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STUDY OF RICCI SOLITONS IN f-KENMOTSU MANIFOLDS WITH THE QUARTER-SYMMETRIC METRIC CONNECTION

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Abstract

In this paper, the non-existence of ξ-projectively flat 3-dimensional f-Kenmotsu manifold with quarter-symmetric metric connection has been established. Moreover, we prove that 3-dimensional f-Kenmotsu manifold with the quarter-symmetric metric connection is an η- Einstein manifold and the Ricci soliton is given as expanding or shrinking under certain restrictions on f.
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1 Introduction

In 1924, the notion of semi-symmetric connections on a manifold was introduced by Friedman and Schouten[7] and the notion of quarter-symmetric connections which are generalization of the semi-symmetric connections was defined and studied by Golab[14]. Kenmotsu, in 1972, studied a class of contact Riemannian manifold together with some special conditions and given it a name as Kenmotsu manifold.

A manifold M, with the structure (ϕ, ξ, η, g) is called normal if $[\phi, \phi] + 2d\eta \bigotimes \xi = 0$ and it is almost cosymplectic if $d\eta = 0$ and $d\phi = 0$. M is cosymplectic if it is normal and almost cosymplectic. Olszak and Rosca [12] studied f-Kenmotsu Manifolds in a geometrical aspect, and gave some curvature conditions. The other mathematicians studied that a Ricci-symmetric f-Kenmotsu Manifold is an Einstein Manifold. Later on, authors, in 2010, also proved that Ricci semi- symmetric α- Kenmotsu manifolds are Einstein manifolds. By f-Kenmotsu Manifolds we mean an almost contact metric manifold which is locally conformal almost cosymplectic and normal.

In 1983, the concept of Ricci solitons in contact geometry was studied by Sharma and Sinha [15]. Later, in contact metric manifold Crasmareanu [4], Bejan [2] and others deeply studied Ricci solitons.

In 2012, Ricci solitons on Kenmotsu manifolds were studied exclusively by Nagraja and Premlatha [11] and a study on quarter-symmetric metric connection were done by Sular, Özgur and De [13] and De and De [6] in different ways.

Section 1 is introductory and in section 2, we have some fundamental notions used in this study. Section 3 deals with the introduction of f-Kenmotsu Manifold. In the next section 4, we study f-Kenmotsu manifold with quarter-symmetric metric connection and proved that this manifold is not always ξ - projective flat. In the last section we prove that f-Kenmotsu manifold with the quarter-symmetric metric connection is η-Einstein manifold and the Ricci soliton defined on this manifold is classified with respect to the values of f and λ.

2 Preliminaries

Let us consider a 3-dimensional differentiable manifold M with an almost contact metric structure (ϕ, ξ, η, g), where ϕ is a (1,1) tensor field, ξ is a vecter field, η is a 1-form and g is Riemannian metric, satisfying

$$\begin{aligned}\phi^2 X &= -X + \eta(X)\xi,\\ \eta \circ \phi &= 0,\\ \phi\xi &= 0,\\ \eta(\xi) &= 1,\\ g(X, \xi) &= \eta(X),\end{aligned}$$

$$g(X, \phi Y) = -g(\phi X, Y),$$
$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.1}$$

for any vector fields $X, Y \in \chi(M)$. Then M is called an almost contact manifold. For an almost contact manifold M, we have

$$(\nabla_X \phi)Y = \nabla_X \phi Y - \phi(\nabla_X Y), \tag{2.2}$$
$$(\nabla_X \eta)Y = \nabla_X \eta Y - \eta(\nabla_X Y). \tag{2.3}$$

Let $\{e_1, e_2, e_3, ..., e_n\}$ be orthonormal basis of $T_p(M)$. R be Riemannian curvature tensor, S be Ricci curvature tensor, Q be Ricci operator, then $\forall X, Y \in \chi(M)$ it follows that [5]

$$S(X, Y) = \sum_{i=1}^{n} g(R(e_i, X)Y, e_i), \tag{2.4}$$

$$QX = -\sum_{i=1}^{n} R(e_i, X)e_i, \tag{2.5}$$

$$S(X, Y) = g(QX, Y). \tag{2.6}$$

In f-Kenmotsu manifold, if the Ricci tensor S satisfy the condition

$$S(X, Y) = \alpha g(X, Y) + \beta\eta(X)\eta(Y) \tag{2.7}$$

α, β be certain scalars, then the manifold M is said to be η-Einstein manifold. If $\beta = 0$, the manifold is Einstein manifold.

In a 3-dimensional Riemannian manifold, the curvature tensor R is defind as

$$\begin{aligned} R(X, Y)Z \quad = \quad & S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY \\ & -\frac{\tau}{2}[g(Y, Z)X - g(X, Z)Y]. \end{aligned} \tag{2.8}$$

where S is the Ricci tensor, Q is Ricci operator and τ is the scalar curvature.

Now, let M be an n-dimensional Riemannian manifold with the Riemannian connection ∇. A linear connection $\bar{\nabla}$ is said to be a quarter-symmetric connection on M if the torsion tensor $\bar{T}$ of the connection $\bar{\nabla}$ satisfies

$$\bar{T}(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y, \tag{2.9}$$

where $\bar{T} \neq 0$ and η is a 1-form.

If moreover $\bar{\nabla} g = 0$, then the connection is called quarter-symmetric metric connection.

If $\bar{\nabla} g \neq 0$, the connection is called quarter-symmetric non-metric connection[17].

For $n \geq 1$, the manifold M is locally projectively flat iff the projective curvature tensor P vanishes. We define the projective curvature tensor P as

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{2n}[S(Y, Z)X - S(X, Z)Y]. \tag{2.10}$$

for any $X, Y, Z \in \chi(M)$ where S is the Ricci tensor and R is the curvature tensor of M.

If $P(X, Y)\xi = 0$ for any $X, Y \in \chi(M)$, the manifold M is called ξ-projective flat[16].

A Ricci Soliton is defined on a Riemannian manifold (M, g) as a natural generalization of an Einstein metric. We define Ricci Soliton as a triple (g, V, λ) with g a Riemannian metric, V a vector field and λ be a real scalar such that

$$L_V g + 2S + 2\lambda g = 0 \tag{2.11}$$

where L_V denotes the Lie derivative operator along the vector field V and S is a Ricci tensor of M. The Ricci soliton is said to be shrinking, steady and expanding according as λ is negative, zero and positive respectively.

3 f-Kenmotsu manifolds

A 3-dimensional almost contact manifold M with the structure (ϕ, ξ, η, g) is an f-Kenmotsu manifold if the covariant derivative of ϕ satisfies [17],

$$(\nabla_X \phi)Y = f[g(\phi X, Y)\xi - \eta(Y)\phi X] \tag{3.1}$$

where $f \in C^\infty(M, R)$ such that $df \wedge \eta = 0$.

If $f^2 + f' \neq 0$, where $f'=\xi f$, then M is called Regular [4]. If $f = \alpha = constant \neq 0$, M is called α-Kenmotsu Manifold. If $f = 1$, the manifold is called Kenmotsu manifold.

By (2.1) and (2.3), we have

$$(\nabla_X \eta)Y = fg(\phi X, \phi Y). \tag{3.2}$$

From (3.1), we have [15]

$$\nabla_X \xi = f[X - \eta(X)\xi]. \tag{3.3}$$

Also from (2.6), in 3-dimensional f-Kenmotsu manifold

$$\begin{aligned} R(X,Y)Z &= (\frac{\tau}{2} + 2f^2 + 2f')(X \wedge Y)Z \\ &\quad -(\frac{\tau}{2} + 3f^2 + 3f')[\eta(X)(\xi \wedge Y)Z + \eta(Y)(X \wedge \xi)Z] \end{aligned} \tag{3.4}$$

and

$$S(X,Y) = (\frac{\tau}{2} + f^2 + f')g(X,Y) - (\frac{\tau}{2} + 3f^2 + 3f')\eta(X)\eta(Y). \tag{3.5}$$

Thus from (3.5), we get

$$S(X, \xi) = -2(f^2 + f')\eta(X) \tag{3.6}$$

by (3.4) and (3.5),we get

$$R(X,Y)\xi = -(f^2 + f')[\eta(Y)X - \eta(X)Y] \tag{3.7}$$

$$R(\xi, X)\xi = -(f^2 + f')(\eta(X)\xi - X), \tag{3.8}$$

$$QX = (\frac{\tau}{2} + f^2 + f')X - (\frac{\tau}{2} + 3f^2 + 3f')\eta(X)\xi. \tag{3.9}$$

From (2.10) and using (3.6) and (3.7), we have

Theorem 3.1. *A 3-dimensional f-Kenmotsu manifold is always ξ-projectively flat.*

4 f-Kenmotsu Manifolds with the quarter-symmetric metric connection

Let ∇ be a Riemannian connection of f-Kenmotsu manifold and $\bar{\nabla}$ be a linear connection then this linear connection $\bar{\nabla}$ defined as

$$\bar{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y, \tag{4.1}$$

where $X, Y \in \chi(M)$ be any vector field and η be 1-form, is called the quarter-symmetric metric connection. Now, using (2.2),(3.1) and (4.1) we have

$$(\bar{\nabla}_X \phi)Y = f[g(\phi X, Y)\xi - \eta(Y)\phi X], \tag{4.2}$$

for any vector field $X, Y \in \chi(M)$, where ϕ be (1,1) tensor field, ξ is a vector field, η is 1-form and $f \in C^\infty(M, \mathbf{R})$ so that $df \wedge \eta = 0$. As a result of $df \wedge \eta = 0$, we have

$$df = f', \quad X(f) = f'\eta(X), \tag{4.3}$$

where $f'=\xi f$ [11].

If $f = 0$, manifold is cosymplectic. If $f = \alpha \neq 0$, then the manifold is α-Kenmotsu. An f-Kenmotsu manifold with quarter -symmetric metric connection is called regular if $f^2 + f' - 2f\phi \neq 0$.

By (2.2),(4.2) we have

$$\bar{\nabla}_X \xi = f[X - \eta(X)\xi]. \tag{4.4}$$

Using (2.2),(2.1) and (3.2), we get

$$(\bar{\nabla}_X \eta)Y = fg(\phi X, \phi Y). \tag{4.5}$$

We define the curvature tensor $\bar{R}$ of f-Kenmotsu manifold M with respect to quarter-symmetric metric connection $\bar{\nabla}$ as

$$\bar{R}(X,Y)\xi = \bar{\nabla}_X\bar{\nabla}_Y\xi - \bar{\nabla}_Y\bar{\nabla}_X\xi - \bar{\nabla}_{[X,Y]}\xi. \tag{4.6}$$

Using (4.1), (4.4) and (3.3), we obtain

$$\begin{aligned}\bar{\nabla}_X\bar{\nabla}_Y\xi &= X(f)Y - \eta(Y)X(f)\xi + f\nabla_X Y - f\eta(X)\phi Y - f^2\eta(Y)X \\ &\quad + f^2\eta(X)\eta(Y)\xi - fX\eta(Y)\xi,\end{aligned} \tag{4.7}$$

$$\begin{aligned}\bar{\nabla}_Y\bar{\nabla}_X\xi &= Y(f)X - \eta(X)Y(f)\xi + f\nabla_Y X - f\eta(Y)\phi X - f^2\eta(X)Y \\ &\quad + f^2\eta(Y)\eta(X)\xi - fY\eta(X)\xi\end{aligned} \tag{4.8}$$

and

$$\bar{\nabla}_{[X,Y]}\xi = f\nabla_X Y - f\nabla_Y X - fX\eta(Y)\xi + fY\eta(X)\xi. \tag{4.9}$$

Using (4.9) and (4.8) in (4.6), we have

$$\begin{aligned}\bar{R}(X,Y)\xi &= X(f)Y - Y(f)X - \eta(Y)X(f)\xi + \eta(X)Y(f)\xi - f\eta(X)\phi Y \\ &\quad + f\eta(Y)\phi X - f^2\eta(Y)X + f^2\eta(X)Y.\end{aligned} \tag{4.10}$$

Now using (4.3) in (4.10), we have

$$\bar{R}(X,Y)\xi = -(f^2+f')(\eta(Y)X - \eta(X)Y) + f(\eta(Y)\phi X - \eta(X)\phi Y). \tag{4.11}$$

From (4.11), we get

$$\bar{R}(\xi,Y)\xi = -(f^2+f')(\eta(Y)\xi - Y) - f\phi Y, \tag{4.12}$$

and

$$\bar{R}(X,\xi)\xi = -(f^2+f')(X - \eta(X)\xi) + f\phi X. \tag{4.13}$$

In (4.11) taking inner product with Z, we get

$$\begin{aligned}g(\bar{R}(X,Y)\xi,Z) &= -(f^2+f')(\eta(Y)g(X,Z) - \eta(X)g(Y,Z)) \\ &\quad + f(\eta(Y)g(\phi X,Z) - \eta(X)g(\phi Y,Z)).\end{aligned} \tag{4.14}$$

Now we have,

Lemma 4.1. *Let M be 3-dimensional f-Kenmotsu manifold with the quarter-symmetric metric connection.If $\bar{S}$ be Ricci curvature and $\bar{Q}$ be Ricci operator. Then*

$$\bar{S}(X,\xi) = -2(f^2+f')\eta(X), \tag{4.15}$$

and

$$\bar{Q}\xi = -2(f^2+f')\xi. \tag{4.16}$$

Proof. Contracting (4.14) with Y and Z, taking summation over $i = 1, 2, 3, ..., n$ and using (2.4) the proof of (4.15) is completed. Also by (2.6) and (2.1) in (4.15), we get (4.16).

Lemma 4.2. *Let M be 3-dimensional f-Kenmotsu manifold with quarter-symmetric metric connection.If $\bar{S}$ be Ricci tensor, τ be scaler curvature tensor and $\bar{Q}$ Ricci operator. Then it follows that*

$$\begin{aligned}\bar{S}(X,Y) &= (\frac{\tau}{2} + f^2 + f')g(X,Y) - (\frac{\tau}{2} + 3f^2 + 3f')\eta(X)\eta(Y) \\ &\quad + fg(\phi X,Y),\end{aligned} \tag{4.17}$$

and

$$\bar{Q}X = (\frac{\tau}{2} + f^2 + f')X - (\frac{\tau}{2} + 3f^2 + 3f')\eta(X)\xi + f\phi X. \tag{4.18}$$

Proof. Contracting with Y in (4.13), we get

$$g(\bar{R}(X,\xi)\xi,Y) = -(f^2+f')(g(X,Y) - \eta(X)\eta(Y)) + fg(\phi X,Y). \tag{4.19}$$

Putting X=ξ, Y=X, Z=Y in (2.8), using (4.15) and taking contraction with ξ, we obtain

$$\begin{aligned}g(\bar{R}(\xi,X)Y,\xi) &= \bar{S}(X,Y) - (2f^2 + 2f' + \frac{\tau}{2})g(X,Y) + (4f^2 + 4f' + \frac{\tau}{2})\eta(X)\eta(Y) \\ &\quad - \frac{\tau}{2}[g(X,Y) - \eta(X)\eta(Y)].\end{aligned} \tag{4.20}$$

With the help of (4.19) and (4.20), we have (4.17) . Now using (4.17)and (2.6), we get

$$g(\bar{Q}X - [(\frac{\tau}{2} + f^2 + f')X - (\frac{\tau}{2} + 3f^2 + 3f')\eta(X)\xi + f\phi X], Y) = 0. \tag{4.21}$$

Since $Y \neq 0$ in (4.21), which leads the proof of (4.18).

Example 4.1 (A 3-dimensional f-Kenmotsu manifold with quarter-symmetric metric connection). Let us consider a 3-dimensional manifold $M = (x, y, z) \in R^3$, z≠0, where (x, y, z) are the standard coordinates in R^3. The vector fields

$$e_1 = z^2 \frac{\partial}{\partial x}, e_2 = z^2 \frac{\partial}{\partial y}, e_3 = \frac{\partial}{\partial z}$$

are linearly independent at each point of M. Let g be the Riemannian metric defined as

$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1, \quad g(e_1, e_2) = g(e_2, e_3) = g(e_3, e_1) = 0.$$

Considering a (1,1) tensor field ϕ defined by

$$\phi(e_1) = -e_2, \quad \phi(e_2) = e_1, \quad \phi(e_3) = 0,$$

then using linearity of g and ϕ, for any Z,$W \in \chi\ (M)$, we get

$$\eta(e_3) = 1,$$
$$\phi^2(Z) = -Z + \eta(Z)e_3,$$
$$g(\phi Z, \phi W) = g(Z, W) - \eta(Z)\eta(W).$$

Now by computation directly, we get

$$[e_1, e_2] = 0, [e_2, e_3] = -\frac{2}{z}e_2, [e_1, e_3] = -\frac{2}{z}e_1.$$

By the use of these above equations we have

$$\nabla_{e_1} e_1 = \frac{2}{z}e_3,\ \nabla_{e_2} e_2 = \frac{2}{z}e_3,\ \nabla_{e_3} e_3 = 0,\ \nabla_{e_2} e_1 = \nabla_{e_1} e_2 = \nabla_{e_3} e_1 = \nabla_{e_3} e_3 = 0. \tag{4.22}$$

Now in this example we consider for quarter-symmetric metric connection. By using (4.1) and (4.21), we have

$$\bar{\nabla}_{e_i} e_i = \frac{2}{z}e_3,\ \bar{\nabla}_{e_3} e_3 = 0,\ \bar{\nabla}_{e_i} e_3 = -\frac{2}{z}e_i,\ \bar{\nabla}_{e_i} e_j = 0,\ \bar{\nabla}_{e_3} e_1 = e_2,\ \bar{\nabla}_{e_3} e_2 = -e_1 \tag{4.23}$$

where $i \neq j = 1,\ 2$. We know that

$$\bar{R}(X, Y)Z = \bar{\nabla}_X \bar{\nabla}_Y Z - \bar{\nabla}_Y \bar{\nabla}_X Z - \bar{\nabla}_{[X,Y]} Z. \tag{4.24}$$

Using (4.23) and (4.24), we get

$$\begin{aligned}
\bar{R}(e_1, e_3)e_3 &= -\frac{2}{z}(\frac{3e_1}{z} + e_2),\\
\bar{R}(e_2, e_3)e_3 &= -\frac{2}{z}(\frac{3e_2}{z} - e_1),\\
\bar{R}(e_i, e_j)e_3 &= 0,\ i, j = 1, 2\\
\bar{R}(e_i, e_j)e_j &= \frac{4}{z^2}e_i,\ i, j = 1, 2\\
\bar{R}(e_1, e_3)e_2 &= -\frac{2}{z}e_3,\\
\bar{R}(e_2, e_3)e_1 &= \frac{2}{z}e_3,\\
\bar{R}(e_3, e_i)e_i &= -\frac{6}{z^2}e_3.
\end{aligned} \tag{4.25}$$

where $i \neq j$=1,2.

Using (2.4) and (4.25), we verify that

$$\bar{S}(e_i, e_i) = -\frac{2}{z^2},\ i = 1, 2, \quad \bar{S}(e_3, e_3) = -\frac{12}{z^2}. \tag{4.26}$$

Now using (2.10),(4.25) and (4.26), we find that

$$\bar{P}(e_1, e_2)e_3 = 0, \quad \bar{P}(e_i, e_3)e_3 = -\frac{2}{z}(\frac{2e_1}{z} + e_2).$$

This leads the following Proposition:

Proposition 4.1 *A 3-dimensional f-Kenmotsu manifold with the quarter-symmetric metric connection is not necessarily ξ-projectively flat.*

5 Ricci Solitons in f-Kenmotsu Manifold with the quarter-symmetric metric connection

Consider a 3-dimensional f-Kenmotsu manifold with the quarter-symmetric metric connection. Let V be pointwise collinear with ξ (i.e.$V = b\xi$, where b is a function). Then

$$(L_V g + 2S + 2\lambda g)(X,Y) = 0,$$

implies

$$\begin{aligned} 0 &= (Xb)\eta(Y) + bg(\bar{\nabla}_X\xi, Y) + (Yb)\eta(X) + bg(X, \bar{\nabla}_Y\xi) + 2\bar{S}(X,Y) \\ &\quad +2\lambda g(X,Y). \end{aligned} \tag{5.1}$$

Using (4.4) in (5.1), we get

$$2bfg(X,Y) - 2bf\eta(X)\eta(Y) + (Xb)\eta(Y) + (Yb)\eta(X) + 2\bar{S}(X,Y) + 2\lambda g(X,Y) = 0. \tag{5.2}$$

Substitute Y with ξ in (5.2), we have

$$Xb + (\xi b)\eta(X) - 4(f^2 + f')\eta(X) + 2\lambda\eta(X) = 0. \tag{5.3}$$

Again substituting X with ξ in (5.3)

$$\xi b = 2(f^2 + f') - \lambda. \tag{5.4}$$

Putting (5.4) in (4.3), we get

$$b = [2(f^2 + f') - \lambda]\eta. \tag{5.5}$$

Applying d on (5.5)

$$0 = db = [2(f^2 + f') - \lambda]d\eta. \tag{5.6}$$

Since $d\eta \neq 0$,we have

$$[2(f^2 + f') - \lambda] = 0. \tag{5.7}$$

Now using (5.5) and (5.7) it is obtained that b is constant. Hence from (5.2), we can verify

$$\bar{S}(X,Y) = -(bf + \lambda)g(X,Y) - bf\eta(X)\eta(Y). \tag{5.8}$$

which results that M is η-Einstein manifold. Thus we have:

Theorem 5.1. *If in a 3-dimensional f-Kenmotsu manifold M with quarter-symmetric metric connection, the metric g is a Ricci soliton and V is a pointwise collinear with ξ, then V is a constant multiple of ξ and M is η-Einstein manifold of the form (5.8) and Ricci Soliton is expanding or shrinking according as $\lambda = 2(f^2 + f')$ is positive or negative.*

6 Conclusion

In this study, we have some curvature conditions for 3-dimensional f-Kenmotsu manifolds with quarter-symmetric metric connection. We have also shown that these manifolds are not always ξ-projective flat. Finally, we have that 3-dimensional f-Kenmotsu manifold with the quarter-symmetric metric connection is also an η-Einstein manifold and the Ricci soliton defined expanding or shrinking on this manifold is named with respect to the values of f and λ.

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