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(Dedicated to Professor G. C. Sharma on His 85^{th} Birth Anniversary Celebrations)

AN EXTENDED GENERALIZED FIBONACCI POLYNOMIAL BASED CODING METHOD WITH ERROR DETECTION AND CORRECTION

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Abstract

A Fibonacci coding method is introduced using Extended Generalized Fibonacci Polynomials in this paper. A new square matrix $Q_m^n(a,b)$, the n^{th} power of $Q_m(a,b)$ of order $m \times m$ is defined whose elements are based on extended Generalized Fibonacci Polynomial. Matrix $Q_m^n(a,b)$ for integer $x \geq 1$, $a \geq 1$ and $b \geq 1$ is considered as the encoding matrix and a matrix $Q_m^{-n}(a,b)$ is considered as decoding matrix. An error-detection and error-correction method is also defined in Extended Generalized Fibonacci polynomials.
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1 Introduction

The Fibonacci sequence is one of the most well-known sequences, with numerous intriguing aspects and major applications in a variety of fields including Mathematics,Statistics, Biology, Physics, Finance, Architecture and Computer Sciences. The Fibonacci sequences and golden ratio have rich history, features and uses. This sequence has been modified in a variety of ways.

The Fibonacci Polynomial [5] and the Extended Generalized Fibonacci Polynomial [8] are two such extensions that will be used in this paper. The Fibonacci Polynomial $F_n(x)$ is defined by the recurrence relation shown below,

$$f_n(x) = \begin{cases} 1 & n = 1; \\ x & n = 2; \\ xf_{n-1}(x) + f_{n-2}(x) & n \geq 3. \end{cases} \tag{1.1}$$

There is no restriction on Fibonacci Polynomials for $n \leq 0$.
one such extension of Fibonacci Polynomial is the Extended Generalized Fibonacci Polynomial which is defined by the recurrence relation

$$g_n(x) = \begin{cases} 1 & n = 1; \\ a(x) & n = 2; \\ a(x)g_{n-1}(x) + b(x)g_{n-2}(x) & n \geq 3. \end{cases} \tag{1.2}$$

where $a(x)$, $b(x)$, $g_0(x)$ and $g_1(x)$ are arbitrary real Polynomials and $n \geq 0$. A non-recursive expression for $g_n(x)$, given below is introduced in [14].

$$g_{n+1}(x) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-i}{i} a^{n-2i} b^i \qquad n \geq 0. \tag{1.3}$$

This expression will appear several times in this paper. The first five Extended Generalized Fibonacci Polynomials are shown below.

$$g_n(x)=\begin{cases}1 & n=1;\\ a(x) & n=2;\\ [a(x)]^2+b(x) & n=3;\\ [a(x)]^3+2a(x)b(x) & n=4;\\ [a(x)]^4+3[a(x)]^2b(x)+[b(x)]^2 & n=5.\end{cases} \tag{1.4}$$

There is no restriction on Extended Generalized Fibonacci Polynomial for $n \leq 0$. In this paper, we set $g_0(x) = 0$ and $g_n(x) = 1$ for $n \leq -1$. It's worth nothing that the classical Fibonacci Polynomial can be created by substituting $a(x) = x$ and $b(x) = 1$ in the Extended Generalized Fibonacci Polynomial and the classical Jacobsthal Polynomial can be created by substituting $a(x) = 1$ and $b(x) = x$ in the Extended Generalized Fibonacci Polynomial. A Square matrices Q_m^n of order $m \times m$, $n \geq 1$, Properties, coding and decoding method, relation between code elements of message matrix and error-detection error-correction method has been introduced in Extended Generalized Fibonacci polynomials. This result,s is an extension of the result's [5]. For simplicity, we denote $g_n(x)$,$a(x)$, $b(x)$, $g_0(x)$ and $g_1(x)$ by g_n, a, b, g_0 and g_1 respectively.

2 Main Results

2.1 Extended Generalized Fibonacci Polynomial matrices of order m

The Extended Generalized Fibonacci Polynomials generated by the matrix given below.

$$Q_2(a,b)=\begin{pmatrix} a & b\\ 1 & 0\end{pmatrix}. \tag{2.1}$$

For any a and b, we have $\det(Q_2(a,b)) = -b$. Setting $g_0 = 0$ and applying induction on $n \geq 1$, it is easily verified that

$$Q_2^n(a,b)=\begin{pmatrix} g_{n+1} & bg_n\\ g_n & bg_{n-1}\end{pmatrix}. \tag{2.2}$$

By using the determinant theorem, we see that $\det(Q_2^n(a,b)) = (-b)^n$. The following defines the $m \times m$ matrix $Q_m(a,b)$.

$$Q_m(a,b)=\begin{pmatrix} a & b & 0 & 0 & \cdots & 0\\ 0 & a & b & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots\\ 0 & 0 & \cdots & 0 & a & b\\ 0 & 0 & \vdots & 0 & 1 & 0\end{pmatrix}_{m\times m}.$$

Thus $Q_m(a,b)$ has a recursive expression and $\det(Q_m(a,b)) = -a^{m-2}b$. The n^{th}, $n \geq 2$, power of $Q_m(a,b)$ is given by the following theorem.

Theorem 2.1. *For $n \geq 2$ and $m \geq 2$, we have*
$Q_m^n(a,b)$

$$=\begin{pmatrix}
\binom{n}{0}a^n & \binom{n}{1}a^{n-1}b & \cdots & \binom{n}{m-3}a^{n-m+3}b^{m-3} & \sum_{i=0}^{\lfloor\frac{n-m+2}{2}\rfloor}\binom{n-i}{i+m-2}a^{n-m+2-2i}b^{i+m-2} & \sum_{i=0}^{\lfloor\frac{n-m+1}{2}\rfloor}\binom{n-1-i}{i+m-2}a^{n-m+1-2i}b^{i+m-1}\\
0 & \binom{n}{0}a^n & \cdots & \binom{n}{m-4}a^{n-m+4}b^{m-4} & \sum_{i=0}^{\lfloor\frac{n-m+3}{2}\rfloor}\binom{n-i}{i+m-3}a^{n-m+3-2i}b^{i+m-3} & \sum_{i=0}^{\lfloor\frac{n-m+2}{2}\rfloor}\binom{n-1-i}{i+m-3}a^{n-m+2-2i}b^{i+m-2}\\
\vdots & \ddots & \ddots & \vdots & \vdots & \vdots\\
0 & 0 & \cdots & \binom{n}{0}a^n & \sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor}\binom{n-i}{i+1}a^{n-1-2i}b^{i+1} & \sum_{i=0}^{\lfloor\frac{n-2}{2}\rfloor}\binom{n-1-i}{i+1}a^{n-2-2i}b^{i+2}\\
0 & 0 & \cdots & 0 & \sum_{i=0}^{\lfloor\frac{n}{2}\rfloor}\binom{n-i}{i}a^{n-2i}b^{i} & \sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor}\binom{n-1-i}{i}a^{n-1-2i}b^{i+1}\\
0 & 0 & \cdots & 0 & \sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor}\binom{n-1-i}{i}a^{n-1-2i}b^{i} & \sum_{i=0}^{\lfloor\frac{n-2}{2}\rfloor}\binom{n-2-i}{i}a^{n-2-2i}b^{i+1}
\end{pmatrix}. \tag{2.3}$$

Proof. For the sake of simplicity, assume $m = 4$. The proof is based on induction. The following equality shows that eq.(2.3) holds for $n = 1$.

$$Q_4^1(a,b) = \begin{pmatrix} \binom{1}{0}a & \binom{1}{1}b & \sum\limits_{i=0}^{-1}\binom{1-i}{i+2}a^{-1-2i}b^{i+2} & \sum\limits_{i=0}^{-1}\binom{-i}{i+2}a^{-2-2i}b^{i+3} \\ 0 & \binom{1}{0}a & \sum\limits_{i=0}^{0}\binom{1-i}{i+1}a^{2i}b^{i+1} & \sum\limits_{i=0}^{-1}\binom{-i}{i+1}a^{-1-2i}b^{i+2} \\ 0 & 0 & g_2(x) & bg_1(x) \\ 0 & 0 & g_1(x) & bg_0(x) \end{pmatrix} = \begin{pmatrix} a & b & 0 & 0 \\ 0 & a & b & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Suppose the statement holds for $n = k$. Therefore, for $n = k+1$ we have,

$$Q_4^{k+1}(a,b) = \begin{pmatrix} a & b & 0 & 0 \\ 0 & a & b & 0 \\ 0 & 0 & a & b \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \binom{k}{0}a^k & \binom{k}{1}a^{k-1}b & \sum\limits_{i=0}^{\lfloor\frac{k-2}{2}\rfloor}\binom{k-i}{i+2}a^{k-2-2i}b^{i+2} & \sum\limits_{i=0}^{\lfloor\frac{k-3}{2}\rfloor}\binom{k-1-i}{i+2}a^{k-3-2i}b^{i+3} \\ 0 & \binom{k}{0}a^k & \sum\limits_{i=0}^{\lfloor\frac{k-1}{2}\rfloor}\binom{k-i}{i+1}a^{k-1-2i}b^{i+1} & \sum\limits_{i=0}^{\lfloor\frac{k-2}{2}\rfloor}\binom{k-1-i}{i+1}a^{k-2-2i}b^{i+2} \\ 0 & 0 & g_{k+1} & bg_k \\ 0 & 0 & g_k & bg_{k-1} \end{pmatrix}$$

$$= \begin{pmatrix} \binom{k}{0}a^{k+1} & \binom{k}{1}a^kb + \binom{k}{0}a^kb & q_{1,3} & q_{1,4} \\ 0 & \binom{k}{0}a^{k+1} & a\sum\limits_{i=0}^{\lfloor\frac{k-1}{2}\rfloor}\binom{k-i}{i+1}a^{k-1-2i} + bg_{k+1} & a\sum\limits_{i=0}^{\lfloor\frac{k-2}{2}\rfloor}\binom{k-1-i}{i+1}a^{k-2-2i}b^{i+2} + b^2g_k \\ 0 & 0 & ag_{k+1} + bg_k & b(ag_k + bg_{k-1}) \\ 0 & 0 & g_{k+1} & bg_k \end{pmatrix},$$

where,

$$\begin{cases} q_{1,3} = a\sum\limits_{i=0}^{\lfloor\frac{k-2}{2}\rfloor}\binom{k-i}{i+2}a^{k-2-2i}b^{i+2} + b\sum\limits_{i=0}^{\lfloor\frac{k-1}{2}\rfloor}\binom{k-i}{i+1}a^{k-1-2i}b^{i+1} \\ q_{1,4} = a\sum\limits_{i=0}^{\lfloor\frac{k-3}{2}\rfloor}\binom{k-1-i}{i+2}a^{k-3-2i}b^{i+3} + b\sum\limits_{i=0}^{\lfloor\frac{k-2}{2}\rfloor}\binom{k-1-i}{i+1}a^{k-2-2i}b^{i+2}. \end{cases}$$

Consider the first row of the last matrix. We need to show that the following four cases

$$\begin{cases} \binom{k}{0}a^{k+1} = \binom{k+1}{0}a^{k+1} \\ \binom{k}{1}a^kb + \binom{k}{0}a^kb = ka^kb + a^kb = \binom{k+1}{1}a^kb \\ q_{1,3} = \sum\limits_{i=0}^{\lfloor\frac{k-1}{2}\rfloor}\binom{k+1-i}{i+2}a^{k-1-2i}b^{i+2} \\ q_{1,4} = \sum\limits_{i=0}^{\lfloor\frac{k-2}{2}\rfloor}\binom{k-i}{i+2}a^{k-2-2i}b^{i+3} \end{cases}, \tag{2.4}$$

hold.

The first two cases of eq.(2.4)are easily verified. We will prove the third case of equation (2.4); fourth case is proved in a similar way. For the third case, there are two cases arise.

Case 1. Suppose k is even, so that $k = 2l$.Therefore

$$\left\lfloor\frac{k-2}{2}\right\rfloor = \left\lfloor\frac{2l-2}{2}\right\rfloor = l-1,$$
$$\left\lfloor\frac{k-1}{2}\right\rfloor = \left\lfloor\frac{2l-1}{2}\right\rfloor = l-1. \tag{2.5}$$

By substituting these relations in the L.H.S. of third case of equation (2.4), we get

$$\begin{aligned} q_{1,3} &= a\sum_{i=0}^{l-1}\binom{2l-i}{i+2}a^{2l-2-2i}b^{i+2} + b\sum_{i=0}^{l-1}\binom{2l-i}{i+1}a^{2l-1-2i}b^{i+1} \\ &= \sum_{i=0}^{l-1}\left[\binom{2l-i}{i+2} + \binom{2l-i}{i+1}\right]a^{2l-1-2i}b^{i+2} \\ &= \sum_{i=0}^{l-1}\binom{2l+1-i}{i+2}a^{2l-1-2i}b^{i+2} \end{aligned}$$

$$= \sum_{i=0}^{\lfloor\frac{k-1}{2}\rfloor} \binom{k+1-i}{i+2} a^{k-1-2i} b^{i+2}.$$

Case 2. Now assuming $k = 2l + 1$, we have

$$\begin{aligned} \left\lfloor \frac{k-2}{2} \right\rfloor &= \left\lfloor \frac{2l-1}{2} \right\rfloor = l-1, \\ \left\lfloor \frac{k-1}{2} \right\rfloor &= \left\lfloor \frac{2l}{2} \right\rfloor = l. \end{aligned} \tag{2.6}$$

By substituting these relations in the L.H.S. of third case of equation (2.4), we get

$$\begin{aligned} q_{1,3} &= a \sum_{i=0}^{l-1} \binom{2l+1-i}{i+2} a^{2l-1-2i} b^{i+2} + b \sum_{i=0}^{l} \binom{2l+1-i}{i+1} a^{2l-2i} b^{i+1} \\ &= \sum_{i=0}^{l-1} \binom{2l+1-i}{i+2} a^{2l-2i} b^{i+2} + \sum_{i=0}^{l} \binom{2l+1-i}{i+1} a^{2l-2i} b^{i+2} \\ &= \sum_{i=0}^{l-1} \left[\binom{2l+1-i}{i+2} + \binom{2l+1-i}{i+1} \right] a^{2l-2i} b^{i+2} + b^{l+2} \\ &= \sum_{i=0}^{l-1} \binom{2l+2-i}{i+2} a^{2l-2i} b^{i+2} + b^{l+2} \\ &= \sum_{i=0}^{l} \binom{2l+2-i}{i+2} a^{2l-2i} b^{i+2} \\ &= \sum_{i=0}^{\lfloor\frac{k-1}{2}\rfloor} \binom{k+1-i}{i+2} a^{k-1-2i} b^{i+2}. \end{aligned}$$

Further, other rows of $Q_4^{k+1}(a,b)$ can also be solved using above process.
This completes the proof. □

Example 2.1. For $m = 6$ and n=5 we have

$$Q_6^5(a,b) = \begin{pmatrix} a^5 & 5a^4b & 10a^3b^2 & 10a^2b^3 & 5ab^4 & b^5 \\ 0 & a^5 & 5a^4b & 10a^3b^2 & 10a^2b^3 + b^4 & 4ab^4 \\ 0 & 0 & a^5 & 5a^4b & 10a^3b^2 + 4ab^3 & 6a^2b^3 + b^4 \\ 0 & 0 & 0 & a^5 & 5a^4b + 6a^2b^2 + b^3 & 4a^3b^2 + 3ab^3 \\ 0 & 0 & 0 & 0 & a^5 + 3a^3b + a^2b + 3b^2 & a^4b + 3a^2b^2 + b^2 \\ 0 & 0 & 0 & 0 & a^4 + 3a^2b^2 + b^2 & a^3b + 2ab^2 \end{pmatrix}.$$

2.2 Properties of Extended Generalized Fibonacci Polynomial

Lemma 2.1. *For $n \geq k$ and $k \geq 1$ we have*

$$g_n = \frac{1}{a^{n-k}} \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{i} g_{2n-k-2i} b^i, \tag{2.7}$$

where g_n is the n^{th} Extended Generalized Fibonacci Polynomial.

Proof. Let k be a fixed number. The proof is by induction on $n \geq k$. Suppose that the equation holds for $k \leq n \leq l$. we show that (2.7) holds for $n = l+1$.
Then by the recurrence relation, we have

$$\begin{aligned} g_{l+1} &= a g_l + b g_{l-1} \\ &= a \left(\frac{1}{a^{l-k}} \sum_{i=0}^{l-k} (-1)^i \binom{l-k}{i} g_{2l-k-2i} b^i \right) + b \left(\frac{1}{a^{l-k-1}} \sum_{i=0}^{l-1-k} (-1)^i \binom{l-1-k}{i} g_{2l-2-k-2i} b^i \right) \end{aligned}$$

$$= \frac{1}{a^{l-k-1}}\left(\sum_{i=1}^{l-k}(-1)^i\binom{l-k}{i}g_{2l-k-2i}b^i + \sum_{i=0}^{l-1-k}(-1)^i\binom{l-1-k}{i}g_{2l-2-k-2i}b^{i+1} + g_{2l-k}\right)$$
$$= \frac{1}{a^{l-k-1}}\left(\sum_{i=0}^{l-k-1}(-1)^{i+1}\binom{l-k}{i+1}g_{2l-2-k-2i}b^{i+1} + \sum_{i=0}^{l-1-k}(-1)^i\binom{l-1-k}{i}g_{2l-2-k-2i}b^{i+1} + g_{2l-k}\right)$$
$$= \frac{1}{a^{l-k-1}}\left(\sum_{i=0}^{l-k-1}(-1)^i\left(\binom{l-1-k}{i} - \binom{l-k}{i+1}\right)g_{2l-2-k-2i}b^{i+1} + g_{2l-k}\right)$$
$$= \frac{1}{a^{l-k-1}}\left(\sum_{i=0}^{l-k-2}(-1)^{i+1}\binom{l-1-k}{i+1}g_{2l-2-k-2i}b^{i+1} + g_{2l-k}\right)$$
$$= \frac{1}{a^{l-k-1}}\left(\sum_{i=1}^{l-k-1}(-1)^i\binom{l-1-k}{i}g_{2l-k-2i}b^i + g_{2l-k}\right)$$
$$= \frac{1}{a^{l-k-1}}\sum_{i=0}^{l-k-1}(-1)^i\binom{l-1-k}{i}g_{2l-k-2i}b^i.$$

Expanding this relation using (1.2), we get

$$= \frac{a^{k+1}}{a^l}\sum_{i=0}^{l-k-1}(-1)^i\binom{l-1-k}{i}g_{2l-k-2i}b^i$$
$$= \frac{a^{k-1}}{a^l}\sum_{i=0}^{l-k-1}(-1)^i\binom{l-k-1}{i}\left(g_{2l-k-2i+2}b^i - 2g_{2l-k-2i}b^{i+1} + g_{2l-k-2-2i}b^{i+2}\right)$$
$$= \frac{a^{k-1}}{a^l}\left(\sum_{i=2}^{l-k-1}(-1)^i\left(\binom{l-k-1}{i} + 2\binom{l-k-1}{i-1} + \binom{l-k-1}{i-2}\right)g_{2l+2-k-2i}b^i + g_{2l+2-k}\right.$$
$$- (l-k-1)g_{2l-k}b - 2g_{2l-k}b - 2(-1)^{l-k+1}g_{k+2}b^{l-k} - (-1)^{l-k+1}(l-k-1)g_{k+2}b^{l-k}$$
$$\left. + (-1)^{l-k+1}g_kb^{l+1-k}\right)$$
$$= \frac{1}{a^{l+1-k}}\sum_{i=0}^{l+1-k}(-1)^i\binom{l+1-k}{i}g_{2l+2-k-2i}b^i.$$

This completed the proof. □

Lemma 2.2. *Binet formula:- The n^{th} Extended generalized Fibonacci polynomial is given by*

$$g_n = \frac{z_1^n - z_2^n}{z_1 - z_2},$$

where z_1, z_2 are the roots of the characteristic equation (1.2) *and $z_1 > z_2$.*

Proof. We can express the recurrence relation (1.2) into the function of roots of z_1 and z_2 and the characteristic equation of recurrence relation (1.2) is $z^2 = az + b$. The roots of the characteristic equation are $z_1 = \frac{a+\sqrt{a^2+4b}}{2}$ and $z_2 = \frac{a-\sqrt{a^2+4b}}{2}$.
Note that $z_2 < 0 < z_1$ and $|z_2| < |z_1|$.Also $z_1 + z_2 = a$, $z_1z_2 = -b$ and $z_1 - z_2 = \sqrt{a^2 + 4b}$.
Therefore, the general terms of Extended Generalized Fibonacci Polynomial may be expressed in the form $g_n = P_1z_1^n + P_2z_2^n$, for some coefficient P_1 and P_2, for the value $n = 0$ and $n = 1$, we have $P_1 = \frac{1}{z_1-z_2} = -P_2$, so that $g_n = \frac{z_1^n-z_2^n}{z_1-z_2}$. □

Lemma 2.3. $\lim_{n\to\infty} \frac{g_n}{g_{n-1}} = z_1$
Where z_1 is the positive root of characteristic equation (1.2).

Proof. By using Binet formula (see, Lemma 2.2), we have

$$\lim_{n\to\infty} \frac{g_n}{g_{n-1}} = \lim_{n\to\infty} \left(\frac{z_1^n - z_2^n}{z_1 - z_2} \times \frac{z_1 - z_2}{z_1^{n-1} - z_2^{n-1}} \right) = \lim_{n\to\infty} \frac{1 - (\frac{z_2}{z_1})^n}{\frac{1}{z_1} - (\frac{z_2}{z_1})^n (\frac{1}{z_2})}$$

and taking into account that $\lim\limits_{n\to\infty} (\frac{z_2}{z_1})^n = 0$, Since $|z_2| < |z_1|$ then we get our result. □

2.3 The inverse Extended Generalized Fibonacci polynomial matrices

Now, by use of lemma 2.1, the next theorem establishes the structure of the inverse Extended Generalized Fibonacci Polynomial Matrix $Q_m^{-n}(a,b)$.

Theorem 2.2. *For $m \geq 2$, $n \geq 1$,$a \neq 0$and $b \neq 0$, the matrix $Q_m^{-n}(a,b)$ is in the form $Q_m^{-n}(a,b) = (AB)_{m\times m}$ where*

$$A = \begin{pmatrix} \frac{\binom{n-1}{0}}{a^n} & -\frac{\binom{n}{1}b}{a^{n+1}} & -\frac{\binom{n+1}{2}b^2}{a^{n+2}} & \cdots & \frac{\binom{n+m-4}{m-3}b^{m-3}}{a^{n-m+3}} \\ 0 & \frac{\binom{n-1}{0}}{a^n} & -\frac{\binom{n}{1}b}{a^{n+1}} & \cdots & \frac{\binom{n+m-5}{m-4}b^{m-4}}{a^{n-m+4}} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & \frac{\binom{n-1}{0}}{a^n} \\ 0 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & 0 \end{pmatrix}_{m\times m-2},$$

$$B = \begin{pmatrix} \frac{(-a)^{m-2}a^{(m-3)(n-2)}}{(-a^{m-2})^n} \sum_{i=0}^{n-2}(-1)^i\binom{n+m-4}{i} g_{2n-3-2i}b^{i+m-n-1} & \frac{(a^{m-3})^{n-1}}{(-a^{m-2})^n} \sum_{i=0}^{n-1}(-1)^i\binom{n+m-3}{i} g_{2n-1-2i}b^{i+m-n-1} \\ \frac{(-a)^{m-3}a^{(m-4)(n-2)}}{(-a^{m-3})^n} \sum_{i=0}^{n-2}(-1)^i\binom{n+m-5}{i} g_{2n-3-2i}b^{i+m-n-2} & \frac{(a^{m-4})^{n-1}}{(-a^{m-3})^n} \sum_{i=0}^{n-1}(-1)^i\binom{n+m-4}{i} g_{2n-1-2i}b^{i+m-n-2} \\ \vdots & \vdots \\ \frac{-a}{(-a)^n} \sum_{i=0}^{n-2}(-1)^i\binom{n-1}{i} g_{2n-3-2i}b^{i+2-n} & \frac{1}{(-a)^n} \sum_{i=0}^{n-1}(-1)^i\binom{n}{i} g_{2n-1-2i}b^{i+2-n} \\ (-1)^n g_{n-1}b^{1-n} & (-1)^{n-1} g_n b^{1-n} \\ (-1)^{n-1} g_n b^{-n} & (-1)^n g_{n+1} b^{-n} \end{pmatrix}_{m\times 2}.$$

Proof. For the simplicity we prove the statement for $m = 3$. We show that $Q_3^n(a,b) \times Q_3^{-n}(a,b) = I_{3\times 3}$ holds for any n, where $I_{3\times 3}$ is the identity matrix of order 3.

$$Q_3^n(a,b) \times Q_3^{-n}(a,b) = \begin{pmatrix} \binom{n}{0}a^n & \sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor} \binom{n-i}{i+1} a^{n-1-2i}b^{i+1} & \sum_{i=0}^{\lfloor\frac{n-2}{2}\rfloor} \binom{n-1-i}{i+1} a^{n-2-2i}b^{i+2} \\ 0 & g_{n+1} & bg_n \\ 0 & g_n & bg_{n-1} \end{pmatrix}$$

$$\times \begin{pmatrix} \frac{\binom{n-1}{0}}{a^n} & \frac{-a}{(-a)^n} \sum_{i=0}^{n-2}(-1)^i\binom{n-1}{i} g_{2n-3-2i}b^{i+2-n} & \frac{1}{(-a)^n} \sum_{i=0}^{n-1}(-1)^i\binom{n}{i} g_{2n-1-2i}b^{i+2-n} \\ 0 & \frac{(-1)^n}{b^{n-1}} g_{n-1} & \frac{(-1)^{n-1}}{b^{n-1}} g_n \\ 0 & \frac{(-1)^{n-1}}{b^n} g_n & \frac{(-1)^n}{b^n} g_{n+1} \end{pmatrix}.$$

Using the relation $g_{n+1}g_{n-1} - g_n^2 = (-1)^n b^{n-1}$, it is easily verified that all the diagonal entries of this matrix are one. Now we have to show that all the other entries of this matrix are zero.for this, consider the elements of first row and second column.

$$q_{(1,2)} = \binom{n}{0} a^n \frac{-a}{(-a)^n} \sum_{i=0}^{n-2} (-1)^i \binom{n-1}{i} g_{2n-3-2i} b^{i+2-n} + \frac{(-1)^n}{b^{n-1}} g_{n-1} \sum_{i=0}^{\lfloor\frac{n-1}{2}\rfloor} \binom{n-i}{i+1} a^{n-1-2i} b^{i+1}$$
$$+ \frac{(-1)^{n-1}}{b^n} g_n \sum_{i=0}^{\lfloor\frac{n-2}{2}\rfloor} \binom{n-1-i}{i+1} a^{n-2-2i} b^{i+2}.$$

For an even integer $n = 2l$, using (2.5) and (2.7), we have

$$\begin{aligned}
q_{(1,2)} &= (-a)\sum_{i=0}^{2l-2}(-1)^i\binom{2l-1}{i}g_{4l-3-2i}b^{i+2-2l} + \frac{g_{2l-1}}{b^{2l-1}}\sum_{i=0}^{l-1}\binom{2l-i}{i+1}a^{2l-1-2i}b^{i+1}\\
&- \frac{g_{2l}}{b^{2l}}\sum_{i=0}^{l-1}\binom{2l-1-i}{i+1}a^{2l-2-2i}b^{i+2}\\
&= (-a)\sum_{i=0}^{2l-2}(-1)^i\binom{2l-1}{i}g_{4l-3-2i}b^{i+2-2l} + \frac{g_{2l-1}}{b^{2l-1}}\sum_{i=0}^{l-1}\binom{2l-i}{i+1}a^{2l-1-2i}b^{i+1}\\
&- \frac{g_{2l-1}}{b^{2l}}\sum_{i=0}^{l-1}\binom{2l-1-i}{i+1}a^{2l-1-2i}b^{i+2} - \frac{g_{2l-2}}{b^{2l}}\sum_{i=0}^{l-1}\binom{2l-1-i}{i+1}a^{2l-2-2i}b^{i+3}\\
&= (-a)\sum_{i=0}^{2l-2}(-1)^i\binom{2l-1}{i}g_{4l-3-2i}b^{i+2-2l} + \frac{g_{2l-1}}{b^{2l}}\sum_{i=0}^{l-1}\binom{2l-1-i}{i}a^{2l-1-2i}b^{i+2}\\
&- \frac{g_{2l-2}}{b^{2l}}\sum_{i=0}^{l-1}\binom{2l-1-i}{i+1}a^{2l-2-2i}b^{i+3}\\
&= (-a)\sum_{i=0}^{2l-2}(-1)^i\binom{2l-1}{i}g_{4l-3-2i}b^{i+2-2l} + \frac{g_{2l-1}}{b^{2l-2}}\sum_{i=0}^{l-1}\binom{2l-1-i}{i}a^{2l-1-2i}b^{i}\\
&- \frac{g_{2l-2}}{b^{2l-2}}\left(g_{2l+1} - a^{2l}\right)\\
&= (-a)\sum_{i=0}^{2l-2}(-1)^i\binom{2l-1}{i}g_{4l-3-2i}b^{i+2-2l} + \frac{g_{2l-1}g_{2l}}{b^{2l-2}} - \frac{g_{2l-2}g_{2l+1}}{b^{2l-2}} + \frac{g_{2l-2}a^{2l}}{b^{2l-2}}\\
&= (-a)\sum_{i=0}^{2l-2}(-1)^i\binom{2l-1}{i}g_{4l-3-2i}b^{i+2-2l} + a + \frac{g_{2l-2}a^{2l}}{b^{2l-2}}\\
&= a\left(1 - \frac{1}{b^{2l-2}}\sum_{i=0}^{2l-2}(-1)^i\binom{2l-1}{i}g_{4l-3-2i}b^{i}\right) + \frac{g_{2l-2}a^{2l}}{b^{2l-2}}\\
&= a\left(1 - \frac{a^{2l-1}g_{2l-2}}{b^{2l-2}} - 1\right) + \frac{g_{2l-2}a^{2l}}{b^{2l-2}}\\
&= 0.
\end{aligned}$$

Now for odd number $n = 2l+1$, using the equations (2.6) and (2.7), we have

$$\begin{aligned}
q_{(1,2)} &= (a)\sum_{i=0}^{2l-1}(-1)^i\binom{2l}{i}g_{4l-1-2i}b^{i+1-2l} - \frac{g_{2l}}{b^{2l}}\sum_{i=0}^{l}\binom{2l+1-i}{i+1}a^{2l-2i}b^{i+1}\\
&+ \frac{g_{2l+1}}{b^{2l+1}}\sum_{i=0}^{l-1}\binom{2l-i}{i+1}a^{2l-1-2i}b^{i+2}\\
&= (a)\sum_{i=0}^{2l-1}(-1)^i\binom{2l}{i}g_{4l-1-2i}b^{i+1-2l} - \frac{g_{2l}}{b^{2l}}\sum_{i=0}^{l}\binom{2l+1-i}{i+1}a^{2l-2i}b^{i+1}\\
&+ \frac{g_{2l}}{b^{2l+1}}\sum_{i=0}^{l-1}\binom{2l-i}{i+1}a^{2l-2i}b^{i+2} + \frac{g_{2l-1}}{b^{2l+1}}\sum_{i=0}^{l-1}\binom{2l-i}{i+1}a^{2l-1-2i}b^{i+3}\\
&= (a)\sum_{i=0}^{2l-1}(-1)^i\binom{2l}{i}g_{4l-1-2i}b^{i+1-2l} - \frac{g_{2l}}{b^{2l-1}}\sum_{i=0}^{l}\binom{2l-i}{i}a^{2l-2i}b^{i} + \frac{g_{2l-1}}{b^{2l-1}}\sum_{i=0}^{l-1}\binom{2l-i}{i+1}a^{2l-1-2i}b^{i+1}\\
&= (a)\sum_{i=0}^{2l-1}(-1)^i\binom{2l}{i}g_{4l-1-2i}b^{i+1-2l} - \frac{g_{2l}g_{2l+1}}{b^{2l-1}} + \frac{g_{2l-1}}{b^{2l-1}}\left(g_{2l+2} - a^{2l+1}\right)
\end{aligned}$$

$$= (a)\sum_{i=0}^{2l-1}(-1)^i\binom{2l}{i}g_{4l-1-2i}b^{i+1-2l} + a - \frac{g_{2l-1}a^{2l+1}}{b^{2l-1}}$$

$$= (a)\left(1+\sum_{i=0}^{2l-1}(-1)^i\binom{2l}{i}g_{4l-1-2i}b^{i+1-2l}\right) - \frac{g_{2l-1}a^{2l+1}}{b^{2l-1}}$$

$$= a\left(1+\frac{a^{2l}g_{2l-1}}{b^{2l-2}} - 1\right) - \frac{g_{2l-1}a^{2l+1}}{b^{2l-1}}$$

$$= 0.$$

Similarly, we can shown that any other non-diagonal entries of the matrix is also zero.
This completes the proof. □

2.4 The Extended Generalized Fibonacci Polynomial based coding algorithm

The Extended Generalized Fibonacci Polynomial coding algorithm is described in detail in this section.For coding and decoding algorithm Extended Generalized Fibonacci polynomials is converted into integer, for that we choose $a \neq 0$ and $b \neq 0$ for integer x such that a and b also gives non zero integer values. The initial message needs to be represented in the form of a square matrix M of order m, referred as the message-matrix, in order to employ this type of coding. This representation has no constraints and the user is free to arrange it how they want. For instance, the message 283954267 can be represented by the message matrix of order 2:

$$M = \begin{pmatrix} 283 & 95 \\ 42 & 67 \end{pmatrix}.$$

The encoding matrix $Q_m^n(a,b)$ is obtained from (2.3). Once the sender and receiver agree on above parameters and an integer n. To get the message matrix E, multiply the encoding matrix by the message matrix M from right side. For example, for $m = 3$ and $n = 2$ we have

$$E = Q_3^2(a,b)M_{3\times3}$$

$$= \begin{pmatrix} a^2 & 2ab & b^2 \\ 0 & a^2+b & ab \\ 0 & a & b \end{pmatrix}\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix}.$$

The elements of E are delivered by the channel in the following order $e_{11}, e_{12}, e_{13}, \cdots, e_{33}$, followed by the value of $det(M)$. Assuming that the send sequence is received without error, the original message matrix is produced by multiplying E and $Q_3^{-2}(a,b)$:

$$M = Q_3^{-2}(a,b)E$$

$$= \begin{pmatrix} \frac{1}{a^2} & -\frac{1}{a} & 1-\frac{b}{a^2} \\ 0 & \frac{1}{b} & -\frac{a}{b} \\ 0 & -\frac{a}{b^2} & \frac{a^2+b}{b^2} \end{pmatrix}\begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}.$$

2.5 A relation among the elements of a code message-matrix

Inside this part, we develop a fascinating relationship between the components of a code message matrix E, which plays an important role in the error-correction process. Let m = 3 for the sake of simplicity. Assume that all values of M are positive and $a, b \geq 1$ for $(x \geq 1)$. Therefore,

$$M = Q_3^{-n}(a,b) \times E$$

$$= \begin{pmatrix} \frac{\binom{n-1}{0}}{a^n} & \frac{-a}{(-a)^n}\sum_{i=0}^{n-2}(-1)^i\binom{n-1}{i}g_{2n-3-2i}b^{i+2-n} & \frac{1}{(-a)^n}\sum_{i=0}^{n-1}(-1)^i\binom{n}{i}g_{2n-1-2i}b^{i+2-n} \\ 0 & \frac{(-1)^n}{b^{n-1}}g_{n-1} & \frac{(-1)^{n-1}}{b^{n-1}}g_n \\ 0 & \frac{(-1)^{n-1}}{b^n}g_n & \frac{(-1)^n}{b^n}g_{n+1} \end{pmatrix} \times \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix}$$

$$= \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}.$$

For the elements of the first columns of M, we have

$$\begin{cases} m_{11} = e_{11} + (-1)^{n-1}ae_{21}\sum_{i=0}^{n-2}(-1)^i\binom{n-1}{i}g_{2n-3-2i}b^{i+2-n} + (-1)^n e_{31}\sum_{i=0}^{n-1}(-1)^i\binom{n}{i}g_{2n-1-2i}b^{i+2-n} \geq 0; \\ m_{21} = \frac{(-1)^n}{b^{n-1}}e_{21}g_{n-1} + \frac{(-1)^{n-1}}{b^{n-1}}e_{31}g_n \geq 0; \\ m_{31} = \frac{(-1)^{n-1}}{b^n}e_{21}g_n + \frac{(-1)^n}{b^n}e_{31}g_{n+1} \geq 0. \end{cases}$$

Using (2.7) for an even integer $n = 2l$, we obtain the following inequalities.

$$\begin{cases} e_{11} - ae_{21}\left(\frac{a^{2l-1}g_{2l-2}}{b^{2l-2}} + 1\right) + be_{31}\left(\frac{a^{2l}g_{2l-1}}{b^{2l-1}} - 1\right) \geq 01 & (a) \\ \frac{e_{21}g_{2l-1}}{b^{2l-1}} - \frac{e_{31}g_{2l}}{b^{2l-1}} \geq 0 & (b) \\ -\frac{e_{21}g_{2l}}{b^{2l}} + \frac{e_{31}g_{2l+1}}{b^{2l}} \geq 0 & (c) \end{cases} . \tag{2.8}$$

From (2.8)(b) and (2.8)(c), we have

$$\frac{g_{2l}}{g_{2l-1}} \leq \frac{e_{21}}{e_{31}} \leq \frac{g_{2l+1}}{g_{2l}}. \tag{2.9}$$

It follows from 2.8(a) that

$$\frac{e_{11}}{e_{31}} \geq a\frac{e_{21}}{e_{31}}\Big(\frac{a^{2l-1}g_{2l-2}}{b^{2l-2}} + 1\Big) - b\Big(\frac{a^{2l}g_{2l-1}}{b^{2l-1}} - 1\Big).$$

This together with (2.9) gives

$$\begin{aligned} \frac{e_{11}}{e_{31}} &\geq a\frac{g_{2l}}{g_{2l-1}}\Big(\frac{a^{2l-1}g_{2l-2}}{b^{2l-2}} + 1\Big) - b\Big(\frac{a^{2l}g_{2l-1}}{b^{2l-1}} - 1\Big) \\ &\geq \frac{a^{2l}}{b^{2l-2}g_{2l-1}}\Big(g_{2l}g_{2l-2} - g_{2l-1}^2\Big) + \frac{ag_{2l}}{g_{2l-1}} + b \\ &\geq -\frac{a^{2l}b}{g_{2l-1}} + \frac{ag_{2l} + bg_{2l-1}}{g_{2l-1}} \\ \frac{e_{11}}{e_{31}} &\geq \frac{g_{2l+1} - a^{2l}b}{g_{2l-1}}. \end{aligned} \tag{2.10}$$

Similarly, dividing (2.8)(a) by e_{11} results in

$$b\frac{e_{31}}{e_{11}}\Big(\frac{a^{2l}g_{2l-1}}{b^{2l-1}} - 1\Big) \geq a\frac{e_{21}}{e_{11}}\Big(\frac{a^{2l-1}g_{2l-2}}{b^{2l-2}} + 1\Big) - 1.$$

It follows from this and (2.9) that

$$b\frac{e_{31}}{e_{11}}\Big(\frac{a^{2l}g_{2l-1}}{b^{2l-1}} - 1\Big) \geq a\frac{g_{2l}}{g_{2l-1}}\Big(\frac{a^{2l-1}g_{2l-2}}{b^{2l-2}} + 1\Big) - 1 \tag{2.11}$$

and hence

$$\frac{e_{11}}{e_{31}} \leq \frac{g_{2l+1} - a^{2l}b}{g_{2l-1}}. \tag{2.12}$$

Using (**??**) and (2.12), we get

$$\frac{e_{11}}{e_{31}} = \frac{g_{2l+1} - a^{2l}b}{g_{2l-1}}. \tag{2.13}$$

For l large enough, we have from equation (2.9) and (2.13)

$$\frac{e_{11}}{e_{31}} \approx \sigma^2, \qquad \frac{e_{21}}{e_{31}} \approx \sigma,$$

where,

$$\sigma = \frac{a + \sqrt{a^2 + 4b}}{2}.$$

Therefore,

$$\frac{e_{11}}{e_{21}} \approx \sigma.$$

Similarly, assuming that (2.8) for $n = 2l + 1$, we obtain,

$$\frac{g_{2l+2}}{g_{2l+1}} \leq \frac{e_{21}}{e_{31}} \leq \frac{g_{2l+1}}{g_{2l}}$$

$$\frac{g_{2l+2} - a^{2l+1}b}{g_{2l}} \leq \frac{e_{11}}{e_{31}} \leq \frac{g_{2l+2} - a^{2l+1}b}{g_{2l}}$$

For l large enough, we have

$$\frac{e_{11}}{e_{31}} \approx \sigma^2, \qquad \frac{e_{21}}{e_{31}} \approx \sigma.$$

Therefore,

$$\frac{e_{11}}{e_{21}} \approx \sigma.$$

The result is that for large values of n, the following equation holds.

$$\frac{e_{11}}{e_{21}} \approx \frac{e_{21}}{e_{31}} \approx \sigma.$$

In general, for $1 \leq i \leq m$ we get

$$\frac{e_{1,i}}{e_{2,i}} \approx \frac{e_{2,i}}{e_{3,i}} \approx \cdots \approx \frac{e_{m-1,i}}{e_{m,i}} \approx \sigma, \tag{2.14}$$

where $e_{i,j}$ is the element of i^{th} row and j^{th} column of message matrix.

2.6 Error-detection and error-correction

The Fibonacci Polynomial based coding error-correction technique has been developed in [5]. This approach is used in Extended Generalized Fibonacci Polynomial based coding method described. We will start with error detection.From $E = Q_m^n(a,b)M$ we have,

$$det(E) = det(M) \times (-a^{m-2}b)^n \tag{2.15}$$

Using determinant theorem, we have $\det(Q_m^n(a,b)) = (-a^{m-2}b)^n$. Relation (2.15) is controlled when an estimation matrix $\hat{E}$ is rebuilt using the received elements. If the relation is satisfied, we claim there was no error. otherwise, either the components of E or $det(M)$ are incorrect. We may presume that the number $det(M)$ was received correctly after sending it many times and utilising majority logic decoding. As a result, relation (2.15) is regarded as criterion for detecting errors. Assume that some of elements of E are incorrect. Of course, this matrix might have one-fold, two-fold,$\cdots$, or m^2-fold errors.

For simplicity consider a 2×2 receiving matrix to demonstrate how to remedy these problem. Three cases are examined.

Case1. Assume that one of the elements was delivered incorrectly. Then one of the four cases below is feasible, where p, q, r and s are the incorrect elements.

$$\begin{pmatrix} p & e_{12} \\ e_{21} & e_{22} \end{pmatrix} \qquad \begin{pmatrix} e_{11} & q \\ e_{21} & e_{22} \end{pmatrix} \qquad \begin{pmatrix} e_{11} & e_{12} \\ r & e_{22} \end{pmatrix} \qquad \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & s \end{pmatrix}.$$

It follows from (2.15) and $det(Q_2^n(a,b)) = (-b)^n$ that

$pe_{22} - e_{12}e_{21} = (-b)^n det(M)$,
$e_{11}e_{22} - qe_{21} = (-b)^n det(M)$,
$e_{11}e_{22} - re_{12} = (-b)^n det(M)$,
$se_{11} - e_{12}e_{21} = (-b)^n det(M)$,
or equivalently

$$p = \frac{(-b)^n det(M) + e_{12}e_{21}}{e_{22}},$$
$$q = \frac{-(-b)^n det(M) + e_{11}e_{22}}{e_{21}},$$
$$r = \frac{-(-b)^n det(M) + e_{11}e_{22}}{e_{12}},$$
$$s = \frac{(-b)^n det(M) + e_{12}e_{21}}{e_{11}}.$$

The above equation provides four alternative single-error variations, but we must select the right variant only from the instance of integer solutions p, q, r and s; moreover, we must select solutions that satisfy the relation (2.14). Note that only numbers that are integers and satisfy (2.14) are the elements of E.If no such elements is obtained from these equations, we must conclude that our single-error hypothesis is false and we have to consider multiple-fold error cases.

Case 2. Suppose that two elements of E was delivered incorrectly as shown below:

$\begin{pmatrix} p & e_{12} \\ q & e_{22} \end{pmatrix}$.

From (2.15) we have $pe_{22} - e_{12}q = (-b)^n det(M)$. Since above equation has many solutions, we have to choose solutions of p and q, which satisfy (2.14). Again only integer solutions are acceptable. It's worth nothing that if the two errors occur in the same row or in one of the matrix's two diagonals, they may be readily fixed by just applying (2.14). Two-fold error do not arise if no integer solution is discovered. If none of the cases above produce solutions that fulfil the criteria, then all of the elements of E have been received incorrectly. Errors cannot be remedied in this case.

Case 3. Assume that three elements of E was delivered incorrectly as shown below

$\begin{pmatrix} p & q \\ r & e_{22} \end{pmatrix}$.

From (2.14), q can be obtained. Now remaining errors can be corrected by case2 solution.

If none of the cases above produce solutions that fulfil the criteria, then all of the elements of E have been received incorrectly. Errors cannot be remedied in this case.

According to the method described in [11], there are consequently 15 error conditions in the elements of E. Since 14 cases between them can be corrected, the approach's correctable probability is equal to $\frac{14}{15} = 0.9333 = 93.33\%$. Capability to fix errors: Because only m^2-fold faults may notbe rectified, As in [2], the method's error-correction capacity is $\frac{2^{m^2}-2}{2^{m^2-1}}$, where m is the message-matrix order. As a result, the probability of decoding mistake is nearly nil for large values of m.

3 Conclusion

We presented a coding scheme based on Extended Generalized Fibonacci polynomials. The encoder matrix for integers $m \geq 2$, $a, b \geq 1$ and $n \geq 1$ is a matrix $Q_m^n(a,b)$, the nth power of $Q_m(a,b)$ with Extended Generalized Fibonacci polynomial elements.Further, established some properties of Extended Generalized Fibonacci polynomial. Each source word is represented by a matrix M that has been encoded into a code message matrix $E = MQ_m^n(a,b)$. The suggested coding scheme was given a basic error-correcting algorithm. We demonstrated that this approach can correct up to $\frac{2^{m^2}-2}{2^{m^2}-1}$ mistakes, implying that the chance of decoding error is nearly nil for large values of m.

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