

COMMON FIXED-POINT THEOREM USING ψ -WEAK CONTRACTION FOR EIGHT SELF-MAPPINGS IN FUZZY METRIC SPACESonu¹, Anil Kumar² and Santosh Kumar³^{1,2}Department of Mathematics, School of Physical Sciences, Starex University, Gurugram, Haryana, India-122413³Department of Mathematics, College of Natural and Applied Sciences, University of Dar es Salaam, TanzaniaEmail: sonuu.yadav007@gmail.com, dranilkumar73@rediffmail.com and drsengar2002@gmail.com

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DOI: <https://doi.org/10.58250/jnanabha.2023.53136>**Abstract**

Applying weakly compatible for eight self-mappings in fuzzy metric space, we demonstrate common fixed-point theorems in this analysis after already formulating the generalised ψ - weak contraction condition, which involves third and fourth components of $\mathcal{M}(x, y, t)$.

2020 Mathematical Sciences Classification: 47H10, 54H25.**Keywords and Phrases:** ψ -Weak Contraction, weakly Compatible mappings, fuzzy metric space.**1 Introduction**

The idea of fuzzy sets was developed by Zadeh [22] in 1965 as a novel approach to depict the ambiguity in daily life. The development of fuzzy mathematics started at this point. In 1975, Kramosil and Michalek [10] defined the fuzzy metric space with the help of continuous t -norm by using the concepts of fuzziness. Fuzzy set theory is used in a wide range of real-world applications, including neural networks, fixed theory, health care, image processing, and control theory. When Zadeh [22] introduced the idea of a fuzzy set, which served as the basis for fuzzy mathematics, it marked a turning point in the history of mathematics.

Fuzzy mathematics has developed rapidly over the past three decades as a result, and recent studies have revealed that practically all fields of mathematics, including arithmetic, topology, graph theory, probability theory, logic, etc., have been fuzzyfied [1, 2, 4, 8, 9, 12, 13]. Communication, image processing, control theory, mathematical programming, neural network theory, stability theory, engineering, and medical sciences are among the applied areas where fuzzy set theory is used (medical genetics, nervous system). It makes sense that fuzzy fixed point theory has become more popular among experts in the discipline and that fuzzy mathematics has opened up new opportunities for fixed point theorists. For more details on this topic, one can see [5, 11, 14, 15, 16, 17, 18, 20]

2 Preliminaries

Definition 2.1 ([19]). Let $(\mathfrak{B}, M, *)$ be a fuzzy metric space and \mathcal{G} and \mathcal{H} be two self-mappings of this space. When $\{x_n\}$ is a sequence in \mathfrak{B} such that $\lim_{n \rightarrow \infty} \mathcal{G}x_n = \lim_{n \rightarrow \infty} \mathcal{H}x_n = u$ for some $u \in \mathfrak{B}$, the mappings \mathcal{G} and \mathcal{H} are known as compatible if $\lim_{n \rightarrow \infty} \mathcal{M}(\mathcal{G}\mathcal{H}x_n, \mathcal{H}\mathcal{G}x_n, t) = 1$, for all $t > 0$.

Jungck [6, 7] presented the idea of weakly compatible mappings in 1986 and demonstrated that weakly compatible maps are compatible maps, despite the possibility that the opposite is also true. Later Subrahmanyam [19] extended the definition as follows:

Definition 2.2 ([19]). If G and H commute at their coincidence sites, they are considered to be weakly compatible.

Definition 2.3 ([3]). If \mathfrak{B} is arbitrary set, $*$ is a continuous t -norm, \mathcal{M} is a fuzzy set in $\mathfrak{B}^2 \times [0, \infty)$, the triplet $(\mathfrak{B}, \mathcal{M}, *)$ meets the following requirements for being a fuzzy metric space:

(i) $\mathcal{M}(x, y, t) > 0$,(ii) $\mathcal{M}(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,

- (iii) $\mathcal{M}(x, y, t) = \mathcal{M}(y, x, t)$,
- (iv) $(\mathcal{M}(x, y, t) * \mathcal{M}(y, z, s)) \leq \mathcal{M}(x, z, t + s)$,
- (v) $\mathcal{M}(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is left continuous for all $x, y, z \in \mathfrak{B}$ and $s, t > 0$,
- (vi) $\lim_{t \rightarrow \infty} \mathcal{M}(x, y, t) = 1$, for all $x, y, \in \mathfrak{B}$.

$\mathcal{M}(x, y, t)$ is a measure of how close together x and y are with regard to t .

Definition 2.4 ([14]). Let $(\mathfrak{B}, \mathcal{M}, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ in \mathfrak{B} is defined as:

- (i) Converge to $x \in \mathfrak{B}$ if $\lim_{n \rightarrow \infty} \mathcal{M}(x_n, x, t) = 1$ for each $t > 0$.
- (ii) Cauchy sequence if $\lim_{n \rightarrow \infty} \mathcal{M}(x_n, x_p, t) = 1$ for all $t > 0$ and $p > 0$.
- (iii) Complete if every Cauchy sequence in \mathfrak{B} is convergent in \mathfrak{B} .

Proposition 2.1 ([6]). Let A and B be compatible mappings of a fuzzy metric space $(\mathfrak{B}, \mathcal{M}, *)$ into itself. If $At = Bt$ for some t in \mathfrak{B} , then $ABt = AAt = BBt = BAAt$.

Proposition 2.2 ([6]). Let A and B be compatible mappings of a fuzzy metric space $(\mathfrak{B}, \mathcal{M}, *)$ into itself.

Suppose that $\lim_n Ax_n = \lim_n Bx_n = t$ for some t in \mathfrak{B} . Then the following holds:

- (i) $Bx_n = At$ if A is continuous at t ;
- (ii) $Ax_n = Bt$ if B is continuous at t ;
- (iii) $ABt = BAAt$ and $At = Bt$ if A and B are continuous at t .

Lemma 2.1 ([19]). Let $(\mathfrak{B}, \mathcal{M}, *)$ be a fuzzy metric space. If there exists $q \in (0, 1)$ such that $\mathcal{M}(x, y, qt) \geq \mathcal{M}(x, y, t)$ for all $x, y \in \mathfrak{B}$, and $t > 0$, then $x = y$.

Lemma 2.2 ([19]). Let $\{y_n\}$ be a sequence in a fuzzy metric space $(\mathfrak{B}, \mathcal{M}, *)$. If there exists $q \in (0, 1)$ such that $\mathcal{M}(y_{(n+2)}, y_{(n+1)}, qt) \geq \mathcal{M}(y_{(n+1)}, y_n, t)$, $t > 0$, $n \in N$, then y_n is a Cauchy sequence in \mathfrak{B} .

Lemma 2.3 ([20]). Let $(\mathfrak{B}, \mathcal{M}, *)$ be a fuzzy metric space. If there is a sequence $\{x_n\} \in X$, such that for every $n \in N$,

$$\mathcal{M}(x_n, x_{(n+1)}, t) \geq \mathcal{M}(x_0, x_1, k^n t)$$

for every $k > 1$, then the sequence is a Cauchy sequence.

3 Main Results

Let Ψ be set of all continuous functions $\psi : [0, 1]^4 \rightarrow [0, 1]$ increasing in any coordinate and $\psi(t, t, t, t) > t$.

Theorem 3.1. Let $(\mathfrak{B}, \mathcal{M}, *)$ be a complete fuzzy metric space. Let $\mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{S}, \mathcal{T}, \mathcal{K}, \mathcal{L}$ and \mathcal{W} are eight self-mappings of a complete fuzzy metric space $(\mathfrak{B}, \mathcal{M}, *)$ into itself satisfying

- (C₁) $\mathcal{TK}(\mathfrak{B}) \subseteq \mathcal{NP}(\mathfrak{B}), \mathcal{WL}(\mathfrak{B}) \subseteq \mathcal{QS}(\mathfrak{B})$,
- (C₂) $\mathcal{QS} = \mathcal{SQ}, \mathcal{NP} = \mathcal{PN}, \mathcal{TK} = \mathcal{KT}, \mathcal{WL} = \mathcal{LW}, (\mathcal{TK})\mathcal{S} = \mathcal{S}(\mathcal{TK}), (\mathcal{WL})\mathcal{P} = \mathcal{P}(\mathcal{WL}), (\mathcal{NP})\mathcal{L} = \mathcal{L}(\mathcal{NP}), (\mathcal{QS})\mathcal{K} = \mathcal{K}(\mathcal{QS})$
- (C₃) One of $\mathcal{NP}(\mathfrak{B}), \mathcal{WL}(\mathfrak{B}), \mathcal{QS}(\mathfrak{B})$ or $\mathcal{TK}(\mathfrak{B})$ is complete ,
- (C₄) The pair $(\mathcal{TK}, \mathcal{QS})$ and $(\mathcal{WL}, \mathcal{NP})$ are weakly compatible,
- (C₅) $M^3(\mathcal{TK}u, \mathcal{WL}v, t)$

$$\geq \psi \left\{ \begin{array}{l} M^2(\mathcal{QS}u, \mathcal{TK}u, ht)M(\mathcal{WL}v, \mathcal{NP}v, ht), \\ M(\mathcal{QS}u, \mathcal{TK}u, ht)M^2(\mathcal{WL}v, \mathcal{NP}v, ht), \\ M(\mathcal{QS}u, \mathcal{TK}u, ht)M(\mathcal{TK}u, \mathcal{WL}v, ht)M(\mathcal{WL}v, \mathcal{NP}v, ht), \\ M(\mathcal{WL}v, \mathcal{NP}v, ht)M(\mathcal{QS}u, \mathcal{NP}v, ht)M(\mathcal{QS}u, \mathcal{TK}u, ht) \end{array} \right\}$$

for all $u, v \in \mathfrak{B}, h > 1$ and $\psi \in \Psi$.

Then $\mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{S}, \mathcal{T}, \mathcal{K}, \mathcal{L}$ and \mathcal{W} have a unique common fixed point in \mathfrak{B} .

Proof. Let $x_0 \in \mathfrak{B}$ be an arbitrary point. By (C₁) we can search a point x_1 such that $\mathcal{TK}(x_0) = \mathcal{NP}(x_1) = y_0$. For this point x_1 one can search a point $x_2 \in \mathfrak{B}$ such that $\mathcal{WL}(x_1) = \mathcal{QS}(x_2) = y_1$. By continuing in this manner, a sequence $\{x_n\}$ can be created, such that $y_{2n} = \mathcal{JK}(x_{2n}) = \mathcal{NP}(x_{2n+1})$,

$$y_{2n+1} = \mathcal{WL}(x_{2n+1}) = \mathcal{QS}(x_{2n+2}), \text{ for each } n \geq 0. \quad (3.1)$$

□

For simplicity, we take $\alpha_m(t) = \mathcal{M}(y_m, y_{m+1}, t)$.

Initially, we establish that $\{y_n\}$ is Cauchy sequence.

Case I. If n is even, considering $u = x_{2n}$ and $v = x_{2n+1}$ in (C_5) , we get

$$M^3(\mathcal{TK}x_{2n}, \mathcal{WL}x_{2n+1}, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{QS}x_{2n}, \mathcal{TK}x_{2n}, ht)M(\mathcal{WL}x_{2n+1}, \mathcal{NP}x_{2n+1}, ht), \\ M(\mathcal{QS}x_{2n}, \mathcal{TK}x_{2n}, ht)M^2(\mathcal{WL}x_{2n+1}, \mathcal{NP}x_{2n+1}, ht), \\ M(\mathcal{QS}x_{2n}, \mathcal{TK}x_{2n}, ht)M(\mathcal{TK}x_{2n}, \mathcal{WL}x_{2n+1}, ht)M(\mathcal{WL}x_{2n+1}, \mathcal{NP}x_{2n+1}, ht), \\ M(\mathcal{WL}x_{2n+1}, \mathcal{NP}x_{2n+1}, ht)M(\mathcal{QS}x_{2n}, \mathcal{NP}x_{2n+1}, ht)M(\mathcal{QS}x_{2n}, \mathcal{TK}x_{2n}, ht) \end{array} \right\}.$$

Using (3.1), we have

$$M^3(y_{2n}, y_{2n+1}, t) \geq \psi \left\{ \begin{array}{l} M^2(y_{2n-1}, y_{2n}, ht)M(y_{2n+1}, y_{2n}, ht), \\ M(y_{2n-1}, y_{2n}, ht)M^2(y_{2n+1}, y_{2n}, ht), \\ M(y_{2n-1}, y_{2n}, ht)M(y_{2n}, y_{2n+1}, ht)M(y_{2n+1}, y_{2n}, ht), \\ M(y_{2n+1}, y_{2n}, ht)M(y_{2n-1}, y_{2n}, ht)M(y_{2n-1}, y_{2n}, ht) \end{array} \right\}.$$

On using $\alpha_{2n}(t) = \mathcal{M}(y_{2n}, y_{2n+1}, t)$ in the above inequality, we have

$$\alpha_{2n}^3(t) \geq \psi \left\{ \begin{array}{l} \alpha_{2n-1}^2(ht)\alpha_{2n}(ht), (\alpha_{2n-1}(ht)\alpha_{2n}^2(ht)), \\ \alpha_{2n-1}(ht)\alpha_{2n}^2(ht), (\alpha_{2n}(ht)\alpha_{2n-1}^2(ht)) \end{array} \right\}. \quad (3.2)$$

We claim that $\alpha_{2n}(ht) \geq \alpha_{2n-1}(ht)$

If $\alpha_{2n}(ht) < \alpha_{2n-1}(ht)$, then (3.2) reduces to

$$\alpha_{2n}^3(t) \geq \psi \left\{ \alpha_{2n}^3(ht), \alpha_{2n}^3(ht), \alpha_{2n}^3(ht), \alpha_{2n}^3(ht) \right\}.$$

Using property of ψ we get

$$\alpha_{2n}^3(t) > \alpha_{2n}^3(ht) \implies \alpha_{2n}(t) > \alpha_{2n}(ht),$$

a contradiction.

Therefore $\alpha_{2n}(ht) \geq \alpha_{2n-1}(ht)$.

Like in similar manner, if n is odd, then we can achieve $\alpha_{2n+1}(ht) \geq \alpha_{2n}(ht)$.

It follows that the sequence $\alpha_n(t)$ is increasing in $[0, 1]$, thus (3.2) reduces to

$$\alpha_{2n}^3(t) \geq \psi \left\{ \alpha_{2n-1}^3(ht), \alpha_{2n-1}^3(ht), \alpha_{2n-1}^3(ht)\alpha_{2n-1}^3(ht) \right\}.$$

Using property of ψ we get

$$\alpha_{2n}^3(t) > \alpha_{2n-1}^3(ht) \implies \alpha_{2n}(t) \geq \alpha_{2n-1}(t).$$

Similarly for an odd integer $2n+1$, we have $\alpha_{2n+1}(t) \geq \alpha_{2n}(ht)$,

Hence $\alpha_n(t) \geq \alpha_{n-1}(ht)$, that is,

$$M(y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, ht) \geq \dots \geq M(y_0, y_1, h^n t).$$

Hence by Lemma 2.3 $\{y_n\}$ is a Cauchy sequence in \mathfrak{B} .

Case II. $\mathcal{NP}(\mathfrak{B})$ is complete. In this case $\{y_{2n}\} = \{\mathcal{NP}x_{2n+1}\}$ is a Cauchy sequence in $\mathcal{NP}(\mathfrak{B})$, which is complete then the sequence $\{y_{2n}\}$ converges to some point $z \in \mathcal{NP}(\mathfrak{B})$. Consequently, the subsequences $\{\mathcal{TK}x_{2n}\}$, $\{\mathcal{QS}x_{2n}\}$, $\{\mathcal{NP}x_{2n+1}\}$, and $\{\mathcal{WL}x_{2n+1}\}$ also converges to the same point z . As $z \in \mathcal{NP}(\mathfrak{B})$, there exists $r \in \mathfrak{B}$ such that $z = \mathcal{NP}r$.

Now we claim that $z = \mathcal{WL}r$. For this putting $u = x_{2n}$ and $v = r$ in (C_5) , we get

$$M^3(\mathcal{TK}x_{2n}, \mathcal{WL}r, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{QS}x_{2n}, \mathcal{TK}x_{2n}, ht)M(\mathcal{WL}r, \mathcal{NP}r, ht), \\ M(\mathcal{QS}x_{2n}, \mathcal{TK}x_{2n}, ht)M^2(\mathcal{WL}r, \mathcal{NP}r, ht), \\ M(\mathcal{QS}x_{2n}, \mathcal{TK}x_{2n}, ht)M(\mathcal{TK}x_{2n}, \mathcal{WL}r, ht)M(\mathcal{WL}r, \mathcal{NP}r, ht), \\ M(\mathcal{WL}r, \mathcal{NP}r, ht)M(\mathcal{QS}x_{2n}, \mathcal{NP}r, ht)M(\mathcal{QS}x_{2n}, \mathcal{TK}x_{2n}, ht) \end{array} \right\}.$$

Taking limit $n \rightarrow \infty$ and using $z = \mathcal{NP}r$ in above inequality we have,

$$M^3(z, \mathcal{WL}r, t) \geq \psi \left\{ \begin{array}{l} M^2(z, z, ht)M(\mathcal{WL}r, z, ht), \\ M(z, z, ht)M^2(\mathcal{WL}r, z, ht), \\ M(z, z, ht)M(z, \mathcal{WL}r, ht)M(\mathcal{WL}r, z, ht), \\ M(\mathcal{WL}r, z, ht)M(z, z, ht)M(z, z, ht) \end{array} \right\},$$

$$M^3(z, \mathcal{W}\mathcal{L}r, t) \geq \psi \left\{ \begin{array}{l} 1.1.M(\mathcal{W}\mathcal{L}r, z, ht), \\ 1.M^2(\mathcal{W}\mathcal{L}r, z, ht), \\ 1.M(z, \mathcal{W}\mathcal{L}r, ht)M(\mathcal{W}\mathcal{L}r, z, ht), \\ M(\mathcal{W}\mathcal{L}r, z, ht).1.1 \end{array} \right\}.$$

Suppose $\mathcal{W}\mathcal{L}r \neq z$, then $M(z, \mathcal{W}\mathcal{L}r, ht) < 1$, using this in above inequality we get

$$M^3(z, \mathcal{W}\mathcal{L}r, t) \geq \psi \{M^3(z, \mathcal{W}\mathcal{L}r, ht), M^3(z, \mathcal{W}\mathcal{L}r, ht), M^3(z, \mathcal{W}\mathcal{L}r, ht), M^3(z, \mathcal{W}\mathcal{L}r, ht)\}.$$

Using property of ψ we get

$$\begin{aligned} M^3(z, \mathcal{W}\mathcal{L}r, t) &> M^3(z, \mathcal{W}\mathcal{L}r, ht) \\ \implies M(z, \mathcal{W}\mathcal{L}r, t) &> M(z, \mathcal{W}\mathcal{L}r, ht), \text{ a contradiction.} \end{aligned}$$

Hence $\mathcal{W}\mathcal{L}r = z$

Thus $\mathcal{W}\mathcal{L}r = z = \mathcal{N}\mathcal{P}r$. Since $(\mathcal{W}\mathcal{L}, \mathcal{N}\mathcal{P})$ are weakly compatible, so we have $\mathcal{W}\mathcal{L}z = \mathcal{N}\mathcal{P}z$.

Next, we will show that $\mathcal{P}z = z$, for this putting $u = x_{2n}$ and $v = \mathcal{P}r$ in (C_5) , we get

$$M^3(\mathcal{T}\mathcal{K}x_{2n}, \mathcal{W}\mathcal{L}\mathcal{P}r, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M(\mathcal{W}\mathcal{L}\mathcal{P}r, \mathcal{N}\mathcal{P}\mathcal{P}r, ht), \\ M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M^2(\mathcal{W}\mathcal{L}\mathcal{P}r, \mathcal{N}\mathcal{P}\mathcal{P}r, ht), \\ M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M(\mathcal{T}\mathcal{K}x_{2n}, \mathcal{W}\mathcal{L}\mathcal{P}r, ht)M(\mathcal{W}\mathcal{L}\mathcal{P}r, \mathcal{N}\mathcal{P}\mathcal{P}r, ht) \\ M(\mathcal{W}\mathcal{L}\mathcal{P}r, \mathcal{N}\mathcal{P}\mathcal{P}r, ht)M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{N}\mathcal{P}\mathcal{P}r, ht)M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht) \end{array} \right\}.$$

From (C_2) $\mathcal{W}\mathcal{L}\mathcal{P} = \mathcal{P}\mathcal{W}\mathcal{L}$ and $\mathcal{N}\mathcal{P} = \mathcal{P}\mathcal{N}$ using in above inequality we get,

$$M^3(\mathcal{T}\mathcal{K}x_{2n}, \mathcal{P}\mathcal{W}\mathcal{L}r, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M(\mathcal{P}\mathcal{W}\mathcal{L}r, \mathcal{P}\mathcal{N}\mathcal{P}r, ht), \\ M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M^2(\mathcal{P}\mathcal{W}\mathcal{L}r, \mathcal{P}\mathcal{N}\mathcal{P}r, ht), \\ M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M(\mathcal{T}\mathcal{K}x_{2n}, \mathcal{P}\mathcal{W}\mathcal{L}r, ht)M(\mathcal{P}\mathcal{W}\mathcal{L}r, \mathcal{P}\mathcal{N}\mathcal{P}r, ht) \\ M(\mathcal{P}\mathcal{W}\mathcal{L}r, \mathcal{P}\mathcal{N}\mathcal{P}r, ht)M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{P}\mathcal{N}\mathcal{P}r, ht)M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht) \end{array} \right\}.$$

Taking limit $n \rightarrow \infty$ and using $\mathcal{W}\mathcal{L}r = z = \mathcal{N}\mathcal{P}r$ in above inequality we have,

$$M^3(z, \mathcal{P}z, t) \geq \psi \left\{ \begin{array}{l} M^2(z, z, ht)M(\mathcal{P}z, \mathcal{P}z, ht), \\ M(z, z, ht)M^2(\mathcal{P}z, \mathcal{P}z, ht), \\ M(z, z, ht)M(z, \mathcal{P}z, ht)M(\mathcal{P}z, \mathcal{P}z, ht) \\ M(\mathcal{P}z, \mathcal{P}z, ht)M(z, \mathcal{P}z, ht)M(z, z, ht) \end{array} \right\}.$$

Suppose $\mathcal{P}z \neq z$, then $M(z, \mathcal{P}z, ht) < 1$, using this in above inequality we get

$$M^3(z, \mathcal{P}z, t) \geq \psi \{M^3(z, \mathcal{P}z, ht), M^3(z, \mathcal{P}z, ht), M^3(z, \mathcal{P}z, ht), M^3(z, \mathcal{P}z, ht)\}.$$

Using property of ψ we get

$$\begin{aligned} M^3(z, \mathcal{P}z, t) &> M^3(z, \mathcal{P}z, ht). \\ \implies M(z, \mathcal{P}z, t) &> M(z, \mathcal{P}z, ht), \text{ a contradiction.} \end{aligned}$$

Hence $z = \mathcal{P}z$.

Thus $\mathcal{P}z = \mathcal{N}\mathcal{P}z = z \implies \mathcal{N}z = z$.

Thus $\mathcal{N}z = \mathcal{P}z = \mathcal{W}\mathcal{L}z = z$.

Next, we will show that $\mathcal{L}z = z$, for this putting $u = x_{2n}$ and $v = \mathcal{L}r$ in (C_5) , we get

$$M^3(\mathcal{T}\mathcal{K}x_{2n}, \mathcal{W}\mathcal{L}\mathcal{L}r, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M(\mathcal{W}\mathcal{L}\mathcal{L}r, \mathcal{N}\mathcal{P}\mathcal{L}r, ht), \\ M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M^2(\mathcal{W}\mathcal{L}\mathcal{L}r, \mathcal{N}\mathcal{P}\mathcal{L}r, ht), \\ M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M(\mathcal{T}\mathcal{K}x_{2n}, \mathcal{W}\mathcal{L}\mathcal{L}r, ht)M(\mathcal{W}\mathcal{L}\mathcal{L}r, \mathcal{N}\mathcal{P}\mathcal{L}r, ht), \\ M(\mathcal{W}\mathcal{L}\mathcal{L}r, \mathcal{N}\mathcal{P}\mathcal{L}r, ht)M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{N}\mathcal{P}\mathcal{L}r, ht)M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht). \end{array} \right\}.$$

From (C_2) $\mathcal{W}\mathcal{L} = \mathcal{L}\mathcal{W}$ and $(\mathcal{N}\mathcal{P})\mathcal{L} = \mathcal{L}(\mathcal{P}\mathcal{N})$ using in above inequality we get,

$$M^3(\mathcal{T}\mathcal{K}x_{2n}, \mathcal{L}\mathcal{W}\mathcal{L}r, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M(\mathcal{L}\mathcal{W}\mathcal{L}r, \mathcal{L}\mathcal{N}\mathcal{P}r, ht), \\ M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M^2(\mathcal{L}\mathcal{W}\mathcal{L}r, \mathcal{L}\mathcal{N}\mathcal{P}r, ht), \\ M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht)M(\mathcal{T}\mathcal{K}x_{2n}, \mathcal{L}\mathcal{W}\mathcal{L}r, ht)M(\mathcal{L}\mathcal{W}\mathcal{L}r, \mathcal{L}\mathcal{N}\mathcal{P}r, ht), \\ M(\mathcal{L}\mathcal{W}\mathcal{L}r, \mathcal{L}\mathcal{N}\mathcal{P}r, ht)M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{L}\mathcal{N}\mathcal{P}r, ht)M(\mathcal{Q}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{K}x_{2n}, ht) \end{array} \right\}.$$

Taking limit $n \rightarrow \infty$ and using $W\mathcal{L}r = z = \mathcal{N}\mathcal{P}r$ in above inequality we have,

$$M^3(z, \mathcal{L}z, t) \geq \psi \left\{ \begin{array}{l} M^2(z, z, ht)M(\mathcal{L}z, \mathcal{L}z, ht), \\ M(z, z, ht)M^2(\mathcal{L}z, \mathcal{L}z, ht), \\ M(z, z, ht)M(z, \mathcal{L}z, ht)M(\mathcal{L}z, \mathcal{L}z, ht), \\ M(\mathcal{L}z, \mathcal{L}z, ht)M(z, \mathcal{L}z, ht)M(z, z, ht) \end{array} \right\}.$$

Suppose $\mathcal{L}z \neq z$, then $M(z, \mathcal{L}z, ht) < 1$, using this in above inequality we get

$$M^3(z, \mathcal{L}z, t) \geq \psi \{M^3(z, \mathcal{L}z, ht), M^3(z, \mathcal{L}z, ht), M^3(z, \mathcal{L}z, ht), M^3(z, \mathcal{L}z, ht)\}.$$

Using property of ψ we get

$$M^3(z, \mathcal{L}z, t) > M^3(z, \mathcal{L}z, ht).$$

$$\implies M(z, \mathcal{L}z, t) > M(z, \mathcal{L}z, ht), \text{ a contradiction.}$$

Hence $z = \mathcal{L}z$.

Thus, $\mathcal{L}z = W\mathcal{L}z = z \implies Wz = z$.

Thus $Nz = Pz = Wz = \mathcal{L}z = z$.

As $W\mathcal{L}(\mathfrak{B}) \subseteq \mathcal{Q}\mathcal{S}(\mathfrak{B})$, there exists $m \in \mathfrak{B}$ such that $z = W\mathcal{L}z = \mathcal{Q}\mathcal{S}m$.

Next, we will show that $\mathcal{T}\mathcal{K}m = z$, for this putting $u = m$ and $v = x_{2n+1}$ in (C_5) , we have

$$\begin{aligned} & M^3(\mathcal{T}\mathcal{K}m, W\mathcal{L}x_{2n+1}, t) \\ & \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{Q}\mathcal{S}m, \mathcal{T}\mathcal{K}m, ht)M(W\mathcal{L}x_{2n+1}, \mathcal{N}\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{Q}\mathcal{S}m, \mathcal{T}\mathcal{K}m, ht)M^2(W\mathcal{L}x_{2n+1}, \mathcal{N}\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{Q}\mathcal{S}m, \mathcal{T}\mathcal{K}m, ht)M(\mathcal{T}\mathcal{K}m, W\mathcal{L}x_{2n+1}, ht)M(W\mathcal{L}x_{2n+1}, \mathcal{N}\mathcal{P}x_{2n+1}, ht), \\ M(W\mathcal{L}x_{2n+1}, \mathcal{N}\mathcal{P}x_{2n+1}, ht)M(\mathcal{Q}\mathcal{S}m, \mathcal{N}\mathcal{P}x_{2n+1}, ht)M(\mathcal{Q}\mathcal{S}m, \mathcal{T}\mathcal{K}m, ht). \end{array} \right\}. \end{aligned}$$

Taking limit $n \rightarrow \infty$ and using $z = W\mathcal{L}z = \mathcal{Q}\mathcal{S}m$ in above inequality we have,

$$M^3(\mathcal{T}\mathcal{K}m, z, t) \geq \psi \left\{ \begin{array}{l} M^2(z, \mathcal{T}\mathcal{K}m, ht)M(z, z, ht), \\ M(z, \mathcal{T}\mathcal{K}m, ht)M^2(z, z, ht), \\ M(z, \mathcal{T}\mathcal{K}m, ht)M(\mathcal{T}\mathcal{K}m, z, ht)M(z, z, ht), \\ M(z, z, ht)M(z, z, ht)M(z, \mathcal{T}\mathcal{K}m, ht). \end{array} \right\}.$$

Suppose $\mathcal{T}\mathcal{K}m \neq z$, then $M(\mathcal{T}\mathcal{K}m, z, ht) < 1$, using this in above inequality we get

$$M^3(\mathcal{T}\mathcal{K}m, z, t) \geq \psi \left\{ \begin{array}{l} M^3(\mathcal{T}\mathcal{K}m, z, ht)M^3(\mathcal{T}\mathcal{K}m, z, ht), \\ M^3(\mathcal{T}\mathcal{K}m, z, ht)M^3(\mathcal{T}\mathcal{K}m, z, ht) \end{array} \right\}.$$

Using property of ψ we get

$$M^3(\mathcal{T}\mathcal{K}m, z, t) > M^3(\mathcal{T}\mathcal{K}m, z, ht).$$

$$\implies M(\mathcal{T}\mathcal{K}m, z, t) > M(\mathcal{T}\mathcal{K}m, z, ht), \text{ a contradiction.}$$

Hence $\mathcal{T}\mathcal{K}m = z$.

Since $(\mathcal{T}\mathcal{K}, \mathcal{Q}\mathcal{S})$ are weakly compatible, so $\mathcal{T}\mathcal{K}$ and $\mathcal{Q}\mathcal{S}$ commute their coincidence point m , then we have $\mathcal{T}\mathcal{K}z = \mathcal{Q}\mathcal{S}z$.

Next we will show that $\mathcal{T}\mathcal{K}z = z$, for this putting $u = z$ and $v = x_{2n+1}$ in (C_5) , we have

$$\begin{aligned} & M^3(\mathcal{T}\mathcal{K}z, W\mathcal{L}x_{2n+1}, t) \\ & \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{Q}\mathcal{S}z, \mathcal{T}\mathcal{K}z, ht)M(W\mathcal{L}x_{2n+1}, \mathcal{N}\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{Q}\mathcal{S}z, \mathcal{T}\mathcal{K}z, ht)M^2(W\mathcal{L}x_{2n+1}, \mathcal{N}\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{Q}\mathcal{S}z, \mathcal{T}\mathcal{K}z, ht)M(\mathcal{T}\mathcal{K}z, W\mathcal{L}x_{2n+1}, ht)M(W\mathcal{L}x_{2n+1}, \mathcal{N}\mathcal{P}x_{2n+1}, ht), \\ M(W\mathcal{L}x_{2n+1}, \mathcal{N}\mathcal{P}x_{2n+1}, ht)M(\mathcal{Q}\mathcal{S}z, \mathcal{N}\mathcal{P}x_{2n+1}, ht)M(\mathcal{Q}\mathcal{S}z, \mathcal{T}\mathcal{K}z, ht) \end{array} \right\}. \end{aligned}$$

Taking limit $n \rightarrow \infty$ and using $\mathcal{T}\mathcal{K}z = \mathcal{Q}\mathcal{S}z$ in above inequality we have

$$M^3(\mathcal{T}\mathcal{K}z, z, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{T}\mathcal{K}z, \mathcal{T}\mathcal{K}z, ht)M(z, z, ht), \\ M(\mathcal{T}\mathcal{K}z, \mathcal{T}\mathcal{K}z, ht)M^2(z, z, ht), \\ M(\mathcal{T}\mathcal{K}z, \mathcal{T}\mathcal{K}z, ht)M(\mathcal{T}\mathcal{K}z, z, ht)M(z, z, ht), \\ M(z, z, ht)M(\mathcal{T}\mathcal{K}z, z, ht)M(\mathcal{T}\mathcal{K}z, \mathcal{T}\mathcal{K}z, ht) \end{array} \right\}.$$

Suppose $\mathcal{T}\mathcal{K}z \neq z$, then $M(\mathcal{T}\mathcal{K}z, z, ht) < 1$, using this in above inequality we get

$$M^3(\mathcal{T}\mathcal{K}z, z, t) \geq \psi \left\{ \begin{array}{l} M^3(\mathcal{T}\mathcal{K}z, z, ht)M^3(\mathcal{T}\mathcal{K}z, z, ht), \\ M^3(\mathcal{T}\mathcal{K}z, z, ht)M^3(\mathcal{T}\mathcal{K}z, z, ht) \end{array} \right\}.$$

Using property of ψ , we get

$$\begin{aligned} M^3(\mathcal{TK}z, z, t) &> M^3(\mathcal{TK}z, z, ht). \\ \implies M(\mathcal{TK}z, z, t) &> M(\mathcal{TK}z, z, ht), \text{ a contradiction.} \end{aligned}$$

Hence $\mathcal{TK}z = z$.

Thus $\mathcal{TK}z = \mathcal{QS}z = z$.

Next we will show that $\mathcal{S}z = z$, for this putting $\mu = \mathcal{S}z$ and $v = x_{2n+1}$ in (C_5) , we have

$$M^3(\mathcal{TK}\mathcal{S}z, W\mathcal{L}x_{2n+1}, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{QSS}z, \mathcal{TK}\mathcal{S}z, ht)M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{QSS}z, \mathcal{TK}\mathcal{S}z, ht)M^2(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{QSS}z, \mathcal{TK}\mathcal{S}z, ht)M(\mathcal{TK}\mathcal{S}z, W\mathcal{L}x_{2n+1}, ht)M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht)M(\mathcal{QSS}z, N\mathcal{P}x_{2n+1}, ht)M(\mathcal{QSS}z, \mathcal{TK}\mathcal{S}z, ht) \end{array} \right\}.$$

From (C_2) $\mathcal{QS} = \mathcal{S}\mathcal{Q}$ and $(\mathcal{TK})\mathcal{S} = \mathcal{S}(\mathcal{TK})$ using in above inequality we have,

$$M^3(\mathcal{STK}z, W\mathcal{L}x_{2n+1}, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{SQS}z, \mathcal{STK}z, ht)M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{SQS}z, \mathcal{STK}z, ht)M^2(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{SQS}z, \mathcal{STK}z, ht)M(\mathcal{STK}z, W\mathcal{L}x_{2n+1}, ht)M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht)M(\mathcal{SQS}z, N\mathcal{P}x_{2n+1}, ht)M(\mathcal{SQS}z, \mathcal{STK}z, ht) \end{array} \right\}.$$

Taking limit $n \rightarrow \infty$ and using $\mathcal{TK}z = \mathcal{QS}z = z$ in above inequality we have,

$$M^3(\mathcal{S}z, z, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{S}z, \mathcal{S}z, ht)M(z, z, ht), \\ M(\mathcal{S}z, \mathcal{S}z, ht)M^2(z, z, ht), \\ M(\mathcal{S}z, \mathcal{S}z, kt)M(\mathcal{S}z, z, kt)M(z, z, ht), \\ M((z, z, kt)M(\mathcal{S}z, z, kt)M(\mathcal{S}z, \mathcal{S}z, ht) \end{array} \right\}.$$

Suppose $\mathcal{S}z \neq z$, then $M(\mathcal{S}z, z, ht) < 1$, using this in above inequality we get

$$M^3(\mathcal{S}z, z, t) \geq \psi \left\{ \begin{array}{l} M^3(\mathcal{S}z, z, ht), M^3(\mathcal{S}z, z, ht), \\ M^3(\mathcal{S}z, z, ht), M^3(\mathcal{S}z, z, ht) \end{array} \right\}$$

using property of ψ we get

$$\begin{aligned} M^3(\mathcal{S}z, z, t) &> M^3(\mathcal{S}z, z, ht) \\ \implies M(\mathcal{S}z, z, t) &> M(\mathcal{S}z, z, ht), \text{ a contradiction.} \end{aligned}$$

Hence $\mathcal{S}z = z$. Then $z = \mathcal{QS}z = \mathcal{Q}z$. Therefore $z = \mathcal{S}z = \mathcal{Q}z = \mathcal{TK}z$.

Next we will show that $\mathcal{K}z = z$, for this putting $u = \mathcal{K}z$ and $v = x_{2n+1}$ in (C_5) , we have

$$M^3(\mathcal{TKK}z, W\mathcal{L}x_{2n+1}, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{QSK}z, \mathcal{TKK}z, ht)M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{QSK}z, \mathcal{TKK}z, ht)M^2(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{QSK}z, \mathcal{TKK}z, ht)M(\mathcal{TKK}z, W\mathcal{L}x_{2n+1}, ht)M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht)M(\mathcal{QSK}z, N\mathcal{P}x_{2n+1}, ht)M(\mathcal{QSK}z, \mathcal{TKK}z, ht) \end{array} \right\}.$$

From C_2 , using $\mathcal{TK} = \mathcal{K}\mathcal{T}$, $(\mathcal{QS})\mathcal{K} = \mathcal{K}(\mathcal{QS})$ in above inequality we have

$$M^3(\mathcal{TKK}z, W\mathcal{L}x_{2n+1}, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{KQS}z, \mathcal{KTK}z, ht)M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{KQS}z, \mathcal{KTK}z, ht)M^2(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(\mathcal{KQS}z, \mathcal{KTK}z, ht)M(\mathcal{KTK}z, W\mathcal{L}x_{2n+1}, ht)M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht), \\ M(W\mathcal{L}x_{2n+1}, N\mathcal{P}x_{2n+1}, ht)M(\mathcal{KQS}z, N\mathcal{P}x_{2n+1}, ht)M(\mathcal{KQS}z, \mathcal{KTK}z, ht) \end{array} \right\}.$$

Taking limit $n \rightarrow \infty$ and using $\mathcal{TK}z = \mathcal{QS}z = z$ in above inequality we have,

$$M^3(\mathcal{K}z, z, t) \geq \psi \left\{ \begin{array}{l} M^2(\mathcal{K}z, \mathcal{K}z, ht)M(z, z, ht), \\ M(\mathcal{K}z, \mathcal{K}z, ht)M^2(z, z, ht), \\ M(\mathcal{K}z, \mathcal{K}z, ht)M(\mathcal{K}z, z, ht)M(z, z, ht), \\ M(z, z, ht)M(\mathcal{K}z, z, ht)M(\mathcal{K}z, \mathcal{K}z, ht) \end{array} \right\}.$$

Suppose $\mathcal{K}z \neq z$, then $M(\mathcal{K}z, z, ht) < 1$, using this in above inequality we get

$$M^3(\mathcal{K}z, z, t) \geq \psi \left\{ \begin{array}{l} M^3(\mathcal{K}z, z, ht), M^3(\mathcal{K}z, z, ht), \\ M^3(\mathcal{K}z, z, ht), M^3(\mathcal{K}z, z, ht) \end{array} \right\}.$$

Using property of ψ we get

$$M^3(\mathcal{K}z, z, t) > M^3(\mathcal{K}z, z, ht) \\ \implies M(\mathcal{K}z, z, t) > M(\mathcal{K}z, z, ht), \text{ a contradiction,}$$

Hence $\mathcal{K}z = z$.

Thus $\mathcal{T}\mathcal{K}z = \mathcal{T}z = z$.

Thus $\mathcal{Q}z = \mathcal{S}z = \mathcal{K}z = z$.

Hence $Nz = \mathcal{P}z = \mathcal{Q}z = \mathcal{S}z = \mathcal{T}z = \mathcal{K}z = \mathcal{L}z = Wz = z$.

Hence z be a unique fixed point of $N, \mathcal{P}, \mathcal{Q}, \mathcal{S}, \mathcal{T}, \mathcal{K}, \mathcal{L}$, and W .

4 Application

A fixed point theorem for a single mapping satisfies an analogue of a Banach contraction principle for an integral type inequality was discovered by Branciari in 2002.

As an application of Theorem 3.1, we now show the following theorem.

Theorem 4.1. *Let $N, \mathcal{P}, \mathcal{Q}, \mathcal{S}, \mathcal{T}, \mathcal{K}, \mathcal{L}$ and W be eight self mappings of a complete fuzzy metric space $(\mathbb{B}, M, *)$ satisfying the conditions $(C_1), (C_2), (C_3), (C_4)$ and the following condition.*

$$\int_0^{M^3(x,y,t)} \psi(w)dw \geq \int_0^{\sigma(u,v)} \psi(w)dw$$

$$\sigma(u, v) = \psi \left\{ \begin{array}{l} M^2(\mathcal{Q}\mathcal{S}u, \mathcal{T}\mathcal{K}u, ht)M(W\mathcal{L}v, N\mathcal{P}v, ht), \\ M(\mathcal{Q}\mathcal{S}u, \mathcal{T}\mathcal{K}u, ht)M^2(W\mathcal{L}v, N\mathcal{P}v, ht), \\ M(\mathcal{Q}\mathcal{S}u, \mathcal{T}\mathcal{K}\mathcal{K}z, ht)M(\mathcal{T}\mathcal{K}u, W\mathcal{L}v, ht)M(W\mathcal{L}v, N\mathcal{P}v, ht) \\ M(W\mathcal{L}v, N\mathcal{P}v, ht)M(\mathcal{Q}\mathcal{S}u, N\mathcal{P}v, ht)M(\mathcal{Q}\mathcal{S}u, \mathcal{T}\mathcal{K}u, ht) \end{array} \right\}$$

for all $u, v \in \mathbb{B}$, where $\psi : [0, 1]^4 \rightarrow [0, 1]$ is increasing in any coordinate and $\psi(t, t, t, t) > t$ for every $t \in [0, 1]$, where $\psi : [0, 1]^4 \rightarrow [0, 1]$ is a "Lebesgue-integrable function" which is summable, nonnegative, and such that, for each $\epsilon > 0$, $\int_0^\epsilon \psi(\omega)d\omega > 0$. Then $N, \mathcal{P}, \mathcal{Q}, \mathcal{S}, \mathcal{T}, \mathcal{K}, \mathcal{L}$ and W have a unique common fixed point in \mathcal{W} .

Proof. The theorem's proof proceeds in a manner similar to that of Theorem 3.1. □

5 Conclusion

For eight self-mappings in fuzzy metric space that contain third and fourth power of the distance measure $\mathcal{M}(x, y, t)$, we demonstrate the common fixed-point theorem.

Authors Contributions.

Each author contributed in the writing of this work. All authors read and approved the final draft.

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