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(Dedicated to Professor G. C. Sharma on His 85^{th} Birth Anniversary Celebrations)

SOME FIXED POINT RESULTS FOR CYCLIC $(\psi, \phi, Z)-$ CONTRACTION IN PARTIAL METRIC SPACES

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Abstract

In this paper, we present a new type of cyclic $(\psi, \phi, Z)-$ contraction which is a combination of cyclic $(\psi, \phi, A, B)-$ contraction and $Z-$contraction in the framework of complete partial metric space with the help of simulation function. We investigate the existence of fixed point result using cyclic $(\psi, \phi, Z)-$ contraction in the setting of complete partial metric space. Also we give an example to clarify the main result.

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Keywords and Phrases: Partial metric spaces, Simulation function, Cyclic mapping, Cyclic (ψ, ϕ, Z)-contraction.

1 Introduction

The idea of partial metric space was introduced by Mathews ([19]) and it is defined as the same point in partial metric does not necessarily need to be zero. In 2003, Kirk ([17]) introduced the notion of cyclic contraction. Karapinar ([14]) explored cyclic contraction in partial metric space in 2012 while Agarwal ([2]) defined a very useful cyclic generalized contractions on the complete partial metric space in the same year. Khojasteh ([16]) introduced new approach in fixed point theory by using a simulation function. This paper inspired us to find a different type of cyclic contraction in complete partial metric space. Many authors have already demonstrated different types of contractions in partial metric spaces (see [4, 5, 6, 7, 8, 11]).

In this paper, we establish a cyclic $(\psi, \phi, Z)-$ contraction in complete partial metric space to determine a unique fixed point.

On the other hand, the concept of simulation function was established in [16] to unify the existing fixed point results.

2 Preliminaries

Definition 2.1 ([16]). *A function $\xi : [0, \infty) \to [0, \infty)$ satisfying the following conditions*

(ξ_1) $\xi(0, 0) = 0$;
(ξ_2) $\xi(t, s) < t - s$ *for all* $t, s > 0$;
(ξ_3) $\{t_n\}, \{s_n\}$ *are sequences in* $(0, \infty)$ *such that* $lim_{n\to\infty} t_n = lim_{n\to\infty} s_n > 0$, *then* $\limsup_{n\to\infty} \xi(t_n, s_n) < 0$, *is called a simulation function.*

Due to the axiom (ξ_2), we have $\xi(t, t) < 0$ for all $t > 0$.

Example 2.1 ([3, 16, 20]). Let $\phi_1 : [0, \infty) \to [0, \infty)$ be a continuous functions with $\phi_i(t) = 0$ if and only if $t = 0$. For $i = 1, 2, 3, 4, 5, 6$, we define the mappings $\xi_i : [0, \infty) \times [0, \infty) \to \mathbb{R}$ as follows

(i) $\xi_1(t, s) = \phi_1(s) - \phi_1(t)$ for all $t, s \in [0, \infty)$, where $\phi_1(t) < t \le \phi_2(t)$ for all $t > 0$;
(ii) $\xi_1(t, s) = s - \frac{f(t,s)}{g(t,s)} t$ for all $t, s \in [0, \infty)$, where $f, g : [0, \infty)^2 \to [0, \infty)$ are two continuous functions with respect to each variable such that $f(t, s) > g(t, s)$ for all $t, s > 0$;
(iii) $\xi_3(t, s) = s - \phi_3(s) - t$ for all $t, s \in [0, \infty)$;
(iv) If $\psi : [0, \infty) \to [0, 1)$ is a function such that $\limsup_{t\to r^+} \psi(t) < 1$ for all $r > 0$ and define

$$\xi_4(t, s) = s\psi(s) - t \text{ for all } s, t \in [0, \infty);$$

(v) If $\eta : [0,\infty) \to [0,\infty)$ is an upper semi-continuous mapping such that $\eta(t) < t$ for all $t > 0$ and $\eta(0) = 0$ and define

$$\xi_5(t,s) = \eta(s) - t \text{ for all } s,t \in [0,\infty);$$

(vi) If $\phi : [0,\infty) \to [0,\infty)$ is a function such that $\int_0^\epsilon \phi(u)du$ exists and $\int_0^\epsilon \phi(u)du > \epsilon$ for each $\epsilon > 0$ and define

$$\xi_6(t,s) = s - \int_0^t \phi(u)du \text{ for all } s,t \in [0,\infty).$$

It is clear that each function $\xi_i (i = 1,2,3,4,5,6)$ forms a simulation function.

Definition 2.2 ([19])**.** *A partial metric on a non empty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y \in X$*
(p_1) $x = y \iff p(x,x) = p(y,y) = p(x,y)$;
(p_2) $p(x,x) \le p(x,y)$;
(p_3) $p(x,y) = p(y,x)$;
(p_4) $p(x,z) \le p(x,y) + p(y,z) - p(y,y)$.
A pair (X,p) is called a partial metric space. Each partial metric on X generates T_0 topology τ_p on X which is the family of $p-$open balls $\{B_p(x,\delta) : x \in X, \delta > 0\}$, where $B_p(x,\delta) = \{y \in X : p(x,y) < p(x,x) + \delta\}$ for all $x \in X$ and $\delta > 0$. If p is partial metric on X, then the function $d_p : X \times X \to R^+$ given by $d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ is a metric on X.

Definition 2.3. *Let (X,p) be a partial metric space. Then*
(1) *A sequence $\{x_n\}$ in a partial metric space (X,p) converges to a point $x \in X$ if and only if*

$$p(x,x) = \lim_{n\to\infty} p(x,x_n);$$

(2) *A sequence $\{x_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if and only if*

$$\lim_{n,m\to\infty} p(x_n,x_m)$$

exists (finite);
(3) *A partial metric space $(X,p)$is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p to a point $x \in X$ such that*

$$p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m);$$

(4) *A subset A of a partial metric space (X,p) is closed if whenever $\{x_n\}$is a sequence in A such that $\{x_n\}$ converges to some $x \in X$, then $x \in A$.*

Definition 2.4. *Let A and B be non-empty subset of a metric space (X,d) and $T : A \cup B \to A \cup B$. Then T is called a cyclic map if $T(A) \subseteq B$ and $T(B) \subseteq A$.*

Theorem 2.1 ([17])**.** *Let A and B be non empty closed subsets of a complete metric space (X,d). Suppose that $T : A \cup B \to A \cup B$ is a cyclic map such that*

$$d(Tx,Ty) \le kd(x,y).$$

If $k \in [0,1)$, then T has a unique fixed point in $A \cap B$.

To see [12], Karapinar and Erhan showed different types of cyclic contractions in usual metric space.

Definition 2.5 ([15])**.** *The function $\phi : [0,\infty) \to [0,\infty)$ is called an altering distance functions if the following conditions are satisfied:*
(1) *ϕ is continuous and non decreasing;*
(2) *$\phi(t) = 0$ if and only if $t = 0$.*

3 Main Results

Definition 3.1. *Let (X,p) be a partial metric space and A, B be a non empty closed subsets of (X,p). A mapping $T : A \cup B \to A \cup B$ is called cyclic (ψ,ϕ,Z)-contraction if*
(i) *$A \cup B$ has a cyclic representation with respect to T, i.e) $T(A) \subseteq B$ and $T(B) \subseteq A$;*
(ii) *If ψ and ϕ are altering distance functions,*

$$\xi(\psi(p(Tx,Ty)), \phi(\max(p(x,Tx), p(y,Ty)))) \ge 0 \quad \forall x \in A \text{ and } y \in B. \tag{3.1}$$

Theorem 3.1. *Let A, B be non empty closed subsets of a complete partial metric space (X, p). if $T : A\cup B \to A \cup B$ is a cyclic (ψ, ϕ, Z)-contraction. Then T has a unique fixed point $v \in A \cap B$.*

Proof. Fix any $x_0 \in A$. We choose $x_1 \in B$, since $T(A) \subseteq B$ such that $Tx_0 = x_1$. Again we choose $x_2 \in A$ such that $Tx_1 = x_2$, since $T(B) \subseteq A$. Continuing on this way, we construct a sequence $\{x_n\}$ in X such that $x_{2n} \in A, x_{2n+1} \in B$, i.e) $x_{2n+1} = Tx_{2n}$ and $x_{2n+2} = Tx_{2n+1}$. if $x_{2n_0+1} = Tx_{2n_0+1}$. Thus x_{2n_0+1} is a fixed point of T in $A \cap B$.

In this above manner we assume that $x_{2n+1} \neq x_{2n+2}$ for all $n \in \mathbb{N}$. If n is even, then $n = 2j$ for some $j \in \mathbb{N}$. Let $x_{2j+1} \neq x_{2j+2}$ and from equation (3.1), we have

$$\xi(\psi(p(Tx_{2j}, Tx_{2j+1})), \phi(\max(p(x_{2j}, Tx_{2j}), p(x_{2j+1}, Tx_{2j+1})))) \geq 0.$$

Using (ξ_2), we have

$$\begin{aligned}\xi(\psi(p(x_{2j+1}, x_{2j+2})), \phi(\max(p(x_{2j}, x_{2j+1}), p(x_{2j+1}, x_{2j+2}))))\\ < \phi(\max(p(x_{2j}, x_{2j+1}), p(x_{2j+1}, x_{2j+2}))) - \psi(p(x_{2j+1}, x_{2j+2})).\end{aligned} \tag{3.2}$$

From the above, we have

$$\psi(p(x_{2j+1}, x_{2j+2})) < \phi(\max(p(x_{2j}, x_{2j+1}), p(x_{2j+1}, x_{2j+2}))), \tag{3.3}$$

if $\max(p(x_{2j}, x_{2j+1}), p(x_{2j+1}, x_{2j+2})) = p(x_{2j+1}, x_{2j+2})$,
$p(x_{2j}, x_{2j+1}) < p(x_{2j+1}, x_{2j+2})$,
$\psi(p(x_{2j}, x_{2j+1})) < \phi(p(x_{2j+1}, x_{2j+2}))$.
Since ϕ is non-decreasing function
$\phi(p(x_{2j+1}, x_{2j+2})) = 0$, hence $p(x_{2j+1}, x_{2j+2}) = 0$.
By (p_1) and (p_2), $x_{2j+1} = x_{2j+2}$,
which is a contradiction to our assumption

$$\max(p(x_{2j}, x_{2j+1}), p(x_{2j+1}, x_{2j+2})) = p(x_{2j}, x_{2j+1})$$

From (3.3), we get

$$\psi(p(x_{2j+1}, x_{2j+2})) < \phi(p(x_{2j}, x_{2j+1})). \tag{3.4}$$

If n is odd, then $n = 2j + 1$ for some $j \in \mathbb{N}$. By equation (3.1), we get

$$\xi(\psi(p(Tx_{2j+1}, Tx_{2j+2})), \phi(\max(p(x_{2j+1}, Tx_{2j+1}), p(x_{2j+2}, Tx_{2j+2})))) \geq 0.$$

Using (ξ_2), we get

$$\psi(p(x_{2j+2}, x_{2j+3})) < \phi(\max(p(x_{2j+1}, x_{2j+2}), p(x_{2j+2}, x_{2j+3}))),$$

if

$$\max(p(x_{2j+1}, x_{2j+2}), p(x_{2j+2}, x_{2j+3})) = p(x_{2j+2}, x_{2j+3})$$

i.e)

$$p(x_{2j+2}, x_{2j+3}) < p(x_{2j+2}, x_{2j+3})$$

$$\psi(p(x_{2j+2}, x_{2j+3})) < \phi(p(x_{2j+2}, x_{2j+3})).$$

Since ϕ is non-decreasing function.
$\phi(p(x_{2j+2}, x_{2j+3})) = 0$ and hence $p(x_{2j+2}, x_{2j+3}) = 0$,by (p_1) and (p_2).
It implies that, $x_{2j+2} = x_{2j+3}$
which contradicts to our assumption
Therefore,

$$\max(p(x_{2j+1}, x_{2j+2}), p(x_{2j+2}, x_{2j+3})) = p(x_{2j+1}, x_{2j+2}),$$

$$\psi(p(x_{2j+2}, x_{2j+3})) < \phi(p(x_{2j+1}, x_{2j+2})). \tag{3.5}$$

From equation (3.4) and (3.5), we get

$$\psi(p(x_{n+1}, x_{n+2})) < \phi(p(x_n, x_{n+1})). \tag{3.6}$$

In the above $\{p(x_n, x_{n+1})/n \in \mathbb{N}\}$ is a non-increasing sequence and hence there exist $r \geq 0$ such that

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = r. \tag{3.7}$$

Let $n \to \infty$ in equation (3.6) and also using the fact ψ and ϕ are continuous, we get $\psi(r) < \phi(r)$. It gives

$$\xi(\psi(r), \phi(r)) \geq 0, \xi(\psi(r), \phi(r)) < \phi(r) - \psi(r).$$

From (ξ_1), $\xi(\psi(r), \phi(r)) = 0$ and hence $\psi(r) = \phi(r) = 0$, by altering distance function, $\psi(r) = \phi(r) = 0$ iff $r = 0$.
By equation (3.7), we get

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = 0, \tag{3.8}$$

by (p_2), we get

$$\lim_{n\to\infty} p(x_n, x_n) = 0, \tag{3.9}$$

since $d_p(x, y) = 2p(x, y)$ for all $x, y \in X$.

$$\lim_{n\to\infty} d_p(x_n, x_{n+1}) = 0. \tag{3.10}$$

Next we show that $\{x_n\}$ is a Cauchy sequence in metric space $(A \cup B, d_P)$. It is sufficient to show that $\{x_{2n}\}$ is a Cauchy sequence in $(A \cup B, d_P)$. Suppose to the contrary $\{x_{2n}\}$ is not a Cauchy sequence in $(A \cup B, d_P)$, there exist $\epsilon > 0$ and two subsequences $\{x_{2n(k)}\}$ and $\{x_{2m(k)}\}$ of $\{x_{2n}\}$ with $m(k) > n(k) > k$. $m(k)$ is the smallest index in $\mathbb{N}$ such that

$$d_p(x_{2m(k)}, x_{2n(k)}) \geq \epsilon, \tag{3.11}$$

this means that

$$d_p(x_{2m(k)}, x_{2n(k)-2}) < \epsilon, \tag{3.12}$$

from equation (3.10), (3.11) and triangle inequality, we get

$$\begin{aligned}
\epsilon &\leq d_p(x_{2m(k)}, x_{2n(k)}) \\
&\leq d_p(x_{2m(k)}, x_{2n(k)-2}) + d_p(x_{2n(k)-2}, x_{2n(k)}) \\
&< \epsilon + d_p(x_{2n(k)-2}, x_{2n(k)-1}) + d_p(x_{2n(k)-1}, x_{2n(k)}).
\end{aligned}$$

As $k \to \infty$ and using (3.8) we have

$$\lim_{k\to\infty} d_p(x_{2m(k)}, x_{2n(k)}) = \epsilon. \tag{3.13}$$

Again from (3.10) and we use triangle inequality we get

$$\begin{aligned}
\epsilon &\leq d_P(x_{2m(k)}, x_{2n(k)}) \\
&\leq d_p(x_{2n(k)}, x_{2n(k)-1}) + d_p(x_{2n(k)-1}, x_{2m(k)}) \\
&\leq d_p(x_{2n(k)}, x_{2n(k)-1}) + d_p(x_{2n(k)}, x_{2m(k)+1}) + d_p(x_{2m(k)+1}, x_{2m(k)}) \\
&\leq d_p(x_{2n(k)}, x_{2n(k)-1}) + d_p(x_{2n(k)-1}, x_{2m(k)}) + 2d_p(x_{2m(k)+1}, x_{2m(k)}) \\
&\leq 2d_p(x_{2n(k)}, x_{2n(k)-1}) + d_p(x_{2m(k)}, x_{2n(k)}) + 2d_p(x_{2m(k)+1}, x_{2m(k)}).
\end{aligned}$$

Using limit $n \to \infty$ in the above inequality and using equation (3.8), (3.10), we get

$$\begin{aligned}
\lim_{k\to\infty} d_p(x_{2m(k)}, x_{2n(k)}) &= \lim_{k\to\infty} d_p(x_{2m(k)+1}, x_{2n(k)-1}) \\
&= \lim_{k\to\infty} d_p(x_{2m(k)+1}, x_{2n(k)}) \\
&= \lim_{k\to\infty} d_p(x_{2m(k)}, x_{2n(k)-1}).
\end{aligned}$$

Since $d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ for all $x, y \in X$, therefore

$$\begin{aligned}
\lim_{k\to\infty} d_p(x_{2m(k)}, x_{2n(k)}) &= \lim_{k\to\infty} d_p(x_{2m(k)+1}, x_{2n(k)-1}) \\
&= \lim_{k\to\infty} d_p(x_{2m(k)+1}, x_{2n(k)}) \\
&= \lim_{k\to\infty} d_p(x_{2m(k)}, x_{2n(k)-1}) \\
&= \frac{\epsilon}{2}.
\end{aligned}$$

By equation (3.1) , we have

$$\begin{aligned}
&\xi(\psi(p(x_{2m(k)+1}, x_{2n(k)-1})), \phi(\max(p(x_{2m(k)}, Tx_{2m(k)}), p(x_{n(k)-2}, Tx_{2n(k)-2})))) \\
&\qquad < \phi(\max(p(x_{2m(k)}, Tx_{2m(k)}), p(x_{n(k)-2}, Tx_{2n(k)-2}))) - \psi(p(x_{2m(k)+1}, x_{2n(k)-1})
\end{aligned}$$

$$\xi(\psi(p(x_{2m(k)+1}, x_{2n(k)-1})) < \phi(\max(p(x_{2m(k)}, Tx_{2m(k)}), p(x_{n(k)-2}, Tx_{2n(k)-2}))).$$

Therefore

$$\xi(\psi(p(x_{2m(k)+1}, x_{2n(k)-1})) = 0.$$

Also $\xi(\psi(p(x_{2m(k)+1}, x_{2n(k)-1})) = 0$ if and only if $x_{2m(k)+1} = x_{2n(k)-1}$, hence $\psi(\frac{\epsilon}{2}) = 0$ iff $\frac{\epsilon}{2} = 0$ and $\epsilon = 0$. It is a contradiction to our assumption, thus $\{x_{2n}\}$ is a Cauchy sequence in $(A \cup B, d_p)$. Since (X, d) is complete and $A \cup B$ is a closed subspace of (X, p), then $(A \cup B, p)$ is complete.
Therefore $\{x_n\}$ converges in the metric space $(A \cup B, d_p)$,

$$\lim_{n\to\infty} d_p(x_n, v) = 0.$$

Hence

$$p(v, v) = \lim_{n\to\infty} p(x_n, v) = \lim_{n,m\to\infty} p(x_n, x_m). \tag{3.14}$$

Since $\{x_n\}$ is Cauchy in $(A \cup B, d_p)$ and $(A \cup B, p)$ if and only if it is Cauchy in $(A \cup B, d_p)$ and $(A \cup B, p)$ is complete iff $(A \cup B, d_p)$ is complete.

$$\lim_{n,m\to\infty} d_p(x_n, x_m) = 0,$$

$$d_p(x_m, x_n) = 2p(x_m, x_n) - p(x_m, x_m) - p(x_n, x_n). \tag{3.15}$$

As $m, n \to \infty$ and using equation (3.9) and equation (3.15) in the above we get

$$\lim_{n,m\to\infty} d_p(x_m, x_n) = 2p(x_m, x_n) = 0.$$

By equation (3.14), we have

$$\lim_{n\to\infty} p(x_n, v) = p(v, v) = 0.$$

Since $p(x_{2n}, v) \to 0$, x_{2n} is belongs to A and A is closed in (X, p),$v \in A$, ie) $v \in A \cap B$.
From definition of p, we have

$$\begin{aligned} p(x_n, Tv) &\leq p(x_n, v) + p(v, Tv) - p(v, v) \\ &\leq p(x_n, v) + p(v, x_n) + p(x_n, Tv) - p(v, v) - p(x_n, x_n). \end{aligned}$$

Taking limit $n \to \infty$ in the above inequality, we get

$$\lim_{n\to\infty} p(x_n, Tv) = p(v, Tv).$$

Now, we claim that $Tv = v$.
Since $x_{2n} \in A$ and $v \in B$ by equation (3.1), we have

$$\begin{aligned} \xi(\psi(p(x_{2n+1}, Tv), \phi(\max(p(x_{2n}, Tx_{2n}), p(v, Tv))))) &< \phi(\max(p(x_{2n}, Tx_{2n}), p(v, Tv)) \\ &- \psi(p(x_{2n+1}, Tv))), \\ \psi(p(x_{2n+1}, Tv)) &\leq \phi(\max(p(x_{2n}, Tx_{2n}), p(v, Tv))) \\ &= \phi(p(v, Tv)). \end{aligned}$$

Since ϕ is an altering distance function, $\phi(v, Tv) = 0 \iff p(v, Tv) = 0$,
ie) $Tv = v$.
Hence v is a fixed point of T.
To prove uniqueness:
Let w be any other fixed point of T in $A \cap B$.It is easy to prove $p(v, w) = 0$.

$$\begin{aligned} \xi(\psi(p(Tv, Tw), \phi(\max(p(v, Tv), p(w, Tw))))) &< \phi(\max(p(v, Tv), p(w, Tw)) \\ &- \psi(p(Tv, Tw) \\ \psi(p(Tv, Tw) &\leq \phi(\max(p(v, Tv), p(w, Tw))). \end{aligned}$$

Thus $\psi(p(Tv, Tw)) = 0$ and hence $p(Tv, Tw) = 0$, $p(v, w) = 0$. Hence $v = w$. □

4 Conclusion

In this paper, the main result determines a fixed point using cyclic $(\psi, \phi, Z)-$ contraction in partial metric spaces. Suppose, if we use this contraction in quasi-partial metric space, it satisfies the conditions (QPM1), (QPM2), (QPM3), (QPM4) in [13]. As a result, this contraction has a unique fixed point in quasi-partial metric space as well.

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