ISSN 0304-9892 (Print) ISSN 2455-7463 (Online)
www.vijnanaparishadofindia.org/jnanabha
***Jñānābha*, Vol. 53(1) (2023), 196-201**
(Dedicated to Professor G. C. Sharma on His 85^{th} Birth Anniversary Celebrations)

A COMMON FIXED POINT THEOREM FOR FOUR LIMIT COINCIDENTLY COMMUTING SELFMAPS OF A S-METRIC SPACE

V. Kiran
Department of Mathematics, Osmania University, Hyderabad, Telangana, India-500007
Email: kiranmathou@gmail.com
(Received : May 19, 2022; In format: June 02, 2022; Revised: March 11, 2023; Accepted: March 16, 2023)

DOI: https://doi.org/10.58250/jnanabha.2023.53122

Abstract

In this paper we prove a common fixed point theorem for four selfmaps of a S -metric space. Also we deduce a common fixed point theorem for four selfmaps of a complete S-metric space. Moreover we show that a common fixed point theorem for four selfmaps of a metric space proved by Brian Fisher follows as a particular case.
2020 Mathematical Sciences Classification: 47H10; 54H25.
Keywords and Phrases: S-metric space, Fixed point, Contractive modulus, Associated sequence for four selfmaps.

1 Introduction

Fixed point theorems are extensively studied in the literature for several reasons and one of the reason is that there are a quite number of problems in integral and differential equations, for which solutions can be equivalently formulated as a fixed point of some operator on a suitable space. In an attempt to generalize fixed point theorems proved for selfmaps of metric spaces, Dhage [2,3] has introduced generalized metric spaces called D-metric space in his *Ph.D.* thesis [1] in the year 1984 which is a landmark in the history of metric fixed point theory in higher dimensional metric spaces. As a probable modification to D-metric spaces, Sedghi, Shobe and Zhou [11] introduced D^*-metric spaces. In 2006, Mustafa and Sims [10] initiated G-metric spaces; while Sedghi, Shobe and Aliouche [12] considered S-metric spaces in 2012. Hereafter we consider, in this paper, only S-metric spaces and common fixed point theorems on such spaces.

The notion of commutativity of self maps on a metric space has been generalized to weakly commuting by Sessa [13], which is further generalized to compatibility by Jungck [9]. These common fixed point theorems on the lines of Sessa [13] and Jungck[9] are further extended to D-metric spaces by Dhage [4,5] and Dhage et al. [6] under the meaningful terminology "coincidently commuting mappings" and "limit coincidently commuting mappings."

In this paper, we establish a common fixed point theorem for four limit coincidently commuting selfmaps of a S-metric space. Further we generalize a common fixed point theorem of Fisher [8].

2 Preliminaries

Definition 2.1 ([12]). *Let X be a non empty set. By S-metric we mean a function $S : X^3 \to [0,\infty)$ which satisfies the following conditions for $x, y, z, w \in X$*
(a) $S(x, y, z) \geq 0$.
(b) $S(x, y, z) = 0$ *if and only if* $x = y = z$.
(c) $S(x, y, z) \leq S(x, x, w) + S(y, y, w) + S(z, z, w)$.
An ordered pair (X, S) is called a S-metric space.

Remark 2.1. It was shown in ([12], Lemma 2.5) that $S(x, x, y) = S(y, y, x)$ for all $x, y \in X$.

Definition 2.2 ([12]). *Let (X, S) be a S-metric space. A sequence $\{x_n\}$ in X is said to converge, if there is a $x \in X$ such that $S(x_n, x_n, x) \to 0$ as $n \to \infty$; that is, for $\epsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, we have $S(x_n, x_n, x) < \epsilon$ and in this case we write $\lim\limits_{n\to\infty} x_n = x$.*

Definition 2.3 ([12]). *Let (X, S) be a S-metric space. A sequence $\{x_n\}$ in X is called a Cauchy sequence if for $\epsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that $S(x_n, x_n, x_m) < \epsilon$ for all $n, m \geq n_0$.*

Definition 2.4 ([12]). *A S-metric space (X, S) is said to be complete if every Cauchy sequence in it converges to some point in X.*

Definition 2.5 ([1]). *Let (X, d) be any metric space then $S_d(x, y, z) = d(x, y) + d(y, z) + d(z, x)$ is a S-metric on X. We call this S-metric as the S-metric induced by d (we denote this by S_d).*

Remark 2.2. Let (X, d) be any metric space and S_d be the S-metric induced by d. For any sequence $\{x_n\}$ in (X, S_d) is a Cauchy sequence if and only if $\{x_n\}$ is a Cauchy sequence in (X, d). Thus (X, S_d) is complete if and only if (X, d) is complete.

Definition 2.6. *Let (X, S) be a S-metric space. If there exists sequences $\{x_n\}$ and $\{y_n\}$ such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$ then $\lim_{n\to\infty} S(x_n, x_n, y_n) = S(x, x, y)$, then we say that $S(x, y, z)$ is continuous in x and y.*

Definition 2.7. *If g and f are self maps of a S-metric space (X, S) such that for every sequence $\{x_n\}$ in X with $\lim_{n\to\infty} gx_n = \lim_{n\to\infty} fx_n = t$ for some $t \in X$ we have $\lim_{n\to\infty} S(gfx_n, gfx_n, fgx_n) = 0$ then g and f are said to be limit coincidently commuting.*

Trivially commuting self maps of a S-metric space are limit coincidently commuting but not conversely.

Definition 2.8 ([7]). *An upper semi-continuous nondecreasing function $\phi : [0, \infty) \to [0, \infty)$ is called D-function if $\phi(0) = 0.\phi$ is called contractive if $\phi(t) < t$ for $t > 0$.*

Definition 2.9. *Let g, f, h and p be self maps of a S-metric space such that $g(X) \subseteq p(X)$ and $f(X) \subseteq h(X)$. Then for any $x_0 \in X$, if $\{x_n\}$ is a sequence in X such that $gx_{2n} = px_{2n+1}$ and $fx_{2n+1} = hx_{2n+2}$ for $n \geq 0$, then $\{x_n\}$ is called an associated sequence of x_0 relative to self maps g, f, h and p.*

3 Main Result

Theorem 3.1. *Let g, f, h and p be self maps of a S- metric space (X, S) satisfying the following conditions*

(i) $g(X) \subseteq p(X)$ and $f(X) \subseteq h(X)$,

(ii) $S(gx, gx, fy) \leq \phi(\mu(x, y))$ for all $x, y \in X$ where ϕ is a contractive D-function and $\mu(x, y) = \max\{S(hx, hx, py), S(hx, hx, gx), S(py, py, fy)\}$ for $x, y \in X$,

(iii) one of g, f, h and p is continuous and

(iv) the pairs (g, h) and (f, p) are limit coincidently commuting. Further if

(v) there exists a point $x_0 \in X$ and an associated sequence $\{x_n\}$ relative to selfmaps such that $gx_0, fx_1, gx_2, fx_3, \cdots, gx_{2n}, fx_{2n+1} \cdots$ converges to some $z \in X$,

then g, f, h and p have a unique common fixed point $z \in X$. Also there is no other common fixed point for g and h; and that there is no other common fixed point for f and p.

Before proving the theorem, we establish some Lemmas which are noteworthy.

Lemma 3.1. *Suppose that g, f, h and p are self maps of a S-metric space satisfying the conditions (i),(ii) and (v) of* Theorem 3.1 *with the pair (g, h) is limit coincidently commuting. Then*
(a) $\lim_{n\to\infty} \mu(hx_{2n}, x_{2n+1}) = S(hz, hz, z)$ whenever h is continuous,
(b) $\lim_{n\to\infty} \mu(gx_{2n}, x_{2n+1}) = S(gz, gz, z)$ whenever g is continuous.

Proof. In view of (v), the sequences $\{gx_{2n}\}$ and $\{fx_{2n+1}\}$ converge to some $z \in X$ and since $gx_{2n} = px_{2n+1}$ and $fx_{2n+1} = hx_{2n+2}$, we have

$$gx_{2n}, fx_{2n+1}, hx_{2n}, p_{2n+1} \to z \text{ as } n \to \infty. \tag{3.1}$$

(a) If h is continuous, then we have

$$h^2x_{2n} \to hz, hgx_{2n} \to hz \text{ as } n \to \infty. \tag{3.2}$$

Also the limit coincidently commutativity of the pair (g, h) implies

$$\lim_{n\to\infty} S(ghx_{2n}, ghx_{2n}, hgx_{2n}) = 0. \tag{3.3}$$

From (3.2) and (3.3) we get

$$ghx_{2n} \to hz \text{ as } n \to \infty. \tag{3.4}$$

Now from (ii), we have

$$\begin{aligned}\mu(hx_{2n}, x_{2n+1}) = \max\{&S(h^2x_{2n}, h^2x_{2n}, px_{2n+1}),\\ &S(h^2x_{2n}, h^2x_{2n}, ghx_{2n}), S(px_{2n+1}, px_{2n+1}, fx_{2n+1})\}.\end{aligned} \tag{3.5}$$

Letting $n \to \infty$ in (3.5) and using the continuity of S(x, y, z) in x and y and (3.1), (3.2) and (3.4), we get $\lim_{n\to\infty} \mu(hx_{2n}, x_{2n+1}) = \max\{S(hz, hz, z), S(hz, hz, hz), S(z, z, z)\} = S(hz, hz, z)$
This proves (a).

(b) If g is continuous, by (3.1) we have

$$g^2x_{2n} \to gz, ghx_{2n} \to gz \text{ as } n \to \infty. \tag{3.6}$$

Therefore in view of (3.3), we get

$$hgx_{2n} \to gz. \tag{3.7}$$

Now we have

$$\begin{aligned}\mu(gx_{2n}, x_{2n+1}) &= \max\{S(hgx_{2n}, hgx_{2n}, px_{2n+1}), S(hgx_{2n}, hgx_{2n}, gx_{2n+1}), S(px_{2n+1}, px_{2n+1}, fx_{2n+1})\}\\ &= \max\{S(gz, gz, z), S(gz, gz, z), S(z, z, z)\}\\ &= S(gz, gz, z).\end{aligned} \tag{3.8}$$

This proves (b). □

Lemma 3.2. *Suppose that g, f, h and p are self maps of a S-metric space (X,S) such that the pair (f,p) is limit coincidently commuting and the conditions (i),(ii) and (v) of* Theorem 3.1, *then*
(a) $\lim_{n\to\infty} \mu(x_{2n}, px_{2n+1}) = S(z, z, pz)$ *whenever p is continuous,*
(b) $\lim_{n\to\infty} \mu(x_{2n}, fx_{2n+1}) = S(z, z, fz)$ *whenever f is continuous.*

Proof. The proof of Lemma 3.2 is similar to the proof of Lemma 3.1 with appropriate changes. □

Proof of Theorem 3.1.

We first establish the existence of a common fixed point in case if h is continuous.
The proof is similar in other cases of condition (iii) of Theorem 3.1 with suitable changes.
Suppose that h is continuous.
Taking $x = hx_{2n}$ and $y = x_{2n+1}$ in condition (ii) of Theorem 3.1, we have

$$S(ghx_{2n}, ghx_{2n}, fx_{2n+1}) \leq \phi(\mu(hx_{2n}, x_{2n+1})). \tag{3.9}$$

Also the continuity of S(x, y, z) in x and y gives

$$S(hz, hz, z) = \lim_{n\to\infty} S(ghx_{2n}, ghx_{2n}, fx_{2n+1}).$$

Therefore by Lemma 3.1, we get

$$\begin{aligned}S(hz, hz, z) &= \limsup_{n\to\infty} S(ghx_{2n}, ghx_{2n}, fx_{2n+1})\\ &\leq \limsup_{n\to\infty} \phi(\mu(hx_{2n}, x_{2n+1}))\\ &= \phi(\limsup_{n\to\infty} \mu(hx_{2n}, x_{2n+1}))\\ &= \phi(\lim_{n\to\infty} \mu(hx_{2n}, x_{2n+1}))\\ &= \phi(S(hz, hz, z)).\end{aligned} \tag{3.10}$$

Hence

$$S(hz, hz, z) \leq \phi(S(hz, hz, z). \tag{3.11}$$

We now claim that $hz = z$.
In fact, if $hz \neq z$, then $S(hz, hz, z) > 0$ so that $\phi(S(hz, hz, z)) < S(hz, hz, z)$ and this contradicts (3.11),

therefore $hz = z$.
Now the continuity of S(x, y, z) in x and y gives

$$\begin{aligned} S(gz, gz, z) &= \lim_{n\to\infty} S(gz, gz, fx_{2n+1}) \\ &= \limsup_{n\to\infty} S(gz, gz, fx_{2n+1}). \end{aligned}$$

Using condition (ii) of Theorem 3.1 and the upper semicontinuity of ϕ in the above, we get

$$\begin{aligned} S(gz, gz, z) &\le \limsup_{n\to\infty} \phi(\mu(z, x_{2n+1})) \\ &= \phi(\limsup_{n\to\infty} \mu(z, x_{2n+1})). \end{aligned} \tag{3.12}$$

But

$$\begin{aligned} \lim_{n\to\infty} \mu(z, x_{2n+1}) &= \lim_{n\to\infty} \max\{S(hz, hz, px_{2n+1}, S(hz, hz, gz), S(px_{2n+1}, px_{2n+1}, fx_{2n+1})\} \\ &= \max\{S(hz, hz, z), S(z, z, gz), S(z, z, z)\} \\ &= S(z, z, gz) = S(gz, gz, z), \text{ since } hz = z, px_{2n+1} \to z \text{ and } fx_{2n+1} \to z \text{ as } n \to \infty. \end{aligned}$$

Therefore we get

$$S(gz, gz, z) \le \phi(S(gz, gz, z)). \tag{3.13}$$

If $gz \neq z$ then $S(gz, gz, z) > 0$ and by the definition of ϕ we get $\phi(S(gz, gz, z) < S(gz, gz, z)$, contradicting (3.13),hence $gz = z$
Thus we have $gz = hz = z$.
Now since $g(X) \subseteq p(X)$, there is a $u \in X$ with $z = gz = pu$ and we have $gz = hz = pu = z$.
We now claim that $fu = z$.
In fact if $fu \neq z$, then $S(z, z, fu) > 0$ and therefore by (ii) of Theorem 3.1 we get

$$\begin{aligned} S(z, z, fu) = S(gz, gz, fu) &\le \phi(\mu(z, u)) \\ &= \phi(\max\{S(gz, gz, pu), S(hz, hz, gz), S(pu, pu, fu)\}) \\ &= \phi(S(z, z, fu)), \end{aligned}$$

since $gz = hz = pu = z$ and above result implies $S(z, z, fu) \le \phi(S(z, z, fu)) < S(z, z, fu)$ which is contradiction. Therefore $fu = z$.
Hence we have $gz = hz = pu = fu = z$.
Now taking $y_n = u$ for all $n \ge 1$, it follows that $fy_n \to fu = z$ and $py_n \to pu = z$ as $n \to \infty$.
Also since the pair (f, p) is limit coincidently commuting,
we have $\lim_{n\to\infty} S(fpy_n, fpy_n, pfy_n) = 0$ which gives $S(fpu, fpu, pfu) = 0$ implies $fpu = pfu$ so that $fz = pz$.
Now by condition (ii) of Theorem 3.1, we have

$$\begin{aligned} S(z, z, fz) = S(gz, gz, fz) &\le \phi(\mu(z, z)) \\ &= \phi(\max\{S(hz, hz, pz), S(hz, hz, gz), S(pz, pz, fz)\} \\ &= \phi(S(hz, hz, fz)) = \phi(S(z, z, fz)), \end{aligned} \tag{3.14}$$

since $gz = hz = z$ and $pz = fz$.
Therefore we get $S(z, z, fz) \le \phi(S(z, z, fz))$ which yields $fz = z$.
Hence $gz = hz = pz = fz = z$, proving z is a common fixed point of g, f, h and p.
Now we prove uniqueness of common fixed point.
If possible let $z'(\neq z)$ be another common fixed point of g, f, h and p.
Then from condition (ii) of Theorem 3.1 we have

$$S(z, z, z') = S(gz, gz, gz') \le \phi(\mu(z, z')). \tag{3.15}$$

Since $\mu(z, z') = S(z, z, z')$ from (ii) of Theorem 3.1, (3.15) gives $S(z, z, z') \le \phi(S(z, z, z'))$ and this will be a contradiction if $z \neq z'$.
Hence z is unique common fixed point of f, g, h and p.
Now we prove that z is unique common fixed point of g and h and of f and p.
Let w be another fixed point of g and h. Then $z = hz = fz = gz = pz$ and $w = hw = gw$.
Now from condition (ii) of Theorem 3.1 we have

$$S(z, z, w) = S(w, w, z) = S(gw, gw, fz) \le \phi(\mu(w, z)), \tag{3.16}$$

since $\mu(w,z) = S(w,w,z)$. Therefore (3.16) gives $S(w,w,z) \le \phi(S(w,w,z))$ and this will be a contradiction if $w \ne z$.
Hence $w = z$
Therefore z is unique common fixed point of g and h. Similarly we can show that z is unique common fixed point of f and p.
Hence Theorem 3.1 is completely proved.

4 Common fixed point Theorem for four self maps of a complete S-metric space

Before proving the main result in this section, first we establish a preparatory Lemma.

Lemma 4.1. *Let (X,S) be a S- metric space and g,f,h and p be self maps of X such that*

(i) $g(X) \subseteq p(X)$ *and* $f(X) \subseteq h(X)$,

(ii) $S(gx,gx,fy) \le c.\mu(x,y)$ *for all* $x,y \in X$ *where* $0 \le c < 1$ *and*
$\mu(x,y) = \max\{S(hx,hx,py), S(hx,hx,gx), S(py,py,fy)\}$.
Further if

(iii) (X,S) *is complete, then for any* $x_0 \in X$ *and for any associated sequence* $\{x_n\}$ *relative to four self maps the sequence* $gx_0, fx_1, gx_2, fx_3, \cdots, gx_{2n}, fx_{2n+1}\cdots$ *converges to some* $z \in X$.

Proof. Suppose that g,f,h and p are self maps of a S-metric space (X,S) for which conditions (i) and (ii) holds. Let a point $x_0 \in X$ and $\{x_n\}$ be any associated sequence of x_0 relative to four selfmaps. Then since $gx_{2n} = px_{2n+1}$ and $fx_{2n+1} = hx_{2n+2}$ for all $n \ge 0$.
Note that

$$\begin{aligned}\mu(x_{2n},x_{2n+1}) &= \max\{S(hx_{2n},hx_{2n},px_{2n+1}), S(hx_{2n},hx_{2n},gx_{2n}), S(px_{2n+1},px_{2n+1},fx_{2n+1})\}\\ &= \max\{S(hx_{2n},hx_{2n},gx_{2n}), S(hx_{2n},hx_{2n},gx_{2n}), S(px_{2n+1},px_{2n+1},fx_{2n+1})\}\\ &= \max\{S(hx_{2n},hx_{2n},gx_{2n}), S(px_{2n+1},px_{2n+1},fx_{2n+1})\}\\ &= \max\{S(hx_{2n},hx_{2n},gx_{2n}), S(gx_{2n},gx_{2n},fx_{2n+1})\}\\ &= \max\{S(gx_{2n},gx_{2n},fx_{2n-1}), S(gx_{2n},gx_{2n},fx_{2n+1})\}.\end{aligned}$$

This together with (ii) of Lemma 4.1 gives

$$\begin{aligned}S(gx_{2n},gx_{2n},fx_{2n+1}) &\le c\mu(x_{2n},x_{2n+1})\\ &\le c\max\{S(gx_{2n},gx_{2n},fx_{2n-1}), S(gx_{2n},gx_{2n},fx_{2n+1})\}\end{aligned}$$

and since $0 \le c < 1$, it follows from the above inequality that

$$\max\{S(gx_{2n},gx_{2n},fx_{2n-1}), S(gx_{2n},gx_{2n},fx_{2n+1})\} = S(gx_{2n},gx_{2n},fx_{2n-1}).$$

Therefore

$$S(gx_{2n},gx_{2n},fx_{2n+1}) \le cS(gx_{2n},gx_{2n},fx_{2n-1}). \tag{4.1}$$

Similarly

$$S(gx_{2n},gx_{2n},fx_{2n-1}) \le cS(gx_{2n-2},gx_{2n-2},fx_{2n-3}). \tag{4.2}$$

From (4.1) and (4.2), we get

$$\begin{aligned}S(gx_{2n},gx_{2n},fx_{2n+1}) &\le c^2 S(gx_{2n-2},gx_{2n-2},fx_{2n-1})\\ &\le c^4 S(gx_{2n-4},gx_{2n-4},fx_{2n-3})\\ &\quad \cdots \quad \cdots \quad \cdots\\ &\quad \cdots \quad \cdots \quad \cdots\\ &\le c^{2n} S(gx_0,gx_0,fx_1) \to 0,\end{aligned}$$

as $c^{2n} \to 0$ as $n \to \infty$ (because $c < 1$), therefore the sequence $gx_0, fx_1, gx_2, fx_3, \cdots, gx_{2n}, fx_{2n+1}\cdots$ is a Cauchy sequence in (X,S) and since X is complete, it converges to a point say $z \in X$, proving lemma. □

Theorem 4.1. *Suppose that (X,S) is a S-metric space satisfying conditions (i) to (v) of* Theorem 3.1. *Further if*
(v)' (X,S) is complete,
then g,f,h and p have a unique common fixed point $z \in X$. Also there is no other common fixed point for g and h and that there is no other common fixed point for f and p.

Proof. In view of Lemma 4.1, the condition (v) of the Theorem follows from Theorem 3.1 because of (v)', hence Theorem 4.1 follows from Theorem 3.1. □

Corollary 4.1 ([8] Theorem 2)**.** *Let g, f, h and p be self maps of a metric space (X, d) satisfying the conditions*

(i) $g(X) \subseteq p(X)$ *and* $f(X) \subseteq h(X)$,

(ii) $d(gx, fy) \leq c.\mu_0(x, y)$ *for all* $x, y \in X$ *where*
$\mu_0(x, y) = \max\{d(hx, py), d(hx, gx), d(py, fy)\}$ *for all* $x, y \in X$ *and* $0 \leq c < 1$,

(iii) one of g, f, h *and* p *is continuous and*

(iv) $gh = hg$ *and* $fp = pf$.
Further if

(v) X *is complete.*

Then the four self maps g, f, h *and* p *have a unique common fixed point. Also there is no other common fixed point for* g *and* h *and that there is no other common fixed for* f *and* p.

Proof. Given that (X, d) is a metric space satisfying conditions (i) to (v) of Corollary 4.1.
Defining $S(x, y, z) = d(x, y) + d(y, z) + d(z, x)$ for $x, y, z \in X$, it follows that (X, S) is a S-metric space. Also condition (ii) can be written as $S(gx, gx, fy) \leq c\mu(x, y)$ for all $x, y \in X$,
where $\mu(x, y) = \max\{S(hx, hx, py), S(hx, hx, gx), S(py, py, fy)\}$ which is the same as condition(ii) of Theorem 4.1.
Since (X, d) is complete, we have (X, S) is complete by Remark 2.2.
Now g, f, h and p are self maps of S-metric space (X, S) satisfying conditions of Theorem 4.1 and hence Corollary 4.1 follows from Theorem 4.1. □

5 Conclusion

we proved a common fixed point theorem for four limit coincidently commuting selfmaps of a S-metric space. Also we deduced a common fixed point theorem for four limit coincidently commuting selfmaps of a complete S-metric space. Moreover a common fixed point theorem for four self maps of a metric space proved by Brian Fisher follows as a particular case of our theorem.

References

[1] B. C. Dhage, *A study of some fixed point theorems*, Ph.D thesis, Marathwada University, Aurangabad, India, 1984.

[2] B. C. Dhage, Generalized metric spaces and mappings with fixed point, *Bull.Calcutta.Math.Soc.*, **84**(4) (1992), 329-336.

[3] B. C. Dhage, A common fixed point principle in D-metric spaces, *Bull.Calcutta.Math.Soc.*, **91**(6) (1999), 475-480.

[4] B. C. Dhage, On common fixed points of pairs of coincidentally commuting mappings in D-metric spaces, *Indian J.Pure Appl.Math.*, **30**(4) (1999), 395-406.

[5] B. C. Dhage, A common fixed point theorem for a pair of limit coincidentally commuting mappings in D-metric spaces, *Math.Sci.Res.Hot-line*, **4**(2) (2000), 45-55.

[6] B. C. Dhage, A. Jenifer and S. M. Kang, On common fixed points of a pairs of a single and a multi-valued coincidentally commuting mappings in D-metric spaces, *Inter.J. Math.Sci.*, **40** (2003), 2519-2539.

[7] B. C. Dhage and S. B. Dhage, Hybrid fixed point theory for nonincreasing mappings in partially ordered metric spaces and applications, *Journal of Nonlinear Analysis and Applications*, **5**(2) (2014), 71-79.

[8] B. Fisher, Common fixed points of four mappings, *Bull.Inst.Math.Academia.Sinica.*, **11** (1983), 103-113.

[9] G. Jungck, Compatible mappings and common fixed points , *Int J.Math.Math.Soc.*, **9** (1986), 771-779.

[10] Z. Mustafa and B. Sims, A new approach to generalized metric spaces, *Journal of Non linear and convex Analysis.*, **7**(2) (2006), 289-297.

[11] S. Shaban, S. Nabi and Z. Haiyun, A common fixed point theorem in D^*- metric spaces,*Fixed point Theory Appl.* **2007** 027906(2007), 1-3.

[12] S. Shaban, S. Nabi and A. Abdelkrim, A generalization of fixed point theorems in S-metric space, *Mat.Vesnik.*, **64**(3) (2012), 258-266.

[13] S. Sessa, On a week commutativity condition of mappings in a fixed point considerations, *Publ.Inst.Math.*, **32** (1982), 149-153.