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(Dedicated to Professor G. C. Sharma on His 85^{th} Birth Anniversary Celebrations)

A STUDY ON *Fuzzy* SEPARATION AXIOMS ($T_i, i = 0, 1, 2$) VIA *Fuzzy* gp^*-OPEN SETS

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Abstract

In this paper we have introduced *Fuzzy* gp^* closure, *Fuzzy* gp^*-interior and separation axioms via *Fuzzy* gp^*-open sets. Also we found out the relationship between *Fuzzy* separation axioms, *Fuzzy* gp^* separation axioms and *Fuzzy* pre separation axioms.

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1 Introduction

In this paper we introduce and study separation properties of *Fuzzy* topological spaces via *Fuzzy* gp^* closed sets and draw a valid implication between the different axioms introduced earlier. *Fuzzy* separation axioms were introduced and studied by Ghanim et al. [3]. Similarly *Fuzzy* pre separation axioms were introduced and many of their properties were established by Singal et al. [11]. In 2011 Lee and Yun [9] introduced and studied *Fuzzy* delta separation axioms based on *Fuzzy* δ-open sets. They investigated the relationship between *Fuzzy* separation axioms and *Fuzzy* δ-separation axioms and showed *Fuzzy* δ-separation axioms are hereditary in *Fuzzy* regular open subspaces. In 2018 Paul et al. [10] studied and introduced separation axioms ($T_i, i = 0, 1, 2$) in the light of *Fuzzy* γ^*-open set via quasi-coincidence, quasi-neighborhood and also established relation between *Fuzzy* separation axioms, *Fuzzy* pre-separation axioms and *Fuzzy* γ^*-separation axioms.

In this paper, we introduce *Fuzzy* separation axioms via *Fuzzy* gp^*-open sets and find out there relation with *Fuzzy* separation axioms and *Fuzzy* pre separation axioms introduced earlier. We find out that every FT_i space [3] is Fgp^*T_i space for $i = 0, 1, 2$ and every FPT_i space [11] is Fgp^*T_i space for $i = 0, 1, 2$. But the converse is not true for both the cases, which we proved by counter examples.

2 Preliminaries

In this paper (Z, τ) always mean *Fuzzy* topological space on which no separation axioms are mentioned unless otherwise explicitly stated. A *Fuzzy* set in topological space (X, τ) is called a *Fuzzy* point iff it takes the value 0 for all $y \in X$ except one, say $x \in X$. If its value at x is λ $(0 < \lambda \leq 1)$ we denote this *Fuzzy* point by x_λ, where the point x is called its support see [11]. From the previous literature, following definitions and remarks play a key role in establishing the main work of this paper.

Definition 2.1 ([5]). *Suppose (Y, τ) is a Fuzzy topological space. Then a subset λ of (Y, τ) is called Fuzzy generalized pre regular weakly closed (briefly Fuzzy gp^*-closed) if $pcl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is a Fuzzy regular semi open set in (Y, τ). Complement of Fuzzy generalized pre regular weakly closed set is called Fuzzy generalized pre regular weakly open (briefly Fuzzy gp^*-open).*

Definition 2.2 ([2]). *A Fuzzy set on X is called a Fuzzy singleton if it takes the value zero (0) for all points x in X except one point. The point at which a Fuzzy singleton takes the non-zero value is called the support and the corresponding element of $(0, 1]$ its value. A Fuzzy singleton with value 1 is called a Fuzzy crisp singleton.*

Definition 2.3 ([3]). *A Fuzzy topological space is said to be FT_0 iff for every pair of Fuzzy singletons P_1 and P_2 with different supports, there exists an open Fuzzy set O such that $p_1 \leq O \leq cop_2$ or $p_2 \leq O \leq cop_1$.*

Definition 2.4 ([3]). *A Fuzzy topological space is said to be* FT_1 *iff for every pair of Fuzzy singletons* p_1 *and* p_2 *with different supports, there exists open Fuzzy sets* O_1 *and* O_2 *such that* $p_1 \leq O_1 \leq cop_2$ *and* $p_2 \leq O_2 \leq cop_1$.

Definition 2.5 ([3]). *A Fuzzy topological space is said to be* FT_2 *(F-Hausdorff) iff for every pair of Fuzzy singletons* p_1 *and* p_2 *with different supports, there exists open Fuzzy sets* O_1 *and* O_2 *such that* $p_1 \leq O_1 \leq cop_2$, $p_2 \leq O_2 \leq cop_1$ *and* $O_1 \leq coO_2$.

Definition 2.6 ([11]). *A Fuzzy topological space is said to be Fuzzy pre-*T_0 *or in short* FPT_0 *if for every pair of Fuzzy singletons* p_1 *and* p_2 *with different supports, there exists a Fuzzy pre-open set u such that either* $p_1 \leq u \leq cop_2$ *or* $p_2 \leq u \leq cop_1$.

Definition 2.7 ([11]). *A Fuzzy topological space* (X, τ) *is said to be Fuzzy pre-*T_1 *or in short* FPT_1 *if for every pair of Fuzzy singletons* p_1 *and* p_2 *with different supports* x_1 *and* x_2*, (* $x_1 \neq x_2$ *), there exists Fuzzy pre-open sets u and v such that* $p_1 \leq u \leq cop_2$ *and* $p_2 \leq v \leq cop_1$.

Definition 2.8 ([11]). *A Fuzzy topological space is said to be Fuzzy pre-Hausdorff or in short* FPT_2 *iff for every pair of Fuzzy singletons* p_1 *and* p_2 *with different supports, there exists two Fuzzy pre-open sets u and v such that* $p_1 \leq u \leq cop_2$, $p_2 \leq v \leq cop_1$ *and* $u \leq cov$.

Remark 2.1 ([5]). Suppose (Y, τ) is a *Fuzzy* topological space and $\lambda \leq Y$. Then we call λ *Fuzzy* gp^*-open if $(1 - \lambda)$ is *Fuzzy* gp^* closed in (Y, τ).

Remark 2.2 ([11]). In *Fuzzy* topological space (Y, τ) every *Fuzzy* closed set is *Fuzzy* pre-closed.

Remark 2.3 ([5]). In *Fuzzy* topological space (Y, τ), every *Fuzzy* open set is *Fuzzy* gp^*-open.

3 Fuzzy gp^*-closure

Definition 3.1. *Suppose* (Y, τ) *is a Fuzzy topological space and* $\alpha \leq Y$*. Then Fuzzy* gp^**-closure (briefly* Fgp^**-cl) and Fuzzy* gp^**-interior (briefly* Fgp^**-int) of* α *are respectively defined as,*

$$\text{Fuzzy } gp^*\text{-}cl(\alpha) = \wedge \{\mu : \alpha \leq \mu, \mu \text{ is Fuzzy } gp^*\text{-closed set in } Y \},$$
$$\text{Fuzzy } gp^*\text{-}int(\alpha) = \vee \{\mu : \alpha \geq \mu, \mu \text{ is Fuzzy } gp^*\text{-open set in } Y\}.$$

Theorem 3.1. *In Fuzzy topological space* (Y, τ) *every Fuzzy pre-closed set is Fuzzy* gp^**-closed.*

Proof. Suppose λ is a *Fuzzy* pre-closed set in (Y, τ) such that $\lambda \leq \mu$, where μ is *Fuzzy* generalized pre-open in (Y, τ). Now as λ is *Fuzzy* pre-closed implying that $pcl(\lambda) = \lambda$. Also by Remark 2.2 every *Fuzzy* closed set is *Fuzzy* pre-closed, implying $cl(\lambda) \leq pcl(\lambda) = \lambda \leq \mu$, whenever $\lambda \leq \mu$ and μ is *Fuzzy* generalized pre-open in (Y, τ). So λ is *Fuzzy* gp^*-closed. □

Theorem 3.2. *Suppose* λ *is a Fuzzy set in Fuzzy space* (Y, τ)*. Then Fuzzy* $gp^* - cl(1 - \lambda)$ *= 1- (Fuzzy* $gp^* - int(\lambda)$*) and Fuzzy* $gp^* - int(1 - \lambda)$ *= 1-(Fuzzy* $gp^* - cl(\lambda)$*).*

Proof. From Remark 2.1, a *Fuzzy* gp^*-open set $p \leq \lambda$ is the complement of *Fuzzy* gp^*-closed set $q \geq 1 - \lambda$. So

Fuzzy gp^*-int(λ) =$\vee\{1 - q : q$ is *fuzzy* gp^* closed and $q \geq 1 - \lambda\}$,
Fuzzy $gp^* - int(\lambda) = 1 - \wedge\{q : q$ is *Fuzzy* gp^* closed and $q \geq 1 - \lambda\}$,
Fuzzy $gp^* - int(\lambda)$ = 1- *Fuzzy* $gp^* - cl(1 - \lambda)$
$\implies gp^* - cl(1 - \lambda)$ = 1-*Fuzzy* $gp^* - int(\lambda)$.

Now, suppose r is a *Fuzzy* gp^*-open set so for *fuzzy* gp^*-closed set s$\geq\lambda$, $r = 1 - s \leq 1 - \lambda$

Fuzzy $gp^* - cl(\lambda) = \wedge\{1 - r : r$ is *fuzzy* gp^*-open and $r \leq 1 - \lambda$ },
Fuzzy gp^*-cl(λ)=1-$\vee\{r : r$ is *Fuzzy* gp^*-open and $r \leq 1 - \lambda\}$,
Fuzzy $gp^* - cl(\lambda)$ = 1- *Fuzzy* $gp^* - int(1 - \lambda)$
$\implies$ *Fuzzy* $gp^* - int(1 - \lambda)$ = 1-Fuzzy $gp^* - cl(\lambda)$.

□

Theorem 3.3. *Suppose (Y, τ) is a Fuzzy topological space and α, μ are Fuzzy subsets of Y. Then*
(a) Fuzzy $gp^* - cl(1_Y) = 1_Y$ and Fuzzy $gp^* - cl(0_Y) = 0_Y$,
(b) $\alpha \leq$ Fuzzy gp^*-$cl(\alpha)$,
(c) suppose $\mu \leq \alpha$ where α is Fuzzy gp^*-closed set. Then Fuzzy gp^*-$cl(\mu) \leq \alpha$,
(d) If $\alpha \leq \mu$ then Fuzzy $gp^* - cl(\alpha) \leq$ Fuzzy $gp^* - cl(\mu)$.

Proof. (a) Since *Fuzzy* $gp^* - cl(1_Y)$ is the intersection i.e. minimum of all *Fuzzy* gp^*-closed sets in Y containing 1_Y and since 1_Y is the minimum *Fuzzy* gp^*-closed set containing 1_Y. So *Fuzzy* $gp^* - cl(1_Y) = 1_Y$. Now *Fuzzy* $gp^* - cl(0_Y)$ is the intersection i.e. minimum of all *Fuzzy* gp *-closed sets in Y containing 0_Y and since 0_Y is the minimum *Fuzzy* gp^*-closed set containing 0_Y, implying *Fuzzy* $gp^* - cl(0_Y) = 0_Y$.

(b) As *Fuzzy* $gp^* - cl(\alpha)$ is the intersection of all *Fuzzy* gp^*-closed sets containing α. So $\alpha \leq$ *Fuzzy* $gp^* - cl(\alpha)$ is obvious.

(c) Suppose $\mu \leq \alpha$, where α is *Fuzzy* gp^*-closed set. Now,

$$\textit{Fuzzy } gp^*\text{-cl}(\mu) = \wedge \{\pi : \mu \leq \pi, \pi \text{ is } \textit{Fuzzy } gp^*\text{-closed set in } Y\}$$

i.e. *Fuzzy* $gp^* - cl(\mu)$ is contained in all *Fuzzy* gp^*-closed sets, so in particular *Fuzzy* $gp^* - cl(\mu) \leq \alpha$.

(d) Suppose $\alpha \leq \mu$, also

$$\textit{Fuzzy } gp^*\text{-cl}(\mu) = \wedge \{\pi : \mu \leq \pi, \pi \text{ is } \textit{Fuzzy } gp^*\text{-closed set in Y}\} \quad \rightarrow (d.1).$$

Now if$\mu \leq \pi$, where π is *Fuzzy* gp^*-closed in Y, then by (c) of this theorem, *Fuzzy* $gp^* - cl(\mu) \leq \pi$. Now by (b) of this theorem $\mu \leq$ *Fuzzy* $gp^* - cl(\mu)$ implies $\alpha \leq \mu \leq \pi$ where π is *Fuzzy* gp^*-closed. So *Fuzzy* $gp^* - cl(\alpha) \leq \pi$ (by (c) of this theorem). Therefore

$$\textit{Fuzzy } gp^*\text{-cl}(\alpha) \leq \wedge \{\pi : \mu \leq \pi, \pi \text{ is } \textit{Fuzzy } gp^*\text{-closed set in Y}\}$$

$$\Longrightarrow \quad \textit{Fuzzy } gp^*\text{-cl}(\alpha) \leq Fuzzy gp^* - cl(\mu) \quad (using (d.1))$$

□

4 Separation Axioms via *Fuzzy gp**-open Set

Definition 4.1. *A Fuzzy topological space (Z, τ) is $Fgp^* - T_0$ if for arbitrary Fuzzy singletons x^1_λ and x^2_μ, their exists a Fuzzy gp^*-open set Z such that $x^1_\lambda \leq Z \leq (1 - x^2_\mu)$ or $x^2_\mu \leq Z \leq (1 - x^1_\lambda)$.*

Theorem 4.1. *A Fuzzy topological space (Z, τ) is $Fgp^* - T_0$ iff Fuzzy-gp^* closure of any two Fuzzy crisp singletons with different supports is distinct.*

Proof. Suppose (Z, τ) is $Fgp^* - T_0$ and x^1, x^2 are two *Fuzzy* crisp singletons with different supports. Now (Z, τ) being $Fgp^* - T_0$ implies that $\exists$ a *Fuzzy-gp** open set Z such that $x^1 \leq Z \leq (1 - x^2)$, implying $x^2 \leq Fgp^* - cl(x^2) \leq 1 - Z$. Since $x^1 \nleq 1 - Z$ so $x^1 \nleq Fgp^* - cl(x^2)$, but $x^1 \leq Fgp^* - cl(x^1)$ implies $Fgp^* - cl(x^1) \neq Fgp^* - cl(x^2)$.

Conversely, suppose x^1 and x^2 be two *Fuzzy* crisp singletons with different supports z_1 and z_2, respectively such that $x^1(z_1) = x^2(z_2) = 1$. Also let l_1 and l_2 be *Fuzzy* singletons with different supports z_1 and z_2, so by hypothesis $1_Z - Fgp^* - cl\{x^1\} \leq 1_Z - \{x^1\}$ and so $(1_Z - Fgp^* - cl(x^1)) \leq (1_Z - \{l_1\})$. Now $(1_Z - Fgp^* - cl(x^1))$ is a Fgp^*-open set such that $l_2 \leq (1_Z - Fgp^* - cl(x^1) \leq (1_Z - \{l_1\})$. Implying (Z, τ) is $Fgp^* - T_0$. □

Definition 4.2. *A Fuzzy topological space (Z, τ) is $Fgp^* - T_1$ if for arbitrary Fuzzy singletons X^1_λ and x^2_μ, their exists Fuzzy gp^* open sets Z_1 & Z_2 such that $x^1_\lambda \leq Z_1 \leq (1 - x^2_\mu)$ and $x^2_\mu \leq Z_2 \leq (1 - x^1_\lambda)$.*

Obviously every $Fgp^* - T_1$ space is a $Fgp^* - T_0$ space.

Theorem 4.2. *A Fuzzy topological space (Z, τ) is $Fgp^* - T_1$ iff every Fuzzy crisp singleton is Fuzzy-gp* closed.*

Proof. Consider (Z, τ) is $Fgp^* - T_1$ and l_1 is a *Fuzzy* singleton with support z_1 such that $l_1(z_1) = 1$. So for any arbitrary *Fuzzy* singleton l_2 with support $z_2 \neq z_1$, their exists *Fuzzy*-gp* open sets α and β such that $l_1 \leq \alpha \leq 1_Z - l_2$ and $l_2 \leq \beta \leq 1_Z - l_1$. Now, as every *Fuzzy* set can be written as the union of *Fuzzy* singletons contained in it [2]. So $1_Z - l_1 = \vee_{l_2 \leq 1_Z - l_1} l_2$. From $1 - l_1(z_1) = 0$ it is clear that $1_Z - l_1 = \vee_{l_2 \leq 1_Z - l_1} \beta$, implying $1_Z - l_1$ is *Fuzzy*-gp* open. Conversely suppose that l_1 and m_1 are *Fuzzy* singletons with support z_1

such that $m_1(z_1) = 1$ and $l_1(z_1) \neq 1$ & l_2,m_2 are *Fuzzy* singletons with support z_2 such that $m_2(z_2) = 1$ and $l_2(z_2) \neq 1$. Now the *Fuzzy* sets $1_Z - m_1$ & $1_Z - m_2$ are *Fuzzy* gp * open sets satisfying $l_1 \leq 1_Z - m_2 \leq 1_Z - l_2$ & $l_2 \leq 1_Z - m_1 \leq 1_Z - l_1$ implying (Z, τ) is $Fgp^* - T_1$. ☐

Definition 4.3. *A Fuzzy topological space (Z, τ) is Fgp^*-Hausdorff or $Fgp^* - T_2$ if for arbitrary Fuzzy singletons X^1_λ and x^2_μ, their exists Fuzzy gp *-open sets Z_1 & Z_2 such that $x^1_\lambda \leq Z_1 \leq (1 - x^2_\mu)$, $x^2_\mu \leq Z_2 \leq (1 - x^1_\lambda)$ and $Z_1 \leq 1 - Z_2$.*
It is obvious that every $Fgp^* - T_2$ space is $Fgp^* - T_1$ space.

Definition 4.4. *A Fuzzy topological space (Z, τ) is Fgp *-Uryshon or $Fgp^* - T_{2\frac{1}{2}}$ if for arbitrary Fuzzy singletons x^1_λ and x^2_μ, their exists Fuzzy gp *-open sets Z_1 & Z_2 such that $x^1_\lambda \leq Z_1 \leq (1 - x^2_\mu)$, $x^2_\mu \leq Z_2 \leq (1 - x^1_\lambda)$ and $Fgp^* - cl(Z_1) \leq 1 - (Fgp^* - cl(Z_2))$.*

Remark 4.1. Every *Fuzzy* pre-open set in fts (Z, τ) is a *Fuzzy gp**-open set in (Z,τ).

Proof. Suppose α is a *Fuzzy* pre-open set in (Z,τ), so $1 - \alpha$ is *Fuzzy* pre-closed. Now by Theorem 3.1 every *Fuzzy* pre-closed set is *Fuzzy gp**-closed, implying $1 - \alpha$ is *Fuzzy gp**-closed & so α is a *Fuzzy gp**-open set in (Z, τ). ☐

Theorem 4.3. *Every FPT_0 space is $Fgp^* - T_0$ space.*

Proof. Suppose (Z, τ) is a FPT_0-space, so by [2] for *Fuzzy* singletons l_1 & l_2 with supports z_1,z_2 ($z_1 \neq z_2$) their exists a *Fuzzy* pre-open set ν such that $l_1 \leq \nu \leq 1_Z - l_2$ or $l_2 \leq \nu \leq 1_Z - l_1$. Now by Remark 4.1 ν is a *Fuzzy gp**-open set satisfying $l_1 \leq \nu \leq 1_Z - l_2$ or $l_2 \leq \nu \leq 1_Z - l_1$. Hence (Z,$\tau$) is a $Fgp^* - T_0$ space. ☐

Remark 4.2. The converse of the above theorem need not be true, for proof the following example is given.

Example 4.1. If $Z = \{z_1, z_2, z_3, z_4\}$ is a space with *Fuzzy* topology $\tau = \{0_Z, 1_Z, l, m, n, o\}$ where $l, m, n, o : Z \to [0, 1]$ are defined as

$$l(z) = \begin{cases} 1 & \text{if } z = z_1 \\ 0 & \text{otherwise,} \end{cases}$$

$$m(z) = \begin{cases} 1 & \text{if } z = z_2 \\ 0 & \text{otherwise,} \end{cases}$$

$$n(z) = \begin{cases} 1 & \text{if } z = z_1, z_2 \\ 0 & \text{otherwise,} \end{cases}$$

$$o(z) = \begin{cases} 1 & \text{if } z = z_1, z_2, z_3 \\ 0 & \text{otherwise.} \end{cases}$$

In this space Z with such kind of topology τ, the *Fuzzy* set p defined below is Fgp^*-open but not *Fuzzy* pre-open, implying that the space (Z, τ) is $Fgp^* - T_0$ but not FPT_0.

$$p(z) = \begin{cases} 1 & \text{if } z = z_1, z_3, z_4 \\ 0 & \text{otherwise/} \end{cases}$$

Theorem 4.4. *All FPT_1 spaces are $Fgp^* - T_1$ spaces.*

Proof. Suppose (Z,τ) is a FPT_1 space, so by the definition of FPT_1 for arbitrary singletons l_1 and l_2, $l_1 \leq \nu_1 \leq 1 - l_2$ & $l_2 \leq \nu_2 \leq 1 - l_1$ where ν_1 and ν_2 are *Fuzzy* pre-open sets. Now by Remark 4.1 ν_1 and ν_2 are *Fuzzy* gp *-open, concluding that (Z, τ) is a $Fgp^* - T_1$ spaces. ☐

Remark 4.3. The converse of the above theorem may not be true as shown in the following example.

Example 4.2. In the *Fuzzy* topological space defined in Example 4.1, the *Fuzzy* sets p & q defined below are Fgp *-open but not *Fuzzy* pre-open, implying that the space (Z, τ) is $Fgp^* - T_1$ but not FPT_1.

$$p(z) = \begin{cases} 1 & \text{if } z = z_1, z_3, z_4 \\ 0 & \text{otherwise,} \end{cases}$$

$$q(z) = \begin{cases} 1 & \text{if } z = z_1, z_4 \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 4.5. *All FPT_2 spaces are $Fgp^* - T_2$ spaces.*

Proof. From the definition of FPT_2 spaces in [11] and from Remark 4.1, the proof is obvious. □

Remark 4.4. The converse of the above theorem need not be true as shown in the given example.

Example 4.3. In the *Fuzzy* topological space defined in Example 4.1, the *Fuzzy* sets r & s defined below are Fgp^*-open but not *Fuzzy* pre-open, implying that the space (Z, τ) is $Fgp^* - T_2$ but not FPT_2.

$$r(z) = \begin{cases} 1 & \text{if } z = z_3 \\ 0 & \text{otherwise,} \end{cases}$$

$$s(z) = \begin{cases} 1 & \text{if } z = z_4 \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 4.6. *Every FT_0 space is $Fgp^* - T_0$ space.*

Proof. Suppose (Z, τ) is a FT_0 -space, so by [12] for *Fuzzy* singletons l_1 & l_2 with different supports, their exists a *Fuzzy* open set ν such that $l_1 \leq \nu \leq 1_Z - l_2$ or $l_2 \leq \nu \leq 1_Z - l_1$. Now from Remark 2.3 every *Fuzzy* open set is *Fuzzy* gp *-open, implying that ν is a *Fuzzy gp**-open set satisfying $l_1 \leq \nu \leq 1_Z - l_2$ or $l_2 \leq \nu \leq 1_Z - l_1$. Hence (Z, τ) is a $Fgp^* - T_0$ space. □

Remark 4.5. The converse of the above theorem need not be true as shown in the following example.

Example 4.4. If $Z = \{z_1, z_2, z_3, z_4, z_5\}$ is a space with *Fuzzy* topology $\tau = \{0_Z, 1_Z, \lambda_1, \lambda_2, \lambda_3\}$ where $\lambda_1, \lambda_2, \lambda_3 : Z \to [0, 1]$ are defined as

$$\lambda_1(z) = \begin{cases} 1 & \text{if } z = z_1, z_2 \\ 0 & \text{otherwise,} \end{cases}$$

$$\lambda_2(z) = \begin{cases} 1 & \text{if } z = z_3, z_4 \\ 0 & \text{otherwise,} \end{cases}$$

$$\lambda_3(z) = \begin{cases} 1 & \text{if } z = z_1, z_2, z_3, z_4 \\ 0 & \text{otherwise.} \end{cases}$$

In this *Fuzzy* topological space, the *Fuzzy* set λ_4 defined below is a *Fuzzy* gp *-open set but not *Fuzzy* open, implying that the space (Z, τ) is $Fgp^* - T_0$ but not FT_0.

$$\lambda_4(z) = \begin{cases} 1 & \text{if } z = z_1, z_2, z_4, z_5 \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 4.7. *Every FT_1 space is $Fgp^* - T_1$ space.*

Proof. The proof is trivial from the definitions of FT_1 and $Fgp^* - T_1$ spaces and from the result that every *Fuzzy* open set is *Fuzzy gp**-open [5]. □

Remark 4.6. The converse that every $Fgp^* - T_1$ space is a FT_1 space is not true, for proof the following example is given

Example 4.5. In *Fuzzy* topological space (Z, τ) defined in Example 4.4, the *Fuzzy* sets λ_4 and λ_5 defined below are *Fuzzy gp**-open sets but not *Fuzzy* open sets, implying the *Fuzzy* space (Z, τ) is a $Fgp^* - T_1$ space but not a FT_1.

$$\lambda_4(z) = \begin{cases} 1 & \text{if } z = z_1, z_2, z_4, z_5 \\ 0 & \text{otherwise,} \end{cases}$$

$$\lambda_5(z) = \begin{cases} 1 & \text{if } z = z_2, z_3, z_4, z_5 \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 4.8. *Every FT_2 space is $Fgp^* - T_2$ space.*

Proof. The proof is straightforward. □

Remark 4.7. The converse of the above theorem need not be true as shown in the following example.

Example 4.6. In *Fuzzy* topological space (Z, τ) defined in Example 4.4, the *Fuzzy* sets λ_4 and λ_5 defined below are *Fuzzy gp**-open sets but not *Fuzzy* open sets, implying the *Fuzzy* space (Z, τ) is a $Fgp^* - T_2$ space but not a FT_2.

$$\lambda_4(z) = \begin{cases} 1 & \text{if } z = z_1, z_2, z_4, z_5 \\ 0 & \text{otherwise.} \end{cases}$$

$$\lambda_5(z) = \begin{cases} 1 & \text{if } z = z_2, z_3, z_4, z_5 \\ 0 & \text{otherwise} \end{cases}$$

From the above discussion, we have the following diagram of implications

$$\begin{array}{ccccc} FT_0 & \Leftarrow & FT_1 & \Leftarrow & FT_2 \\ \Downarrow\not\Uparrow & & \Downarrow\not\Uparrow & & \Downarrow\not\Uparrow \\ Fgp^*-T_0 & \Leftarrow & Fgp^*-T_1 & \Leftarrow & Fgp^*-T_2 \\ \not\Downarrow\Uparrow & & \not\Downarrow\Uparrow & & \not\Downarrow\Uparrow \\ FPT_0 & \Leftarrow & FPT_1 & \Leftarrow & FPT_2 \end{array}$$

5 Conclusion

The main portion of this manuscript is dedicated to *Fuzzy* separation axioms via *Fuzzy gp**-open sets, We introduced these axioms and find out their relation with *Fuzzy* separation axioms and *Fuzzy* pre separation axioms introduced earlier. We can further investigate these spaces and relate the new results with the results already in trending in this area.

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