18]. □

Definition 3.4. *A subset M of a space (X,τ,I) is called Strong $s - I_s$-set if $cl^{*s}(int(cl(M))) = int(M)$.*

Definition 3.5. *A subset M of a space (X,τ,I) is called Strong S_{IS}-set if $M = U \cap V$, where $U \in \tau$ and V is Strong $S - I_s$-set.*

Remark 3.3. a) Every strong $s - I_s$-set is $S - I_s$-set.
b) Every strong S_{IS}-set is S_{IS}-set.
c) Every open set is strong S_{IS}-set.

Proposition 3.1. *For a subset M of a topological space (X,τ,I), the following holds equivalently :*
a) M is open,
b) M is weakly semi-I_s-open and strong S_{IS}-set,
c) M is semi-I_s-open and strong S_{IS}-set.

Proof. By the above remarks we prove this as follows:

If M is a semi-I_s-open set and also a strong S_{IS}-set, then $M \subseteq cl^{*s}(int(cl(M))) = cl^{*s}(int(cl(U \cap V)))$, where $U \in \tau$ and V is strong S_{IS}-set. Hence $M \subset U \cap M \subset U \cap (cl^{*s}(int(cl(U))) \cap cl^{*s}(int(cl(V)))) = U \cap int(V) = int(M)$, shows that M is open □

4 Weakly semi-I_s-open and Weakly semi-I_s-closed functions

Definition 4.1. *Let $f : (M, \tau, I) \to (N, \sigma, J)$ be a function of weakly semi-I_s-open if the image of every open set in (M, τ, I) is weakly semi-I_s-open in (N, σ, J).*

Theorem 4.1. *A function $f : (M, \tau, I) \to (N, \sigma, J)$ is weakly semi-I_s-open iff for each point m of X and each neighbourhood U of m, there exists a weakly semi-I_s-open set V in N containing $f(m)$ such that $V \subset f(U)$.*

Theorem 4.2. *A function $f : (M, \tau, I) \to (N, \sigma, J)$ is weakly semi-I_s-open function such that $F \subset N$ and $G \subset M$ is a closed set containing $f^{-1}(F)$, then there exists a weakly semi-I_s-open set $W \subset N$ containing F such that $f^{-1}(W) \subset G$.*

Definition 4.2. *Let $f : (M, \tau, I) \to (N, \sigma, J)$ be a function of weakly semi-I_s-closed if the image of every closed set in (M, τ, I) is weakly semi-I_s-closed in (N, σ, J).*

Theorem 4.3. *A function $f : (M, \tau, I) \to (N, \sigma, J)$ is weakly semi-I_s-closed function such that $F \subset N$ and $G \subset M$ is a open set containing $f^{-1}(F)$, then there exists a weakly semi-I_s-closed set $W \subset N$ containing F such that $f^{-1}(W) \subset G$.*

Definition 4.3. *A function $f : (M, \tau, I) \to (N, \sigma, J)$ is said to be weakly semi-I_s-continuous if for every $V \in \sigma$, $f^{-1}(V)$ is an ws-I_s-set of (M, τ, I).*

Proposition 4.1. *$f : (M, \tau, I) \to (N, \sigma, J)$ be bijective function then the following condition holds:*
(1) *f^{-1} is weakly semi-I_s-continuous,*
(2) *f is weakly semi-I_s-open,*
(3) *f is weakly semi-I_s-closed.*

Theorem 4.4. *Consider the functions $f : (M, \tau.I) \to (N, \sigma, J)$ and $g : (N, \sigma, J) \to (O, \nu, K)$, whre I, J and K are ideals on M, N and O, respectively. The following statement holds:*
(1) *If f is open and g is weakly semi-I_s-open then $g \circ f$ is weakly semi-I_s-open,*
(2) *If $g \circ f$ is open and g is weakly semi-I_s-continuous injection then f is weakly semi-I_s-open.*

5 Conclusion

In this paper, we obtained several characterization of weakly semi-I_s-open sets. we introduced weakly semi-I_s-open sets and weakly semi-I_s-closed sets using semi local functions. Also we introduced weakly semi-I_s-open functions and weakly semi-I_s-closed functions. We discussed their relationship with various sets.

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