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(Dedicated to Professor D. S. Hooda on His 80^{th} *Birth Anniversary Celebrations)*

TIME TRUNCATED ACCEPTANCE SAMPLING PLAN FOR ODDS EXPONENTIAL LOG-LOGISTIC DISTRIBUTION

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Abstract

In this paper, the multiple deferred state repetitive acceptance sampling plan *(MDSRASP)* is proposed and designed for assuring a 50^{th} percentile lifetime of the products under odds exponential log-logistic *(OELL)* distribution. The optimum parameters of the proposed plan are determined based on satisfying various blends of producer's risk and consumer's risk for definite quality levels in light of 50^{th} percentile. The aim is to minimize the average sample number *(ASN)*, though the constraints are connected with the lot acceptance probability at the acceptable and limiting quality levels. The efficiency of the proposed plan is compared with single sampling plan using *ASN*. Tables are formed to present the outcomes and comparison of the proposed plan with existing sampling plan is done in terms of *ASN*.

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Keywords and Phrases: Producer's risk, Consumer's risk, Multiple deferred state repetitive acceptance sampling plan, Odds exponential log-logistic distribution, Average sample number.

1. Introduction

Inspection based upon the acceptance sampling plans is one of the most regularly used sampling technique in quality control when the item quality relies upon its life time. It is utilized to observe the ideal plan parameters as the minimum sample size and its acceptance number to save the time and cost of testing the lots within the life test experiment. In such life tests the final decision based on the tested units is to accept or reject the lot. The single acceptance sampling plan designed for life time distribution is implemented by certain authors. In this plan, n items are arbitrarily chosen from a lot and put on a life testing experiment for pre-defined time t_o. The lot is examined for t_o units of time and if the number of failed units is larger than the acceptance number c then the lot is rejected. Otherwise, the lot is accepted if aggregate observed failures are c or fewer before time t_o. It was presented by Epstein [8], accepting a truncated life test in which life of the product follows exponential distribution. Tsai and Wu [15] introduced the problem of an acceptance sampling plan for a truncated life test when the lifetime follows the generalized Rayleigh distribution. Balakrishnan et al. [6] developed the single acceptance sampling plan from a truncated life test based on generalized Birnbaum−Saunders distribution. Al-Masri [1] discussed the single acceptance sampling plan in which lifetime of the products are assumed to follow the Inverse Gamma distribution. Al-Nasser et al. [2] introduced acceptance sampling plans for an Ishita distribution based on a truncated life test. Al-Omari [3] described acceptance sampling plans for Sushila distribution based on truncated life tests. Sherman [12] proposed the attribute repetitive group acceptance sampling plan for a normal distribution which gives an optimal sample size corresponding to the consumer's risk. Aslam et al. [4] introduced the repetitive acceptance sampling plan *(RASP)* for Burr type XII. Singh et al. [14] discussed *RASP* in which the significant comparison study is done between *RASP* and some other existing sampling plan for generalized Pareto distribution. Govindaraju and Subramani [9] proposed selection of multiple deferred state *(MDS)* sampling plans for given *AQL* and *LQL*. Balamurali et al. [7] proposed multiple deferred state repetitive groups sampling *(MDSRGS)* plan for Weibull and gamma distribution in which they gave the comparison of average sample numbers with the other existing sampling plan. Aslam et al. [5] discussed about the monitoring of the production process by an attribute control chart based on an *MDS* sampling methodology. In section 2, introduction about the odds exponential log-logistic distribution is given. The design methodology and execution are given in section 3 for the *MDSRASP*. In section 4, a comparative analysis of the proposed plan with other existing plans is done in terms of *ASN* obtained under odds exponential log-logistic distribution. Concluding remarks are given in Section 5.

2. Odds Exponential Log-Logistic *(OELL)* distribution

The new generalized family of distribution *OELL* distribution is introduced by Rosaiah et al. [11]. The cumulative distribution function *(cdf)* and probability density function *(pdf)* of *OELLD* are obtained by

$$F(t;\gamma,\delta,\theta) = 1 - e^{-\frac{1}{\delta}\left(\frac{t}{\gamma}\right)^{\theta}}, t > 0; \gamma, \delta, \theta > 0, \tag{2.1}$$

$$f(t;\gamma,\delta,\theta) = \frac{\theta}{\delta\gamma}\left(\frac{t}{\gamma}\right)^{\theta-1} e^{-\frac{1}{\delta}\left(\frac{t}{\gamma}\right)^{\theta}}, t > 0; \gamma, \delta, \theta > 0, \tag{2.2}$$

where δ, γ are the scale parameter and θ is the shape parameter. According to Kalyani et al. [10], *OELL* distribution plays an important role for life testing in statistics. The q^{th} percentile of *OELL* distribution is given by

$$t_q = \gamma\eta_q, \tag{2.3}$$

where

$$\eta_q = \{-\delta ln(1-q)\}^{\frac{1}{\theta}}.$$

3. Multiple Deferred State Repetitive Acceptance Sampling Plan *(MDSRASP)*

In this section, we consider the following stages describing the design of an *MDSRASP* based on time truncated life test:

Stage I. Take a random sample of n units from the current lot and put it on life test experiment for specified time t_o.

Stage II. Note down the failure units (denoted by d), before the specified experiment time t_o.

Stage III. If the failure units $d \leq c_1$(first acceptance number), then immediately accept that lot; if $d > c_2$ (second acceptance number), then stop that experiment and instantly reject that lot.

Stage IV. If $c_1 < d \leq c_2$, accept the existing lot if succeeding m lots will be accepted provided $d \leq c_1$. Else, repeat the process while taking the decision on the existing lot.

The n, c_1, c_2 and m are four design parameters of the proposed *MDSRASP* where n is the sample size. The implementation of any acceptance sampling plan can be shown by its Operating Characteristic *(OC)* function. The *OC* function of the *MDSRASP* by Singh et al. [13] under the Inverse Weibull distribution for a truncated life test is given by the following equation:

$$P_A(p) = \frac{P(d \leq c_1) + P(c_1 < d \leq c_2)(P(d \leq c_2))^m}{1 - P(c_1 < d \leq c_2)(1 - P(d \leq c_1)^m)}, \tag{3.1}$$

where $P(d \leq c_1) + P(c_1 < d \leq c_2)(P(d \leq c_2))^m$ is the lot acceptance probability based on the proposed plan and $P(c_1 < d \leq c_2)(1 - P(d \leq c_1)^m)$ the probability of the sampling plan will be repeated. The lot rejecting probability based on first sample is obtained from $1 - P(d \leq c_2)$. In binomial distribution,

$$P(d \leq c_1) = \sum_{d=0}^{c_1} {}^{n}C_d p^d (1-p)^{n-d}, \tag{3.2}$$

$$P(c_1 < d \leq c_2) = \sum_{d=c_1+1}^{c_2} {}^{n}C_d p^d (1-p)^{n-d}, \tag{3.3}$$

$p(t;\gamma,\delta,\theta)$ derived from (2.1) and (2.3) is the probability of failure of a unit before the termination time point t_0 such that

$$p = 1 - e^{-\frac{1}{\delta}\left(\frac{a\eta_q}{\frac{t_q}{t_q^0}}\right)^{\theta}}. \tag{3.4}$$

For accessibility the time termination t_0 express as a multiple the specified length t_q^0, like $t_0 = at_q^0$, a is a termination constant. The average sample number *(ASN)* of the *MDSRASP* is defined as follows:

$$ASN = \frac{n}{1 - P(c_1 < d \leq c_2)(1 - P(d \leq c_1)^m)}. \tag{3.5}$$

Thus, the design parameters for the *MDSRASP* with least sample size will be attained by solving the following optimization problem: Minimize $\frac{1}{2}\{ASN(p_1) + ASN(p_2)\}$

Subject to

$$P_A(p_1) \leq \beta,$$

$$P_A(p_2) \geq 1 - \alpha,$$

where

$$ASN(p_1) = \frac{n}{1-(\sum_{d=c_1+1}^{c_2} {}^nC_d(p_1)^d(1-p_1)^{n-d})((1-\sum_{d=0}^{c_1} {}^nC_d(p_1)^d(1-p_1)^{n-d})^m)},$$

$$ASN(p_2) = \frac{n}{1-(\sum_{d=c_1+1}^{c_2} {}^nC_d(p_2)^d(1-p_2)^{n-d})((1-\sum_{d=0}^{c_1} {}^nC_d(p_2)^d(1-p_2)^{n-d})^m)}$$

and $n > 1, m \geq 1, c_2 > c_1 \geq 0$.

Here p_1 is the probability of failure corresponding to percentile ratio $\frac{t_q}{t_q^0} = 1$ at consumer's risk (β), and p_2 is the probability failure corresponding to median ratio $\frac{t_q}{t_q^0} = 2, 4, 6, 8, 10$ at producer's risk (α). The optimal design parameters of the proposed plan under the *OELL* distribution with the value of shape parameters $\theta = 1$ at producer risk $\alpha = 0.05$, four different levels of $\beta = 0.25, 0.10, 0.05, 0.01$ and various values of time termination ratio $a = 0.5, 1.0$ at $\frac{t_q}{t_q^0} = 2, 4, 6, 8, 10$ are presented in Table 3.1. Table 3.1, reveals that when $\theta = 1$, $\frac{t_q}{t_q^0} = 2, 4, 6, 8, 10$, $\beta = 0.25, 0.10, 0.05, 0.01$ and when the value of a is increased from 0.5 to 1.0 the *ASN* decreases and also when the values $\frac{t_q}{t_q^0}$ increases from 2 to 10, then the *ASN* is decreases in all the cases. Table 3.2 presented the probabilities of acceptance at consumer's risk and producer's risk under *OELL* distribution when $\theta = 1$.

***Table* 3.1:** Optimal design parameters of the proposed MDSRASP under *OELL* distribution when $\theta = 1$

β	$\frac{t_q}{t_q^0}$	$a = 0.5$					$a = 1.0$				
		n	c_1	c_2	m	ASN	n	c_1	c_2	m	ASN
0.25	2	20	3	6	3	34.59	15	5	7	2	19.68
	4	10	1	2	2	11.82	4	0	2	1	7.47
	6	1	0	1	1	7.36	3	0	1	2	4.10
	8	6	0	1	1	7.27	3	0	1	2	4.01
	10	6	0	1	1	7.22	3	0	1	2	3.97
0.10	2	34	5	9	1	48.81	19	5	9	1	29.79
	4	14	1	3	2	18.71	5	0	2	1	8.24
	6	10	0	2	1	13.94	5	0	2	1	7.66
	8	8	0	1	1	9.28	4	0	1	2	4.96
	10	8	0	1	1	9.18	4	0	1	2	4.87
0.05	2	33	4	9	1	56.57	25	7	11	1	33.75
	4	16	1	3	1	19.63	9	1	3	1	11.25
	6	11	0	2	1	14.86	6	0	2	1	8.38
	8	11	0	2	1	14.27	6	0	2	1	8.0
	10	10	0	1	1	11.11	5	0	1	1	5.65
0.01	2	57	8	14	1	79.29	36	10	15	1	45.67
	4	22	1	4	1	28.86	12	1	4	1	16.38
	6	21	1	3	1	23.20	11	1	3	1	12.32
	8	15	0	2	1	18.23	8	0	2	1	9.90
	10	14	0	2	2	17.52	8	0	2	1	9.50

Table **3.2:** Probabilities of acceptance at consumer's risk and producer's risk

β	$\frac{t_q}{t_q^0}$	$a = 0.5$		$a = 1.0$	
		$P_A(p_1)$	$P_A(p_2)$	$P_A(p_1)$	$P_A(p_2)$
0.25	2	0.2488	0.9586	0.2411	0.9516
	4	0.2182	0.9535	0.2452	0.9812
	6	0.2249	0.9562	0.2074	0.9619
	8	0.2249	0.9748	0.2074	0.9787
	10	0.2249	0.9836	0.2074	0.9863
0.10	2	0.0920	0.9568	0.0853	0.9639
	4	0.0798	0.9712	0.0841	0.9541
	6	0.0668	0.9807	0.0841	0.9865
	8	0.0936	0.9542	0.0845	0.9578
	10	0.0936	0.9701	0.0845	0.9729
0.05	2	0.0496	0.9551	0.0419	0.9541
	4	0.0477	0.9557	0.0313	0.9504
	6	0.0415	0.9137	0.0307	0.9735
	8	0.0424	0.9887	0.0307	0.9888
	10	0.0341	0.9529	0.0426	0.9578
0.01	2	0.0099	0.9566	0.0084	0.9505
	4	0.0071	0.9559	0.0047	0.9547
	6	0.0080	0.9694	0.0072	0.9721
	8	0.0072	0.9698	0.0052	0.9695
	10	0.0094	0.9858	0.0052	0.9843

4. Comparative Study

In this section, the effective comparison of the proposed plan is discussed with the existing $RASP$ [10] and Single acceptance sampling plan ($SASP$) [10]. The comparison of $RASP$ and $SASP$ also presented by Singh et al. [14] in terms of ASN. Table 4.1, presented that ASN of the $MDSRASP$ for $\theta = 1$ is least when compared the ASN of $RASP$ and $SASP$ for almost all 50^{th} percentile ratios and levels of consumer's risk. At $\alpha = 0.05$, $\beta = 0.25$, $a = 0.5$ and $\frac{t_q}{t_q^0} = 2$. The ASN of the $MDSRASP$ is 34.59, but the ASN of $RASP$ is 41.24 and ASN of $SASP$ is 54. Thus, the proposed sampling plan will be more effective than $RASP$ and $SASP$ in reducing the inspection cost and time of the life test experiment under the $OELL$ distribution. In Table 3.2 the probabilities corresponding to producer's risk and consumer's risk are given for four different values of β and two values of time termination ratio. Five different values of median ratio $\frac{t_q}{t_q^0}$ are assumed and it is observed that probabilities corresponding to consumers risk decreases whereas the probability corresponding to producers risk increases with rise in the value of median ratio. Same configuration is visible for all values of consumer's risk.

Table **4.1:** Comparison of ASN of the proposed plan with [10] for *OELL* distribution when $\theta = 1$

β	$\frac{t_q}{t_q^0}$	$a = 0.5$			$a = 1.0$		
		$MDSRASP$	RASP [10]	SASP [10]	MDSRASP	RASP [10]	SASP [10]
0.25	2	34.59	41.24	54	19.68	25.50	31
	4	11.82	16.24	17	7.47	10.24	12
	6	7.36	13.18	13	4.10	4.80	7
	8	7.27	8.70	13	4.01	4.80	7
	10	7.22	8.70	9	3.97	4.80	5
0.10	2	48.81	58.51	81	29.79	36.35	48
	4	18.71	20.71	30	8.24	11.91	17
	6	13.94	15.26	21	7.66	8.37	12
	8	9.28	15.26	17	4.96	5.33	9
	10	9.18	10.09	17	4.87	5.33	9
0.05	2	56.57	66.92	-	33.75	39.97	-
	4	19.63	23.95	38	11.25	13.78	21
	6	14.86	15.89	24	8.38	8.93	13
	8	14.27	15.89	24	8.00	8.93	13
	10	11.11	15.89	20	5.65	8.93	11
0.01	2	79.29	84.43	-	45.67	48.04	-
	4	28.86	28.86	55	16.38	15.04	30
	6	23.20	21.67	36	12.32	12.03	19
	8	18.23	16.81	31	9.90	9.31	17
	10	17.52	16.81	31	9.50	9.31	17

5. Conclusion

In this paper, the $MDSRASP$ is discussed for time truncated life test under $OELL$ distribution in which the 50^{th} percentile is considered as a quality feature of the units. The tables for optimal design parameters and ASN of the proposed plan are presented. The comparison of proposed plan with $RASP$ and $SASP$ reveals that the proposed plan is more beneficial for the manufacturer as it reduces the time of experiment and cost as well because it gives smaller ASN.

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