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# SOME BIANCHI TYPE III STRING COSMOLOGICAL MODELS FOR PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY WITH AN ALTERNATE APPROACH

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**Abstract**

Some Bianchi type III string cosmological models for perfect fluid distribution with an alternate approach are discussed, assuming a condition between energy density and string tension density. We have also assumed a relation between metric potentials to get deterministic solution. The physical and geometrical aspects of the models are also discussed.

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## 1. Introduction

The string theory plays an important role in the study of physical situation at the very early stage of formation of universe. The general relativistic treatment of strings was initiated by Letelier [4,5] and Stachel [8]. Letelier [4] has obtained the solution to Einstein's field equation for a cloud of strings with spherical, plane and cylindrical symmetry. Then in 1983 , he solved Einstein's field equation for a cloud of massive string and obtained cosmological model in Bianchi type I and kantowski-sachs space time. Tikekar and Patel [11], following the techniques used by Letelier and Stachel, obtained some exact Bianchi type III cosmological solution of massive strings in the presence of magnetic field.

An exact cosmological solution to the Einstein Maxwell equation for the case of Bianchi type III symmetry with 'stiff' matter and an electromagnetic field construction, behaviour of the solution near the singularity and later stages of expansion is investigated by Lorenz [6]. Wang [13-15] has also discussed LRS Bianchi type I and Bianchi type III cosmological models for the cloud of string with bulk viscosity.

Yadav, Rai, Pradhan [16] have found the integrability of cosmic string in Bianchi type III space time in presence of bulk viscous fluid by applying a new technique. Bianchi type III string cosmological model with bulk viscous fluid in general relativity is studied by Bali and Pradhan [2], they used condition $B = C^n$ between metric potential. Rani, Singh and Sharma [7] investigated Bianchi Type III magnetized massive string cosmological model for perfect fluid distribution in general relativity. Adhav, Dawande, Thakare and Raut [1] investigated Bianchi Type-III magnetized wet dark fluid cosmological model in general relativity.

Recently, Chirde, Hatkar and Katore[3] have discussed Bianchi type I cosmological model with perfect fluid and string in $f(T)$ theory of gravitation. Also Singh and Rani [9] has investigated Bianchi type-III cosmological models in Lyra's geometry in the Presence of Massive Scalar Field. Also Sahoo, Mishra, Sahoo and Pacif [10] has investigated Bianchi type string cosmological models in $f(R, T)$ gravity. Tyagi et al. [12] have discussed some Bianchi type string cosmological model for perfect fluid.

Motivated from above, in this paper we have investigated some Bianchi type III string cosmological models for perfect fluid in general relativity with different approach. To get determinate solution we have assumed an equation of state between energy density and string tension density. The physical and geometrical aspects of model are also discussed.

## 2. The Metric and Field Equations

We consider Bianchi type III metric of the form

$$ds^2 = -\mathrm{d}t^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 dz^2, \tag{2.1}$$

where $A, B$ and $C$ are functions of $t$ alone.

The Einstein's field equations for a cloud of massive string are,

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j. \tag{2.2}$$

The energy momentum tensor for a cloud of string is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j. \tag{2.3}$$

As given by Letelier [4,5] Stachel [8] and Banerjee [10].

Here $\rho$ is the energy density for a cloud of massive string with particle attach to them.

Thus we write,

$$\rho = \rho_p + \lambda. \tag{2.4}$$

$\rho_p$ Being the particle energy density, $\lambda$ the string tension density, $v^i$ the four velocities for the cloud of particles and $x^i$ the four vector which represents the strings direction which is essentially the direction of anisotropy. Thus we have,

$$v_i v^i = -x_i x^i = -1 \tag{2.5}$$

$$v_i x^i = 0. \tag{2.6}$$

In commoving coordinates we have,

$$v^i = (0, 0, 0, 1).$$

We choose $z$ direction as the direction of string,

$$x^i = \left(0, 0, \frac{1}{C}, 0\right).$$

The surviving component of the mixed Ricci tensor for the line element (2.1) as follows:

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = \rho. \tag{2.7}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = 0. \tag{2.8}$$

$$\frac{\dot{A}}{A} + \frac{\dot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 0. \tag{2.9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \lambda. \tag{2.10}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0. \tag{2.11}$$

Here the dot denotes the differentiation with respect to $t$. Introducing the new time variable dt = AdT above field equations (2.7), (2.8), (2.9), (2.10) and (2.11) reduce to,

$$\frac{A'B'}{AB} + \frac{A'C'}{AC} + \frac{B'C'}{BC} - 1 = \rho A^2. \tag{2.12}$$

$$\frac{B''}{B} + \frac{C''}{C} - \frac{A'B'}{AB} - \frac{A'C'}{AC} + \frac{B'C'}{BC} = 0. \tag{2.13}$$

$$\frac{A''}{A} + \frac{C''}{C} - \frac{A'^2}{A^2} = 0. \tag{2.14}$$

$$\frac{A''}{A} + \frac{B''}{B} - \frac{A'^2}{A^2} - 1 = \lambda A^2. \tag{2.15}$$

$$\frac{A'}{A} - \frac{B'}{B} = 0. \tag{2.16}$$

Using (2.12), (2.13), (2.14) and (2.15), we get

$$\frac{B''}{B} + \frac{C''}{C} + 2\frac{B'C'}{BC} - 1 = \rho A^2 \tag{2.17}$$

and

$$\frac{B''}{B} - \frac{C''}{C} - 1 = \lambda A^2. \tag{2.18}$$

For complete determination of set, we assume

$$\rho = k\lambda. \tag{2.19}$$

Taking relation between metric potential, equation (2.19) reduces to

$$\frac{2\,A''}{A} + \frac{2A'^2}{A^2} - 1 + k = 0. \tag{2.20}$$

**Case I.** $K = 1$ then we get $\rho = \lambda$

Using above condition in equation (2.20), we get

$$A = k_1 T + k_2$$
$$B = l\,(k_1 T + k_2)\,.$$

Using suitable transformation and above solution the metric (2.1) takes the form

$$C = m\,(k_1 T + k_2)\,.$$
$$ds^2 = -\tau^2 dT^2 + \tau^2 dX^2 + l^2\tau^2 dY^2 + m^2\tau^2 dZ^2. \tag{2.21}$$

## 3. Some physical and geometrical features

The rest energy density ($\rho$), the string tension density ($\lambda$), the scalar of expansion ($\theta$), shear ($\sigma$), spatial volume ($R$) and the deceleration parameter ($q$) for the model (2.21) are given by,

$$\theta = \frac{3k_1}{\tau^2},$$
$$\rho = \frac{1}{\tau^2}\left[\frac{3k_1}{\tau^2} - 1\right],$$
$$\lambda = \frac{1}{\tau^2}\left[\frac{3k_1}{\tau^2} - 1\right],$$
$$\sigma^2 = \frac{2k_1^2}{3\tau^4},$$
$$R = (lm)^{\frac{1}{3}}\tau,$$
$$q = \frac{1}{\tau}.$$

## 4. Conclusion for model (2.21)

The model starts expanding with a big bang at $\tau = 0$. The expansion in the model decreases slowly and it stops when $\tau \to \infty$. Also ' $\tau \to 0$ and $\rho \to \infty$ which shows there is a massive mass at $\tau = 0$.

The energy condition $\rho \geq 0$ leads to

$$\frac{1}{\tau^2}\left[\frac{3k_1^2}{\tau^2} - 1\right] \geq 0,$$
$$\frac{3k_1^2}{\tau^2} - 1 \geq 0,$$

$\frac{1}{\tau^2} \geq \frac{1}{3k_1^2}$ also model has point type singularity at $\tau = 0$ as $g_{11} \to 0, g_{22} \to 0, g_{33} \to$ and 0 as $\tau \to 0$.

Since $\text{Lim}_{T\to\theta} \frac{\sigma}{\theta} \neq 0$

Hence model does not approach isotropy for large values of $T$.

**Case II.** When $K = -1$ then we get $\rho = -\lambda$.

Using above condition in equation (2.20), we get

$$A = 2\sinh(T/\sqrt{3}),$$
$$B = 2l\sinh(T/\sqrt{3}),$$
$$C = 2m\sinh(T/\sqrt{3}).$$

Using suitable transformation and above solution the metric (2.1) takes the form

$$ds^2 = -4\sinh^2\frac{T}{\sqrt{3}}\mathrm{dT}^2 + 4\ell^2\sinh^2\frac{T}{\sqrt{3}}\,\mathrm{dX}^2 + 4\,\mathrm{m}^2\sin\mathrm{h}^2\frac{T}{\sqrt{3}}\,\mathrm{dY}^2 + 4\,\mathrm{m}^2\cosh^2\frac{T}{\sqrt{3}}\mathrm{dz} \tag{4.1}$$

## 5. Some physical and geometrical features

The rest energy density ($\rho$), the string tension density ($\lambda$), the scalar of expansion ($\theta$), shear ($\sigma$), spatial volume ($R$) and the deceleration parameter ($q$) for the model (4.1) are given by

$$\theta = \frac{\sqrt{3}}{2\sinh\frac{T}{\sqrt{3}}}\coth\frac{T}{\sqrt{3}}$$

$$\rho = \frac{1}{4\sin^2 h\frac{T}{\sqrt{3}}}\left[\cos^2 h\frac{T}{\sqrt{3}} - 1\right]$$

$$\lambda = -\frac{1}{4\sin^2 h\frac{T}{\sqrt{3}}}\left[\cos^2 h\frac{T}{\sqrt{3}} - 1\right]$$

$$\sigma^2 = \frac{1}{6}\coth^2\frac{1}{\sqrt{3}}T\,\mathrm{cosech}^2\frac{1}{\sqrt{3}}T,$$

$$R = 2(\mathrm{lm})^{\frac{1}{3}}\sin\frac{h}{\sqrt{3}}T,$$

$$q = \left(1 - \frac{1}{\sqrt{3}}\tan\frac{h}{\sqrt{3}}\right)\tan\frac{h}{\sqrt{3}}T.$$

## 6. Conclusion for model (4.1)

In case II the model starts expanding with a big bang at $T = 0$. The model decreases slowly.

Also $\tau \to 0$ and $\rho \to \infty$ which shows there is a massive mass at $\tau = 0$.

The energy condition $\rho \geq 0$ leads to, $\cosh^2\frac{T}{\sqrt{3}} - 1 \geq 0$.

Since $\mathrm{Lim}_{T\to\theta}\frac{\sigma}{\theta} \neq 0$, therefore, model does not approach isotropy for large values of $T$.

When $T = 3\sqrt{3}$ then $q > 0$, hence the model is decelerating.

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