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(Dedicated to Professor D. S. Hooda on His 80^{th} Birth Anniversary Celebrations)

LEARNING EFFECTS ON RETAILER ORDERING POLICY FOR IMPERFECT QUALITY ITEMS UNDER TRADE CREDIT FINANCING WITH PRICING STRATEGIES

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Abstract

Nowadays customer service, pricing strategies and trade credit financing scheme are effective, essential and survival parameters for any kind of business setup in the market. In this study we have developed retailer's ordering policy for imperfect production in which we have applied the learning effect on inspection process on each and every batch of imperfect product. To stimulate sales of product and to study the effects of trade credit financing scheme on retailers business policy we have applied trade credit financing scheme on retailers ordering policy.

In this paper, we have developed, an economical order quantity (*EOQ*) model for retailer's price sensitive demand of product under two stage trade credit financing scheme.In the trade credit financing scheme we have assumed that the supplier offers to the retailer a fixed credit period of payment and the retailer also offers t a fixed credit period of payment o his customers. An optimal total profit function per unit time has been formulated under the different trade credit financing periods of payment with different costs and related parameters. A numerical example has been designed to verify the optimum results also we have done sensitive analysis through tables and graphically.
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1. Introduction

The traditional economical order quantity (*EOQ*) inventory model was formulated first by Harris [6]. It was a time and demand depended square root formula. Latter, several research articles have been designed to expand the fundamental (*EOQ*) model by considering various kinds of assumptions.

Ebert [3] developed a systematic device for approximating optimal aggregate scheduling under condition of changing productivity. Systematic changes in productivity constant can increase productivity of production system Muth and Spremann [23] introduced the classical square root formula on the class of stationary lot sizing problem with learning effect into the production cost.

Salameh *et al.* [28] suggested a economical production inventory model which was formulated under the learning curve effect in a finite production rate. Cheng [1] developed an EMQ model which incorporates with declining setup cost after effects of learning for large batch size in calculating the optimal manufacturing quantities and number of orders. Salameh and Jaber [27] extended the traditional (*EOQ/EPQ*) model by considering the imperfect quality items when using the (*EOQ/EPQ*) formulae. They also considered the assumption that at the end of 100 percent screening process the poor-quality items are sold as a single batch with low price.

Jaber and Guiffrida [9] Suggested an (*EPQ*) model with rework for imperfect quality items using (*WLC*). For this they proposed two different cases: (1) there is learning process with production, whereas there is no learning process in the rework process. (2) there is learning process with production and rework both. Eroglu and Ozdemir [4] developed an (*EOQ*) model in which they assumed that each ordered lot contains some defective items incorporating shortages at retailers end. They analyzed effect of percentage defective on optimal solution. They also assumed that, after 100 percent screening of each lot the good and defective items are separated in two collection of imperfect quality and scrap items.

Jaber *et al.* [8] extended the work of Salameh and Jaber [27] by assuming the percentage defective per lot decreases according to a learning curve (*LC*), which was experimentally validated by data from the automotive industry. Jaber *et al.* [10] developed a inventory model in which they assumed that the manufacturing process is interrupted during the maintenance of machinery to control the quality of product. In this article they developed two cases: (1) learning process is applied only regular production whereas no learning in reworks and (2) learning process is applied on regular production and reworks both.

Pan [24] investigated the impact of learning curve effects on setup cost by using the (*CRI*) uninterrupted review inventory model under the controllable lead time with the mixture of backorder and partial lost sales. Lin [21] investigated the marketing and manufacturing problem for a monopolist firm in which they assumed the demand depends on price, quality and cumulative sales. They specify several optimal policies which are beneficial for policy makers and could obtain insight into the consequence of their decisions.

Khedlekar et al.[14], developed three layer supply chain inventory model for price and suggested retail price dependent demand which consisted manufacturer, multi supplier and multi retailers. Khedlekar et al. [15], developed a production model for deteriorating product considering disruption in production with backlogging. Nigwal et al. [16], developed a multi-layer, multi-channel reverse supply chain inventory model for used product which consisted re-manufacturer, multi collector and multi-retailer.

Yoo *et al.* [7] focused on the problem that not only production process but also inspection processes are often not perfect, thereby generating defects and inspection errors. For this they developed a profit-maximizing (*EMQ*) model by incorporating both imperfect production process and two-way imperfect inspection process. Maity *et al.* [22] developed an integrated production-recycling system in a finite planning time in which they considered a dynamic demand rate is satisfied by production and recycling rate. Also they applied and studied Learning curve effects to reduction of setup cost.Sui *et al.* [29] proposed a model for Vendor-Managed Inventory (*VMI*) system in place of traditional retailer managed inventory with learning approach in which the supplier makes decisions of inventory management for the retailer.

Khan *et al.* [17] extended the work of Salameh and Jaber [27] by incorporating the case of learning in the inspection process. The model is more realistic than Salameh and Jabers [27] work because they also considered situations of lost sales and backorders. Based on model of Salameh and Jaber [27], Wahab and Jaber [31] presented a model for the optimal lot sizes of an item with imperfect quality by incorporating different holding cost for good and defective items. Jaber and Khan [11] developed a model to maximize a combination of performance of average processing time and process yield with respect to the number of batches. In this they incorporated the effect of varying the learning curve parameters in production and in rework.

Das *et al.* [2] presented a production-inventory model for deteriorating items in an imprecise situations characterized by inflation and timed value of money by considering with constant demand. They also considered that the planning time of the business activity time is random in nature and follows exponential distribution with a known mean. Khan *et al.* [18] extended the model of Salameh and Jaber [27] by incorporating the inspection error during the screening process by considering the probability of inspection errors is assumed to be known. An inventory model is developed by Raouf *et al.* [25] by considering an imperfect inspection process to describe the defective proportion of a received lot.

Rezaei and Salimi [26] developed a model in which they analyzed the problem under the following two different assumptions: (1) there is a relationship among the selling price, purchasing price and customer's demand (2) there is no relationship among the selling price, purchasing price and customer's demand. Konstantaras *et al.* [19] developed an (*EOQ*) model for imperfect quality items considering with shortages, in which they assumed that the fraction of perfect quality in each shipment increases due to learning effect. An inventory model for imperfect quality items has been developed by Jaggi *et al.* [12] under the condition of permissible delay in payments. In this article they allowed shortage with fully backlogged, which are ignored during screening process.

Teng *et al.* [30] developed an (*EPQ*) model from the seller's point of view in which they determined his/her optimal production lot size with trade credit financing period simultaneously. Finally, they concluded following that (i) trade credit financing stimulate not only sales but also increses opportunity cost and reduces default risk. Kumar *et al.* [20] studied the effect of learning on the economical ordering policy for deteriorating items allowing with shortages and partial backlogging. They concluded that due to learning effect the ordering cost is partly constant and partly decrease in each cycle.

Givi *et al.* [5] developed a Human Reliability Analysis (*HRA*) model that estimates the human error rate while performing an collectively job under the influence of learning -forgetting and fatigue-recovery. This model is able to measure the human error rate dynamically with time. Jayaswal *et al.* [13] developed trade credit financing inventory model for imperfect quality items under the effects of learning on ordering quantity. They derived total profit function per cycle by incorporating various costs and related parameters for the retailer. Yadav *et al.* [33], developed two layer supply chain model to optimize the profit function for imperfect quality items under the asymmetric information with market expenditure dependent demand.

Generally, in the traditional economic order quantity model, It is assumed that the retailer has pay to the supplier as soon as the items/products are received. But in the practice, supplier expects to stimulate his products and he tries to increase the sales of his product and therefore he offers to the retailer a certain delay period of payment. In the practice we call it a credit period of payment. In this article, we consider two stages trade credit financing periods, in which firstly, the supplier offers to the retailer a permissible delay period of payment and the retailer also offers to his customers a permissible delay period of payment without interest. Furthermore, in a real life every production system may produce something defective items. Due to defective units of items the total profit of retailer may be impacted. The defective units of items may be detected by applying the screening process after delivery of batches. For this, we consider screening process on each batches of imperfect quality items on retailer's end.

Learning curve (*LC*) or Experience curve (*EC*) was derived first by Wright [32] in 1936. It is a mathematical tool which relates the learning variables and cumulative quantity of units. In this paper, we have studied the impact of learning on screening process for imperfect quality items. Sigmoid function is the ideal shape of all other learning curves and in this article, we have used Sigmoid function which is formulated as $\alpha(n) = \left(\frac{a}{g+e^{bn}}\right)$, where $\alpha(n)$ is defective percentage rate of item in the single batch and n number of order b, $g > 0$ and $a > 0$ are model parameters.

***Table* 1.1:** Comparative table for contribution of different authors:

Authors	Learning Effects	Screening	Trade Credit Financing	Pricing
Wright (1936)	Yes	No	No	No
Ebert (1976)	Yes	No	No	Yes
Muth and Spremann (1983)	Yes	No	No	No
Salameh *et al.*(1993)	Yes	No	No	No
Cheng (1994)	Yes	No	No	No
Salameh and Jaber (2000)	Yes	Yes	No	No
Jaber *et al.*(2004)	Yes	Yes	No	No
Eroglu and Ozdemir(2007)	Yes	Yes	No	No
Jaber and Guiffrida (2008)	Yes	Yes	No	No
Pan (2008)	Yes	No	No	No
Lin (2008)	Yes	No	No	Yes
Jaber *et al.* (2008)	Yes	Yes	No	No
Yoo *et al.* (2009)	Yes	Yes	No	No
Sui, *et al.* (2010)	Yes	Yes	No	No
Khan *et al.* (2010)	Yes	Yes	No	No
Wahab and Jaber (2010)	Yes	Yes	No	No
Jaber and Khan (2010)	Yes	No	No	No
Das *et al.* (2010)	Yes	No	No	No
Khan *et al.* (2011)	No	Yes	No	No
Konstantaras *et al.* (2012)	Yes	Yes	No	No
Jaggi *et al.* (2013)	No	Yes	Yes	No
Teng *et al.* (2013)	Yes	No	No	No
Kumar *et al.* (2013)	Yes	No	No	No
Givi *et al.* (2015)	Yes	No	No	No
Jayaswal *et al.* (2019)	Yes	Yes	Yes	No
This paper	Yes	Yes	Yes	Yes

2. The Mathematical Model

2.1. Notations

Following notations are used in this model:

ϕ_n : Lot size for the n^{th} batch,

D : Demand rate of items in units per unit time for perfect quality items, Where, $D = \alpha - \beta p$,

C_s : Setup cost per order,

C_p : Purchasing cost per unit of items,

h : Inventory holding cost per unit time,

p : Selling price per unit of perfect quality items,
v : Selling price (On discounted price) per unit of defective items ($p > v$),
$\alpha(n)$: Percentage of defective units of item per lot,
T_n : Length of cycle for shipment per order,
χ : Screening rate per unit time ($D < \chi$),
S_c : Screening cost per unit,
t_n : Screening time of ϕ_n in planing time T_n, where, $t_n = \frac{\phi_n}{\chi} < T_n$,
I_e : Interest rate per unit \$ earned by retailer,
I_p : Interest rate per unit \$ paid by retailer,
SR : Sales revenue,
TC : Total cost,
$\Pi(p, \phi_n)$: Retailer's total profit per unit time,
l : Permissible delay period of payment offered per cycle time by supplier to the retailer,
m : Permissible delay period of payment offered per cycle time by retailer to customers,

2.2. Assumptions

The following assumptions are made in this model

- $D = \alpha - \beta p$, is the demand rate per unit time,
- The supplier provides a fixed and predetermined credit period to settle the accounts to the retailer,
- Selling price p and optimal lot size ϕ_n, are decision variables,
- No scrap is obtained during the screening process,
- Holding costs are constant,
- Screening procedure and demand of items occur simultaneously ($D < \chi$),
- Stock out situation is neglected,
- Lead time is zero,
- Supplier's supplying capacity is infinite,
- It has been assumed that each lot size contains perfect and imperfect items both,
- It has been assumed that the price of the perfect quality items is greater than that of the imperfect quality items,
- It has been assumed that the earned interest rate is less than the payable interest rate,
- It has been assumed that the retailer offers a permissible delay period of payment to his customers without interest to stimulate the sales,
- It has been assumed that $l, m \in [0, T_n]$, only.

2.3. The Mathematical formulation of model

In this section we designed a mathematical model for imperfect quality items with price dependent demand and trade-credit financing scheme. The working procedure of this mathematical model is depicted in the Figure 2.1. Initially, at the time $t = 0$ the batch size contains ϕ_n units of items and a batch of n^{th} shipment contains $\alpha(n)$ percentage of defective units of items. After completion of screening procedure at a rate χ, units per unit time, at time $t = t_n$, the imperfect quality items are equal to $\phi_n \alpha(n)$, which are immediately sold at discounted price v per unit in a single lot. The remaining inventory level of perfect quality items at any time t, is governed by the following differential equation:

$$\frac{dI}{dt} = -D = -(\alpha - \beta p), \ 0 \leq t \leq T_n, \tag{2.1}$$

with the boundary conditions: $I(0) = (1 - \alpha(n))\phi_n$ and $I(T_n) = 0$.

Solution of this equation gives:

$$I(t) = -(\alpha - \beta p)t + (1 - \alpha(n))\phi_n \tag{2.2}$$

at time $t = T_n$, the T_n can be determined by the following formula

$$T_n = \frac{(1 - \alpha(n))\phi_n}{(\alpha - \beta p)} \tag{2.3}$$

and according to the assumptions the screening time t_n is given by the following formula

$$t_n = \frac{\phi_n}{\chi}. \tag{2.4}$$

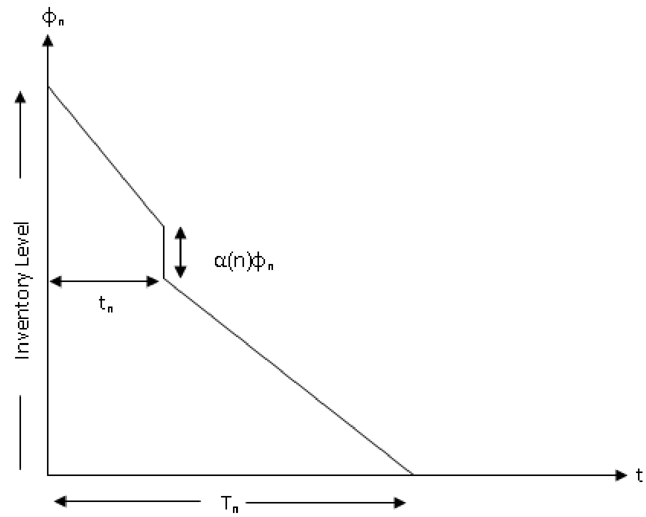

***Figure* 2.1:** Inventory Level Chart

Now we derive the various components of total profit. The Sales revenue must be equal to the sum of the selling of perfect quality items and the selling of imperfect quality items i.e

$$SR = p(1-\alpha(n))\phi_n + v\alpha(n)\phi_n, \tag{2.5}$$

$$\text{Ordering Cost} = C_s, \tag{2.6}$$

$$\text{Purchaging Cost} = C_p\phi_n, \tag{2.7}$$

$$\text{Screening Cost} = S_c\phi_n, \tag{2.8}$$

$$\text{Inventory Holding Cost} = h\left[(1-\alpha(n))\phi_n T_n - (\alpha-\beta p)\frac{T_n^2}{2} + \frac{\phi_n^2\alpha(n)}{\chi}\right], \tag{2.9}$$

Now the total expenditure per cycle is given by:

$$TC = C_s + C_p\phi_n + S_c\phi_n + h\left[(1-\alpha(n))\phi_n T_n - (\alpha-\beta p)\frac{T_n^2}{2} + \frac{\phi_n^2\alpha(n)}{\chi}\right], \tag{2.10}$$

The supplier offers to the retailer a permissible credit period l of payment to inspire sales and retailer also offers to his customer a permissible credit period n of payment to inspire his sales of items. As result, depending on the credit period and for $l, m \in (0, T_n)$, $t_n \neq T_n$, there are four separate cases available for the purchaser (retailer and customers).

$$(1)\ \ t_n \geq l \geq m, \quad (2)\ \ t_n \geq m \geq l,$$
$$(3)\ \ l \geq m \geq t_n, \quad (4)\ \ m \geq l \geq t_n.$$

Hence, the retailer's whole profit $\Pi_j(p, \phi_n)$, j=1,2,3, 4 per unit time can be defined as:

$$\Pi_j(p, \phi_n) = \frac{\text{SR-TC+(Earned Interest)-(Paid Interest)}}{T_n}, \quad \text{where } j\text{=1, 2, 3, 4.} \tag{2.11}$$

According to the credit periods l and m, the earned interest and paid interest for four different cases are estimated as follows:

Case 2.1. $t_n \geq l \geq m$

As per restriction of this case the credit period l is greater than the credit period m, therefore retailer earns the interest till the period 0 to l.Obviously after the end of credit period l, retailer is not able to make all the payment of supplier, because they do not able to sale all the units of item till the credit period l, consequently the retailer has to pay interest along with basic dues to the supplier. Let I_e earned interest rate per unit time and I_p paid interest rate per unit time, then

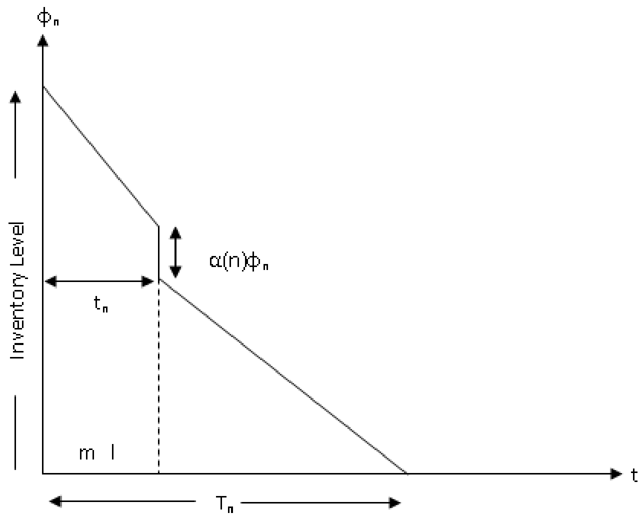

***Figure* 2.2:** Inventory Level Chart for Case 2.1

$$EI_e = I_e p(\alpha - \beta p)\left(\frac{l^2 - m^2}{2}\right), \tag{2.12}$$

$$PI_p = I_p C_p(\alpha - \beta p)(1 - \alpha(n))\phi_n \frac{(T_n - l)^2}{2} + C_p I_p \alpha(n)\phi_n(t_n - l), \tag{2.13}$$

$$SR_1 = p(1 - \alpha(n))\phi_n + v\alpha(n)\phi_n + I_e p(\alpha - \beta p)\left(\frac{l^2 - m^2}{2}\right), \tag{2.14}$$

$$TC_1 = C_s + C_p\phi_n + S_c\phi_n + h\left[(1 - \alpha(n))\phi_n T_n - (\alpha - \beta p)\frac{T_n^2}{2} + \frac{\phi_n^2\alpha(n)}{\chi}\right], \tag{2.15}$$

$$+ I_p C_p(\alpha - \beta p)(1 - \alpha(n))\phi_n \frac{(T_n - l)^2}{2} + C_p I_p \alpha(n)\phi_n(t_n - l),$$

$$\Pi_1(p, \phi_n) = \frac{SR_1 - TC_1}{T_n}, \tag{2.16}$$

$$\Pi_1(p, \phi_n) = p(\alpha - \beta p) + \frac{v\alpha(n)(\alpha - \beta p)}{(1 - \alpha(n)} + \frac{(\alpha - \beta p)^2 l^2 (I_e p - I_p c)}{2(1 - \alpha(n)\phi_n} - \frac{C_s(\alpha - \beta p)}{(1 - \alpha(n)}$$
$$- \frac{(C_p + S_c - C_p I_p \alpha(n))l(\alpha - \beta p) - C_p I_p(1 - \alpha(n)(\alpha - \beta p)l}{(1 - \alpha(n)}$$
$$- \left[\frac{h\left(\frac{(1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{\alpha(n)}{\chi}\right) + \frac{C_p I_p (1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{C_p I_p \alpha(n)}{\chi}}{(1 - \alpha(n)}\right]\phi_n(\alpha - \beta p) - \frac{p I_e(\alpha - \beta p)^2 m^2}{2(1 - \alpha(n))\phi_n}.$$

Case 2.2. $t_n \geq m \geq l$

As per restriction of this case the credit period l is less than the credit period m. Obviously after the end of credit period l, retailer is not able to make all the payment of supplier, because they do not able to sale all the units of item till the credit period l, consequently the retailer has to pay interest along with basic dues to the supplier. Let I_p paid interest rate per unit time, then

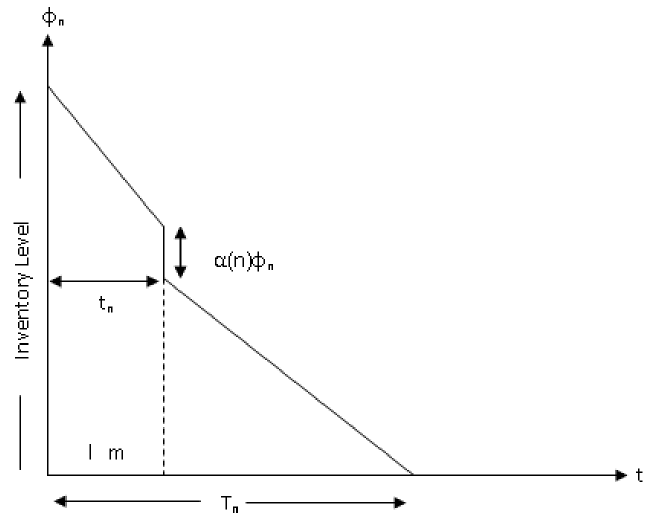

***Figure* 2.3:** Inventory Level Chart for Case 2.2

$$PI_p = I_p C_p(\alpha - \beta p)(1 - \alpha(n))\phi_n \frac{(T_n - l)^2}{2} + vI_p\alpha(n)\phi_n(t_n - l), \tag{2.17}$$

$$SR_2 = p(1 - \alpha(n))\phi_n + v\alpha(n)\phi_n, \tag{2.18}$$

$$TC_2 = C_s + C_p\phi_n + S_c\phi_n + h\left[(1 - \alpha(n))\phi_n T_n - (\alpha - \beta p)\frac{T_n^2}{2} + \frac{\phi_n^2\alpha(n)}{\chi}\right]$$

$$+I_p C_p(\alpha - \beta p)(1 - \alpha(n))\phi_n \frac{(T_n - l)^2}{2} + vI_p\alpha(n)\phi_n(t_n - l), \tag{2.19}$$

$$\Pi_2(p, \phi_n) = \frac{SR_2 - TC_2}{T_n} \tag{2.20}$$

$$= p(\alpha - \beta p) + \frac{v\alpha(n)(\alpha - \beta p)}{(1 - \alpha(n)} - \frac{C_p I_p(\alpha - \beta p)^2 l^2}{2(1 - \alpha(n)\phi_n} + \frac{C_s(\alpha - \beta p)}{(1 - \alpha(n)\phi_n}$$
$$- \frac{(C_p + S_c - C_p I_p\alpha(n)l)(\alpha - \beta p) - C_p I_p((1 - \alpha(n))l(\alpha - \beta p)}{(1 - \alpha(n)}$$
$$- \left[\frac{h\left(\frac{(1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{\alpha(n)}{\chi}\right) + \frac{C_p I_p(1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{C_p I_p\alpha(n)}{\chi}}{(1 - \alpha(n)}\right]\phi_n(\alpha - \beta p).$$

Case 2.3. $l \geq m \geq t_n$

As per restriction of this case the credit period l is greater than the credit period m. Obviously after the end of credit period l, retailer is not able to make all the payment of supplier, because they do not able to sale all the units of item till the credit period l, consequently the retailer has to pay interest along with basic dues to the supplier. Let I_e earned interest per unit time and I_p paid interest rate per unit time, then

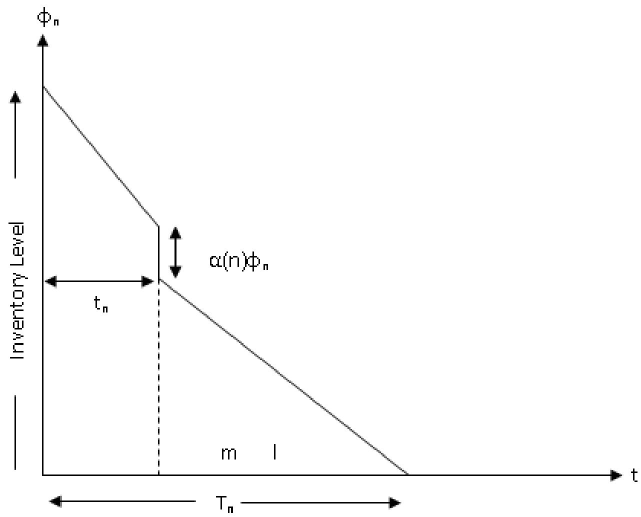

***Figure* 2.4:** Inventory Level Chart for Case 2.3

$$EI_e = I_e p(\alpha - \beta p)\left(\frac{l^2 - m^2}{2}\right) + vI_e\alpha(n)\phi_n(l - t_n), \tag{2.21}$$

$$PI_p = I_p C_p(\alpha - \beta p)(1 - \alpha(n))\phi_n \frac{(T_n - l)^2}{2}, \tag{2.22}$$

$$SR_3 = p(1 - \alpha(n))\phi_n + v\alpha(n)\phi_n + I_e p(\alpha - \beta p)\left(\frac{l^2 - m^2}{2}\right) + vI_e\alpha(n)\phi_n(l - t_n), \tag{2.23}$$

$$TC_3 = C_s + C_p\phi_n + S_c\phi_n + h\left[(1 - \alpha(n))\phi_n T_n - (\alpha - \beta p)\frac{T_n^2}{2} + \frac{\phi_n^2\alpha(n)}{\chi}\right]$$

$$+I_p C_p(\alpha - \beta p)(1 - \alpha(n))\phi_n \frac{(T_n - l)^2}{2}, \tag{2.24}$$

$$\Pi_3(p, \phi_n) = \frac{SR_3 - TC_3}{T_n}, \tag{2.25}$$

$$= p(\alpha - \beta p) + \frac{v\alpha(n)(\alpha - \beta p)}{(1 - \alpha(n)} + \frac{(\alpha - \beta p)^2 l^2(I_e p - I_p c)}{2(1 - \alpha(n)\phi_n} - \frac{C_s(\alpha - \beta p)}{(1 - \alpha(n)\phi_n}$$
$$- \frac{(C_p + S_c - C_p I_p\alpha(n)l)(\alpha - \beta p) - C_p I_p(1 - \alpha(n)(\alpha - \beta p)l}{(1 - \alpha(n)}$$
$$- \left[\frac{h\left(\frac{(1-\alpha(n))^2}{2(\alpha-\beta p)} + \frac{\alpha(n)}{\chi}\right) + \frac{C_p I_p(1-\alpha(n))^2}{2(\alpha-\beta p)} + \frac{C_p I_p\alpha(n)}{\chi}}{(1 - \alpha(n)}\right]\phi_n(\alpha - \beta p) - \frac{pI_e(\alpha - \beta p)^2 m^2}{2(1 - \alpha(n))\phi_n}.$$

Case 2.4. $m \geq l \geq t_n$

As per restriction of this case the credit period l is less than the credit period m. Obviously after the end of credit period l, retailer is not able to make all the payment of supplier, because they do not able to sale all the units of item till the credit period l, consequently the retailer has to pay interest along with basic dues to the supplier. Let I_e earned interest rate per unit time and I_p paid interest rate per unit time, then

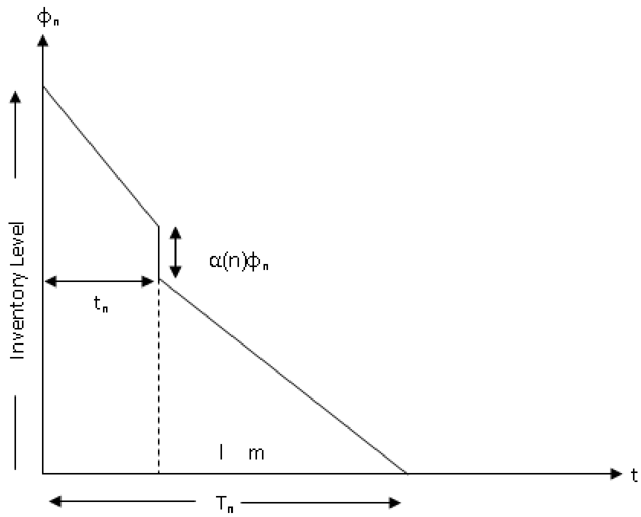

***Figure* 2.5:** Inventory Level Chart for Case 2.4

$$EI_e = I_e p(\alpha - \beta p)\frac{l^2}{2} + vI_e\alpha(n)\phi_n(l - t_n), \tag{2.26}$$

$$PI_p = I_p C_p(\alpha - \beta p)(1 - \alpha(n))\phi_n \frac{(T_n - l)^2}{2}, \tag{2.27}$$

$$SR_4 = p(1 - \alpha(n))\phi_n + v\alpha(n)\phi_n + I_e p(\alpha - \beta p)\frac{l^2}{2} + vI_e\alpha(n)\phi_n(l - t_n), \tag{2.28}$$

$$TC_4 = C_s + C_p\phi_n + S_c\phi_n + h\left[(1 - \alpha(n))\phi_n T_n - (\alpha - \beta p)\frac{T_n^2}{2} + \frac{\phi_n^2\alpha(n)}{\chi}\right]$$
$$+I_p C_p(\alpha - \beta p)(1 - \alpha(n))\phi_n \frac{(T_n - l)^2}{2}, \tag{2.29}$$

$$\Pi_4(p, \phi_n) = \frac{SR_4 - TC_4}{T_n} \tag{2.30}$$

$$= p(\alpha - \beta p) + \frac{v\alpha(n)(\alpha - \beta p)}{(1 - \alpha(n)} + \frac{(\alpha - \beta p)^2 l^2 (I_e p - I_p c)}{2(1 - \alpha(n)\phi_n} - \frac{C_s(\alpha - \beta p)}{(1 - \alpha(n)\phi_n}$$
$$- \frac{(C_p + S_c - C_p I_p \alpha(n) l)(\alpha - \beta p) - C_p I_p (1 - \alpha(n)(\alpha - \beta p) l}{(1 - \alpha(n)}$$
$$- \left[\frac{h\left(\frac{(1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{\alpha(n)}{\chi}\right) + \frac{C_p I_p (1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{C_p I_p \alpha(n)}{\chi}}{(1 - \alpha(n)}\right]\phi_n(\alpha - \beta p).$$

2.4. Optimality criteria

In this subsection we will find the optimal value of $\phi_n = \phi_n^*$, $p = p^*$, to optimize the profit function $\Pi_j(p, \phi_n)$ (j=1,2,3,4) for all the above four cases and we will check in which case gives the better results.

2.4.1. Optimality criteria for Case 2.1

Proposition 2.1. *Retailer's order quantity ϕ_n and retailing price p have an optimum point (p^*, ϕ_n^*).*

Proof. The profit function $\Pi_1(\phi_n, p)$ will be an optimum at point (p^*, ϕ_n^*) if $\frac{\partial \Pi_1(\phi_n, p)}{\partial \phi_n}$ and $\frac{\partial \Pi_1(p, \phi_n)}{\partial p}$, are vanishes at point (p^*, ϕ_n^*). Therefore,

$$-\frac{(\alpha - \beta p)^2 l^2 (I_e p - I_p C_p)}{2(1 - \alpha(n)\phi_n^2} + \frac{C_s(\alpha - \beta p)}{(1 - \alpha(n)\phi_n^2} + \frac{I_e p(\alpha - \beta p)^2 m^2}{2(1 - \alpha(n)\phi_n^2}$$

$$-\left[\frac{h\left(\frac{(1-\alpha(n)^2}{2(\alpha-\beta p)}+\frac{\alpha(n)}{\chi}\right)+\frac{CI_p(1-\alpha(n)^2}{2(\alpha-\beta p)}+\frac{C_pI_p\alpha(n)}{\chi}}{(1-\alpha(n)}\right](\alpha-\beta p)=0. \tag{2.31}$$

$$(\alpha-2\beta p)-\frac{v\alpha(n)\beta}{(1-\alpha(n)}+\frac{(\alpha-\beta p)^2l^2I_e}{2(1-\alpha(n)\phi_n}-\frac{2(\alpha-\beta p)l^2\beta(I_ep-I_pc)}{2(1-\alpha(n)\phi_n}$$
$$+\frac{C_s\beta}{(1-\alpha(n)\phi_n}+\frac{(C_p+S_c-C_pI_p\alpha(n))l\beta-C_pI_p(1-\alpha(n)\beta l}{(1-\alpha(n)}$$
$$-\left[\frac{h\alpha(n)\beta\phi_n}{\chi(1-\alpha(n))}+\frac{C_pI_p\alpha(n)\beta\phi_n}{\chi(1-\alpha(n))}\right]+\frac{I_em^2\left[(\alpha-\beta p)^2-2\beta p(\alpha-\beta p)\right]}{2(1-\alpha(n))}=0, \tag{2.32}$$

solution of the above system of two equations will gives an optimal value of $(p,\,\phi_n)$

Proposition 2.2. *The profit function $\Pi_1(p,\phi_n)$ is jointly concave for the value of ϕ_n, p if*

$$\left[D^2I_ep(l^2-m^2)-C_pI_pD^2l^2-2C_sD\right]\left[\beta\eta\phi_n+2D\beta l^2I_e-\beta^2l^2(I_ep-I_pC_p)+(2\alpha\beta-3\beta^2p)\phi_n\right]$$

-

$$-\frac{1}{4n^2}\left[\eta\left(I_e\eta^2(p-m^2)-2\eta I_PC_p-2C_p\beta\right)+C_pI_pD^2\phi_n^2\beta\right]^2>0.\ \textit{where } \eta=1-\alpha(n)$$

Proof. The second order partial derivatives of $\Pi_1(\phi_n,p)$ are:

$$\frac{\partial^2\Pi_1(\phi_n,p)}{\partial\phi_n^2}=\frac{D^2l^2(I_ep-I_pC_p)-2C_sD-I_epD^2m^2}{\eta\phi_n^3} \tag{2.33}$$

$$\frac{\partial^2\Pi_1(\phi_n,p)}{\partial\phi_n\partial p}=\frac{-D^2I_ep-2DI_pC_p\beta-2C_s\beta+I_eD^2m^2}{2\eta\phi^2}+\frac{(h+C_pI_p)\eta\beta}{D}+\frac{(h+C_pI_p)\alpha(n)h\beta}{\chi\eta} \tag{2.34}$$

$$\frac{\partial^2\Pi_1(\phi_n,p)}{\partial p^2}=-2\beta-\frac{(1-\beta)D\beta l^2I_e}{\eta\phi_n}+\frac{l^2\beta^2(I_ep-I_pC_p)}{\eta\phi_n}-\frac{I_em^2(2\alpha\beta-3\beta^2p)}{\eta}. \tag{2.35}$$

After simplification of above terms, the jointly concavity condition $rt-s^2>0$, of $\Pi_1(\phi_n,p)$ is satisfied with respect to ϕ_n and p if

$$\left[D^2I_ep(l^2-m^2)-C_pI_pD^2l^2-2C_sD\right]\left[\beta\eta\phi_n+2D\beta l^2I_e-\beta^2l^2(I_ep-I_pC_p)+(2\alpha\beta-3\beta^2p)\phi_n\right]$$
$$-\frac{1}{4n^2}\left[\eta\left(I_e\eta^2(p-m^2)-2\eta I_PC_p-2C_p\beta\right)+C_pI_pD^2\phi_n^2\beta\right]^2>0.\ \text{where } \eta=1-\alpha(n). \tag{2.36}$$

2.4.2. Optimality criteria for case 2.2

Proposition 2.3. *Retailer's order quantity ϕ_n and retailing price p have an optimum point (p^*,ϕ_n^*).*

Proof. The profit function $\Pi_2(p,\phi_n)$ will be an optimum at point (p,ϕ_n^*) if $\frac{\partial\Pi_2(p,\phi_n)}{\partial\phi_n}=0$ and $\frac{\partial\Pi_2(p,\phi_n)}{\partial p}=0$, will be vanish at point (p^*,ϕ_n^*). Therefore,

$$\frac{(\alpha-\beta p)^2l^2I_pC_p}{2(1-\alpha(n)\phi_n^2}+\frac{C_s(\alpha-\beta p)}{(1-\alpha(n)\phi_n^2},$$
$$-\left[\frac{h\left(\frac{(1-\alpha(n)^2}{2(\alpha-\beta p)}+\frac{\alpha(n)}{\chi}\right)+\frac{CI_p(1-\alpha(n)^2}{2(\alpha-\beta p)}+\frac{CI_p\alpha(n)}{\chi}}{(1-\alpha(n)}\right](\alpha-\beta p)=0, \tag{2.37}$$

$$(\alpha-2\beta p)-\frac{v\alpha(n)\beta}{(1-\alpha(n))}+\frac{(\alpha-\beta p)l^2\beta I_pC_p}{(1-\alpha(n)\phi_n}$$
$$+\frac{C_s\beta}{(1-\alpha(n)\phi_n}+\frac{(C_p+S_c-C_pI_p\alpha(n)l)\beta-C_pI_p(1-\alpha(n)\beta l}{(1-\alpha(n)}$$
$$-\left[\frac{h\alpha(n)\beta\phi_n}{\chi(1-\alpha(n))}+\frac{C_pI_p\alpha(n)\beta\phi_n}{\chi(1-\alpha(n))}\right]=0. \tag{2.38}$$

Solution of the above system of two equations will gives an optimal value of $(p,\,\phi_n)$.

Proposition 2.4. *The profit function $\Pi_2(\phi_n, p)$ is jointly concave for the value of p, ϕ_n if*

$$\left[(D^2l^2I_pC_p - 2C_sD)(2\beta\eta\phi_n + \beta^2l^2I_pC_p)\right] + \left[Dl^2I_pC_p\beta - C_s\beta - (h + I_pC_p)(\eta\phi_n^2 + \alpha(n))\beta\right]^2 > 0.$$

Proof. The second order partial derivatives of $\Pi_2(\phi_n, p)$ are:

$$\frac{\partial^2\Pi_2(\phi_n, p)}{\partial\phi_n^2} = -\frac{D^2l^2I_pC_p + 2C_sD}{\eta\phi_n^3}, \tag{2.39}$$

$$\frac{\partial^2\Pi_2(\phi_n, p)}{\partial\phi_n\partial p} = -\frac{Dl^2I_pC_p\beta - C_s\beta}{\eta\phi_n^2} + \frac{(h + C_pI_p)(1 - \alpha(n))\beta}{D} + \frac{(h + C_pI_p)\alpha(n)\beta}{\eta\chi}, \tag{2.40}$$

$$\frac{\partial^2\Pi_2(\phi_n, p)}{\partial p^2} = -2\beta - \frac{\beta^2l^2I_pC_p}{\eta\phi_n}. \tag{2.41}$$

After simplification of above terms, the jointly concavity condition $rt - s^2 > 0$, of $\Pi_2(\phi_n, p)$ is satisfied with respect to ϕ_n, and p if

$$\left[(D^2l^2I_pC_p - 2C_sD)(2\beta\eta\phi_n + \beta^2l^2I_pC_p)\right] + \left[Dl^2I_pC_p\beta - C_s\beta - (h + I_pC_p)(\eta\phi_n^2 + \alpha(n))\beta\right]^2 > 0.$$

2.4.3. 2.4.3. Optimality criteria for case 2.3

Proposition 2.5. *Retailer's order quantity ϕ_n and retailing price p have an optimum point (p^*, ϕ_n^*).*

Proof. The profit function $\Pi_3(\phi_n, p)$ will be an optimum at point (p^*, ϕ_n^*) if $\frac{\partial\Pi_3(\phi_n,p)}{\partial\phi_n} = 0$ and $\frac{\partial\Pi_3(\phi_n,p)}{\partial p} = 0$, will vanish at point (ϕ_n^*, p^*). Therefore,

$$\begin{aligned}&-\frac{(\alpha - \beta p)^2l^2(I_ep - I_pC_p)}{2(1 - \alpha(n)\phi_n^2} + \frac{C_s(\alpha - \beta p)}{(1 - \alpha(n)\phi_n^2} + \frac{pI_e(\alpha - \beta p)^2m^2}{2(1 - \alpha(n)\phi_n^2}\\ &-\left[\frac{h\left(\frac{(1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{\alpha(n)}{\chi}\right) + \frac{CI_p(1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{CI_p\alpha(n)}{\chi}}{(1 - \alpha(n)}\right](\alpha - \beta p) = 0,\end{aligned} \tag{2.42}$$

$$\begin{aligned}&(\alpha - 2\beta p) - \frac{v\alpha(n)\beta}{(1 - \alpha(n))} + \frac{(\alpha - \beta p)^2l^2I_e}{2(1 - \alpha(n))\phi_n} + \frac{C_s\beta}{(1 - \alpha(n)\phi_n}\\ &\frac{(\alpha - \beta p)l^2\beta(I_ep - I_pC_p)}{(1 - \alpha(n))\phi_n} + \frac{(C_p + S_c - vI_e\alpha(n)l)\beta - C_pI_p(1 - \alpha(n)l)\beta}{(1 - \alpha(n)}\\ &-\left[\frac{h\alpha(n)\beta\phi_n}{\chi(1 - \alpha(n))} + \frac{vI_e\alpha(n)\beta\phi_n}{\chi(1 - \alpha(n))}\right] + \frac{pI_e(\alpha - \beta p)\beta m^2}{(1 - \alpha(n)\phi_n} = 0.\end{aligned} \tag{2.43}$$

Solution of the above system of two equations will gives an optimal value of (p, ϕ_n).

Proposition 2.6. *Proposition 2.6. The profit function $\Pi_3(\phi_n, p)$ is jointly concave for the value of p, ϕ_n if*

$$\left[D^2I_pp(l^2 - m^2) - D^2l^2I_pC_p - 2C_sD\right]\left[2\beta\eta\phi_n + \beta I_eD(2l^2 - m^2) - \beta^2I_ep(l^2 - m^2) + l^2\beta^2I_pC_p\right]$$

$$\left[D^2l^2I_e - 2Dl^2(I_ep - I_pC_p)\beta + 2C_s\beta + (2pI_em^2\beta - I_eD^2m^2) - \frac{2\beta(h + C_pI_p)\alpha(n)\phi_n^2}{\chi}\right]^2 > 0,$$

where, $\eta = (1 - \alpha(n))$, and $D = \alpha - \beta p$.

Proof. The second order partial derivatives of $\Pi_3(\phi_n, p)$ are:

$$\frac{\partial^2\Pi_3(p, \phi_n)}{\partial\phi_n^2} = \frac{D^2I_ep(l^2 - m^2) - D^2l^2I_pC_p - 2C_sD}{\eta\phi_n^3}, \tag{2.44}$$

$$\frac{\partial^2\Pi_3(p, \phi_n)}{\partial\phi_n\partial p} = -\left(\frac{DI_e(l^2 - m^2) - 2Dl^2(I_ep - I_pC_p\beta) + 2(C_s + pDI_em^2)\beta}{2\eta\chi}\right) + \frac{\beta(h + I_cC_p)\alpha(n)}{\eta\chi}, \tag{2.45}$$

$$\frac{\partial^2\Pi_3(p, \phi_n)}{\partial p^2} = -2\beta - \frac{2D\beta l^2I_e}{\eta\phi_n} + \frac{\beta^2I_ep(l^2 - m^2)}{\eta\phi_n} - \frac{l^2\beta^2I_pC_p}{\eta\phi_n} + \frac{I_eD\beta^2m^2}{\eta\phi_n}. \tag{2.46}$$

After simplification of above terms, the jointly concavity condition $rt - s^2 > 0$, of $\Pi_2(p, \phi_n)$ is satisfied with respect to ϕ_n, and p if

$$\left[D^2 I_p p(l^2 - m^2) - D^2 l^2 I_p C_p - 2C_s D\right]\left[2\beta\eta\phi_n + \beta I_e D(2l^2 - m^2) - \beta^2 I_e p(l^2 - m^2) + l^2\beta^2 I_p C_p\right]$$
$$\left[D^2 l^2 I_e - 2Dl^2(I_e p - I_p C_p)\beta + 2C_s\beta + (2pI_e m^2\beta - I_e D^2 m^2) - \frac{2\beta(h + C_p I_p)\alpha(n)\phi_n^2}{\chi}\right]^2 > 0,$$

where, $\eta = (1 - \alpha(n))$, and $D = \alpha - \beta p$.

2.4.4. Optimality criteria for case 2.4

Proposition 2.7. *Retailer's order quantity ϕ_n and retailing price p have an optimum point (p^*, ϕ_n^*).*

Proof. The profit function $\Pi_4(p, \phi_n)$ will be optimum at point (p^*, ϕ_n^*) if $\frac{\partial \Pi_4(\phi_n, p)}{\partial \phi_n} = 0$ and $\frac{\partial \Pi_4(p, \phi_n)}{\partial p} = 0$, must be vanishes at the point (p^*, ϕ_n^*). Therefore,

$$-\frac{(\alpha - \beta p)^2 l^2 (I_e p - I_p C_p)}{2(1 - \alpha(n)\phi_n^2} + \frac{C_s(\alpha - \beta p)}{(1 - \alpha(n)\phi_n^2}$$
$$-\left[\frac{h\left(\frac{(1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{\alpha(n)}{\chi}\right) + \frac{CI_p(1-\alpha(n)^2}{2(\alpha-\beta p)} + \frac{CI_p\alpha(n)}{\chi}}{(1 - \alpha(n)}\right](\alpha - \beta p) = 0, \tag{2.47}$$

$$(\alpha - 2\beta p) - \frac{v\alpha(n)\beta}{(1 - \alpha(n))} + \frac{(\alpha - \beta p)^2 l^2 I_e}{2(1 - \alpha(n))\phi_n} + \frac{C_s\beta}{(1 - \alpha(n)\phi_n}$$
$$\frac{(\alpha - \beta p) l^2 \beta (I_e p - I_p C_p)}{(1 - \alpha(n))\phi_n} + \frac{(C_p + S_c - vI_e\alpha(n)l)\beta - C_p I_p(1 - \alpha(n)l)\beta}{(1 - \alpha(n)}$$
$$-\left[\frac{h\alpha(n)\beta\phi_n}{\chi(1 - \alpha(n))} + \frac{vI_e\alpha(n)\beta\phi_n}{\chi(1 - \alpha(n))}\right] = 0 \tag{2.48}$$

Solution of the above system of two equations will gives an optimal value of $(\phi_n,\ p)$.

Proposition 2.8. *The profit function $\Pi_3(p, \phi_n)$ is jointly concave for the value of p, ϕ_n if*

$$\left[D^2 l^2 (I_e p - I_p C_p) - 2C_s D\right]\left[l^2\beta^2(I_e p - I_p C_p) - 2\beta\eta\phi_n - 2Dl^2 I_e\beta\right]$$
$$-\left[2Dl^2\beta(I_e p - I_p C_p) - 2C_s D + \frac{2\beta(h + I_e C_p)\alpha(n)\phi_n^2}{\chi} - D^2 l^2 I_e\right]^2 > 0,$$

where $D = \alpha - \beta p$*and* $\eta = 1 - \alpha(n)$.

Proof. The second order partial derivatives of $\Pi_4(p, \phi_n)$ are:

$$\frac{\partial^2 \Pi_4(p, \phi_n)}{\partial \phi_n^2} = \frac{D^2 l^2 (I_e p - C_p I_p) - 2C_s D}{\eta\phi^3} \tag{2.49}$$

$$\frac{\partial^2 \Pi_4(p, \phi_n)}{\partial \phi_n \partial p} = \frac{2Dl^2\beta(I_e p - I_p C_p) - 2C_s D + \frac{2\phi_n^2\beta(h + C_p I_p)\alpha(n)}{\chi}}{2\eta\phi_n^3} \tag{2.50}$$

$$\frac{\partial^2 \Pi_4(\phi_n, p)}{\partial p^2} = -\frac{2\beta\eta\phi_n + 2Dl^2\beta I_e - l^2\beta^2(I_e p - I_p C_p)}{\eta\phi_n}. \tag{2.51}$$

After simplification of above terms, the jointly concavity condition $rt - s^2 > 0$, of $\Pi_4(p, \phi_n)$ is satisfied with respect to ϕ_n, and p if

$$\left[D^2 l^2 (I_e p - I_p C_p) - 2C_s D\right]\left[l^2\beta^2(I_e p - I_p C_p) - 2\beta\eta\phi_n - 2Dl^2 I_e\beta\right]$$
$$-\left[2Dl^2\beta(I_e p - I_p C_p) - 2C_s D + \frac{2\beta(h + I_e C_p)\alpha(n)\phi_n^2}{\chi} - D^2 l^2 I_e\right]^2 > 0,$$

where $D = \alpha - \beta p$and $\eta = 1 - \alpha(n)$.

3. Algorithm

Stage 3.1: Find out $\phi_n^* = \phi_n$ and $p^* = p$ (say) by solving the equations (2.31) and (2.32) and substituting the values of $\phi_n^* = \phi_n$, $p^* = p$ into the equations (2.3) and (2.4) and compute the values of T_n and t_n respectively. If $t_n \geq l \geq m$, then maximum total profit is derived from the equation (2.16).

Stage 3.2: Find out $\phi_n^* = \phi_n$ and $p^* = p$ (say) by solving the equations (2.37) and (2.38) and substituting the values of $\phi_n^* = \phi_n$, $p^* = p$ into the equations (2.3) and (2.4) and compute the values of T_n and t_n respectively. If $t_n \geq m \geq l$, then maximum profit is derived from the equation (2.20).

Stage 3.3: Find out $\phi_n^* = \phi_n$ and $p^* = p$ (say) by solving the equations (2.42) and (2.43) and substituting the values of $\phi_n^* = \phi_n$, $p^* = p$ into the equations (2.3) and (2.4) and compute the values of T_n and t_n respectively. If $l \geq m \geq t_n$, then maximum profit is derived from the equation (2.25).

Stage 3.4: Find out $\phi_n^* = \phi_n$ and $p^* = p$ (say) by solving the equations (2.47) and (2.48) and substituting the values of $\phi_n^* = \phi_n$, $p^* = p$ into the equations (2.3) and (2.4) and compute the values of T_n and t_n respectively. If $m \geq l \geq t_n$, then maximum profit is derived from the equation (2.30).

Stage 3.5: Compare the calculated profit for Case 2.1, Case 2.2, Case 2.3 and Case 2.4 at m, l for choosing an optimal value of ϕ_n and p associated with maximum profit.

4. Numerical Examples

4.1. Case 2.1

We have considered the following data set of input parameters as: $\alpha = 4000$ units/unit time, $\beta = 0.5$, $C_s = \$100$, $h = \$0.5$ unit/unit time, $C_p = \$3500$ /unit, $v = \$2500$ per/unit, $\chi = 5000$ units, $S_c = \$0.5$/unit, $I_e = 0.003$/unit time, $I_p = \$0.004$/unit time, $\alpha(n) = 0.03994$, $n = 2$, $a = 40$, $b = 1$, $g = 999$ as Jaber *et al.* [8], $l = 0.03$/per planning, $m = 0.025$/per planning.

By using of the proposed algorithm we get the optimal order quantity (*EOQ*) $\phi_n = 130$ units per unit time, $p = 5771$ per unit and after substituting the optimum value of ϕ_n and p into the equation (**??**) we get the retailer's profit $\Pi(\phi_n, p) = 2482766$ per unit time, screening time $t_n = 0.026$ per unit time and time interval $T_n = 0.112$. At the optimum point $\phi_n = 126$, $p = 5771$ the necessary and sufficient condition of optimality of profit function is satisfied, i.e. $\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n^2} = -0.108 < 0$ and $\left(\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n^2}\right)\left(\frac{\partial^2\Pi(\phi_n,p)}{\partial p^2}\right) - \left(\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n\partial p}\right)^2 = 0.190 > 0$.

4.2. Case 2.2

We have considered the following data set of input parameters as: $\alpha = 4000$ units/unit time, $\beta = 0.5$, $C_s = \$100$, $h = \$0.5$ unit/ unit time, $C_p = \$3500$ /unit, $v = \$2500$ per/unit, $\chi = 5000$ units, $S_c = \$0.5$/unit, $I_e = 0.003$/unit time, $I_p = \$0.004$/unit time, $\alpha(n) = 0.03994$, $n = 2$, $a = 40$, $b = 1$, $g = 999$ as Jaber *et al.* [8], $m = 0.03$/per planning, $l = 0.025$/per planning.

By using of the proposed algorithm we get the optimal order quantity (*EOQ*) $\phi_n = 178$ units per unit time, $p = 3958$ per unit and after substituting the optimum value of ϕ_n and p into the equation (**??**) we get the retailer's profit $\Pi(\phi_n, p) = 840580$ per unit time, screening time $t_n = 0.035$ per unit time and time interval $T_n = 0.084$. At the optimum point $\phi_n = 178$, $p = 3958$ the necessary and sufficient condition for the profit function is satisfied, i.e. $\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n^2} = -0.005 < 0$ and $\left(\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n^2}\right)\left(\frac{\partial^2\Pi(\phi_n,p)}{\partial p^2}\right) - \left(\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n\partial p}\right)^2 = 0.0053 > 0$.

4.3. Case 2.3

We have considered the following data set of input parameters as: $\alpha = 4000$ units/unit time, $\beta = 0.5$, $C_s = \$100$, $h = \$0.5$ unit/ unit time, $C_p = \$3500$ /unit, $v = \$2500$ per/unit, $\chi = 5000$ units, $S_c = \$0.5$/unit, $I_e = 0.003$/unit time, $I_p = \$0.004$/unit time, $\alpha(n) = 0.03994$, $n = 2$, $a = 40$, $b = 1$, $g = 999$ as Jaber *et al.* [8], $l = 0.045$/per planning, $m = 0.04$/per planning.

By using of the proposed algorithm we get the optimal order quantity (*EOQ*) $\phi_n = 133$ units per unit time, $p = 5822$ per unit and after substituting the optimum value of ϕ_n and p into the equation (**??**) we get the retailer's profit $\Pi(\phi_n, p) = 2481794$ per unit time, screening time $t_n = 0.026$ per unit time and time interval $T_n = 0.118$. At the optimum point $\phi_n = 133$, $p = 552$ the necessary and sufficient condition for the profit function is satisfied, i.e. $\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n^2} = -0.104 < 0$ and $\left(\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n^2}\right)\left(\frac{\partial^2\Pi(\phi_n,p)}{\partial p^2}\right) - \left(\frac{\partial^2\Pi(\phi_n,p)}{\partial\phi_n\partial p}\right)^2 = 0.093 > 0$.

4.4. Case 2.4

We have considered the following data set of input parameters as: $\alpha = 4000$ units/unit time, $\beta = 0.5$, $C_s = \$100$, $h = \$0.5$ unit/ unit time, $C_p = \$3500$ /unit, $v = \$2500$ per/unit, $\chi = 5000$ units, $S_c = \$0.5$/unit, $I_e = 0.003$/unit time, $I_p = \$0.004$/unit time, $\alpha(n) = 0.03994$, $n = 2$, $a = 40$, $b = 1$, $g = 999$ as Jaber *et al.* [8], $l = 0.03$/per planning, $m = 0.025$/per planning.

By using of the proposed algorithm we get the optimal order quantity (EOQ) $\phi_n = 125$ units per unit time, $p = 5822$ per unit and after substituting the optimum value of ϕ_n and p into the equation (**??**) we get the retailer's profit $\Pi(\phi_n, p) = 2481845$ per unit time, screening time $t_n = 0.025$ per unit time and time interval $T_n = 0.11$. At the optimum point $\phi_n = 126$, $p = 5771$ the necessary and sufficient condition for the profit function is satisfied, i.e. $\frac{\partial^2 \Pi(\phi_n,p)}{\partial \phi_n^2} = -0.112 < 0$ and $\left(\frac{\partial^2 \Pi(\phi_n,p)}{\partial \phi_n^2}\right)\left(\frac{\partial^2 \Pi(\phi_n,p)}{\partial p^2}\right) - \left(\frac{\partial^2 \Pi(\phi_n,p)}{\partial \phi_n \partial p}\right)^2 = 0.097 > 0$.

5. Sensitivity Analysis

We performed the sensitivity analysis with respect to key parameters to show the sensitivity of model and to determine the situation, which gives the optimal results. We consider the various values of key parameters like trade credit periods m, l interest earn rate I_e, interest payable rate I_p, defective percentage items $\alpha(n)$, number of shipment (n), the impact on the optimal lot size ϕ_n, selling price p, time period T and total profit $\Pi(\phi_n, p)$ are given in the following tables.

***Table* 5.1:** Effect of learning rate on the profit for Case 2.1:

No. of Shipment (n)	Learning Rate b=1 Case 2.1 $\Pi(\phi_n, p)$	b=1.2 Case 2.1 $\Pi(\phi_n, p)$	b=1.4 Case 2.1 $\Pi(\phi_n, p)$
1	2482766	2482795	2482830
2	2482990	2483163	2483418
3	2483588	2484344	2485658
4	2485139	2487865	2492871
5	2488869	2496420	2507499

***Table* 5.2:** Effect of number of shipment on order quantity, defective percentage, selling price and profit for Case 2.1:

No. of Shipment (n)	Order quantity ϕ_n	Defective percentage per order	Selling price p	Profit $\Pi(\phi_n, p)$
1	130.67	0.039931	5771.25	2482766
2	130.65	0.039746	5771.10	2482990
3	130.61	0.039251	5770.83	2483588
4	130.50	0.037965	5770.14	2485139
5	130.23	0.034861	5768.47	2488869

***Table* 5.3:** Effect of credit periods on order quantity, time interval, screening time, selling price and profit for Case 2.1:

Credit periods (n)	Order quantity ϕ_n	Time Interval T_n	Screening time t_n	Selling price p	Profit $\Pi(\phi_n, p)$
with l, m	130.67	0.1125	0.0261	5771.20	2482766
without l	131.70	0.1134	0.0263	5771.42	2482340
without m	126.83	0.1092	0.0254	5771.26	2482710
without l, m	127.90	0.1102	0.0255	5771.47	2482287

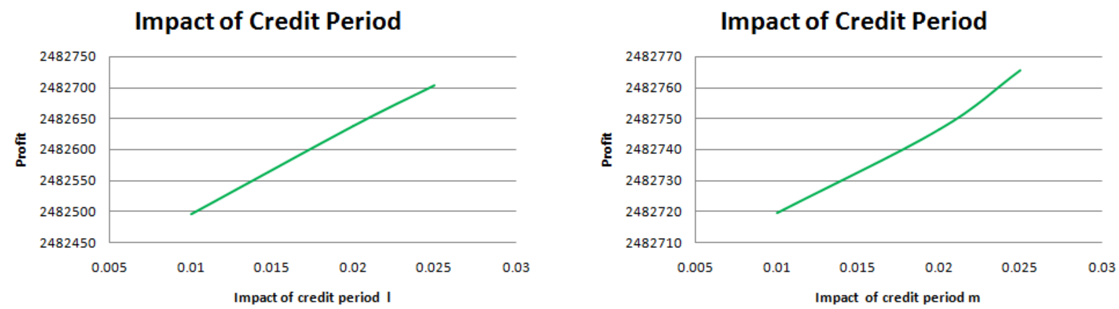

***Figure* 5.1:** Impact of credit period l and m on profit for Case 2.1

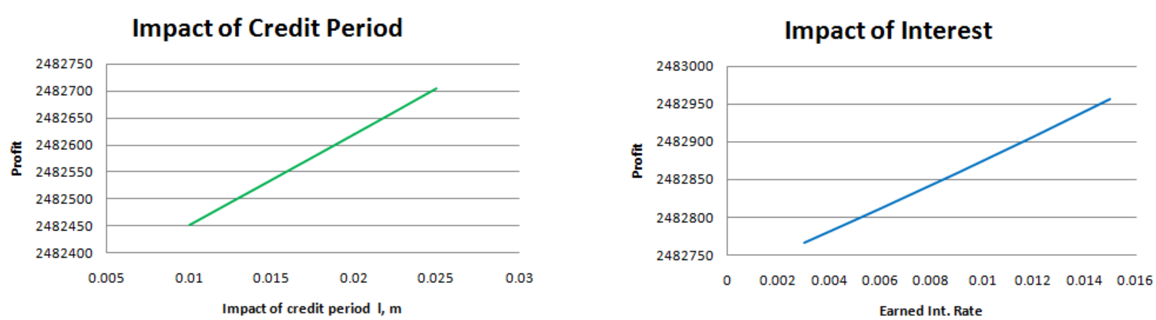

***Figure* 5.2:** Impact of credit period m, l both and earned interest on profit for Case 2.1

From all the data Tables (5.1-5.3) and Figures (5.1-5.3) and based on computational results, we can see that, (since we are seeking that situation in which total expenditure will be minimum and total profit will be maximum). We have seen that Case 2.1 is most favorable with respect to profit for $l, m \in [0, t_n]$. Further we have analyzed this Case 2.1 with respect to key parameters. From the Figures (5.1-5.3), we can see that $(l,m) \to t_n$ the earned interest I_e ϕ_n, t_n and T_n are increasing and paid interest I_p, p decreasing. Figure 5.3. The total profit also increases exponentially if increases the number of shipments.

Data Table 5.1 shows that if as the learning exponential parameter increases from $b = 1.00$ to $b = 1.40$along with the number of shipment, then the total profit increases and decreases the percentage of defective items simultaneously. From the sensitivity analysis we have observed that learning effects, trade credit financial policy and selling price strategies are all important tools which are helpful to gain more and more profit.

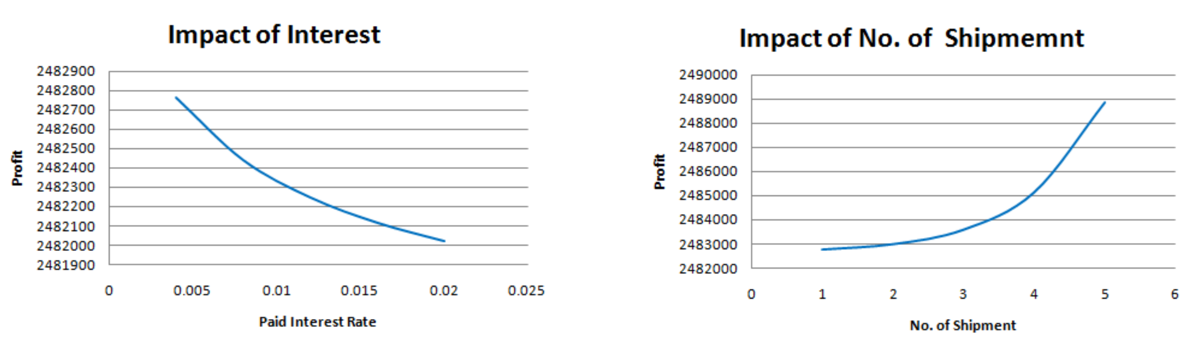

***Figure* 5.3:** Impact of paid interest and no. of shipment on profit for Case 2.1

6. Conclusion

This article proposed an economical order quantity model for imperfect quality items by considering three real assumptions. These three assumptions are:

(1) we have applied learning effects on screening process to separate the good and defective units of items,
(2) we have used two stage trade credit financing to promote the retailer's business and
(3) we have also used pricing strategies to determine optimal selling price. A comprehensive sensitivity has done to reflects the importance of key parameters and assumed assumptions. We have shown that from in the above four cases of credit periods Case 2.1 is more beneficial for any kind of business setup. We have optimized total profit, selling price and order quantity with respect to key elements like credit periods, number of shipment, earned interest and paid interest. Retailer can earn more profit by increasing efficiency of workers, credit periods and ordering smaller size of batch.

6.1. Limitations and Future Scope

1. Study hasn't considered rework process on imperfect quality units of items, one can be extended this study by incorporating this concept,
2. Study hasn't considered shortage of item at retailer's end, and due to shortage it also hasn't considered partially or fully backlog of demand. One can be extended this study by incorporating shortage along with backlog of demand,
3. The present study is developed for non-deteriorating items. One can be extended this study for deteriorating items,
4. Study hasn't considered competitive environment for retailer. One can be extended this model by incorporating competitive environment at retailers end.
5. Study focused on only retailer's ordering policy and pricing strategies, one can be extended it for economical production quantity and pricing strategies.

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