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GENERATING DIOPHANTINE TRIPLES RELATING TO FIGURATE NUMBERS WITH THOUGHT-PROVOKING PROPERTY

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Abstract

In this article, the process for constructing integer triples consisting of some figurate numbers specifically Star number, Centered square number and Centered hexagonal number where the arithmetic mean of any pair of the elements remains a perfect square is scrutinized.

Also, Python program for conforming all the triples sustaining the desired constraint for numerical values is established.
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1. Introduction

Let s be an integer. A set of positive integers $\{l_1, l_2, \ldots, l_s\}$ is said to have the property $D(r)$ if $l_i l_j + r$ is a perfect square for all $1 \leq i < j \leq s$. This set is also named as a Diophantine $s-$ tuples. Deshpande [3], originated a family of Diophantine triples. Muriefah, Fadwa and Al-Rashed [4] discovered the extendability of the Diophantine triple $\{1, 5, c\}$. Pandichelvi and others [5, 6, 7, 8] found various triples involving special numbers satisfying different properties. For general assessment, one can refer to [1, 2, 9].

In this paper, the procedure for creating integer triples consisting of some figurate numbers explicitly Star number, Centered square number and Centered hexagonal number in which the arithmetic mean of any couple of the members stands for a square is studied. Also, such discovered triples filling the desired restriction for algebraic values are checked by Python program.

2. Process of Scrutiny

The next two sections explain the procedure for finding the triple that includes some interesting numbers such that the arithmetic mean of any two elements is a perfect square.

2.1. Triples with Centered Hexagonal number and Star number

Let $CH(n)$ and $S(n)$ be n^{th} Centered Hexagonal number and Star number respectively which are demarcated by

$$CH(n) = 3n^2 - 3n + 1.$$
$$S(n) = 6n^2 - 6n + 1.$$

Approve that

$$A(n) = CH(4n-1) = 48n^2 - 36n + 7, \tag{2.1}$$
$$B(n) = S(n+1) = 24n^2 - 12n + 1. \tag{2.2}$$

Supposing that the arithmetic means of $A(n)$ and $B(n)$ is a perfect square say α^2.
In Mathematical Statement, it is emblazoned as

$$\frac{A(n) + B(n)}{2} = \alpha^2. \tag{2.3}$$

Let $C(n)$ be the third non-zero integer such that the ensuing condition is valid for all n.

$$\frac{A(n)+C(n)}{2}=\beta^2, \tag{2.4}$$

$$\frac{B(n)+C(n)}{2}=\gamma^2. \tag{2.5}$$

Interpretation of (2.4) and (2.5) yields the succeeding combination of $A(n)$ and $B(n)$

$$\frac{A(n)-B(n)}{2}=\beta^2-\gamma^2. \tag{2.6}$$

To treasure the third element in an essential triple, let us adopt that

$$\beta=\Delta+1 \text{ and } \gamma=\Delta. \tag{2.7}$$

Employing (2.1) and (2.2) as well as the above adoptions for β and γ in (2.6), the equivalent value of Δ and hence γ is calculated by

$$\gamma=\Delta=6n^2-6n+1. \tag{2.8}$$

Implementing (2.2) and (2.8) in (2.5), the third element in the necessary triple is provoked by

$$C(n)=72n^4-144n^3+72n^2-12n+1. \tag{2.9}$$

Consequently,

$$\left\{48n^2-36n+7, 24n^2-12n+1, 72n^4-144n^3+72n^2-12n+1\right\}$$

is a triple in which the arithmetic mean of any two members is a square of an integer.

2.2. Triples with Centered square number and Star number

Describe the n^{th} Centered square number by

$$CS(n)=2n^2-2n+1.$$

To discover an alternative triple satisfying the similar condition as in the previous section, let us deliberate the already specified numbers as follows.

$$D(n)=S(2n)=24n^2-12n+1, \tag{2.10}$$

$$E(n)=CS(2n)=8n^2-4n+1. \tag{2.11}$$

Commence that

$$\frac{D(n)+E(n)}{2}=\delta^2. \tag{2.12}$$

Let $F(n)$ be an additional non-zero element together with the succeeding constraints

$$\frac{D(n)+F(n)}{2}=\mu^2, \tag{2.13}$$

$$\frac{E(n)+F(n)}{2}=\rho^2. \tag{2.14}$$

Making simple algebraic calculations in (2.12) and (2.13) offers that

$$\frac{D(n)-E(n)}{2}=\mu^2-\rho^2. \tag{2.15}$$

Further, introduce the resulting modifications to verdict the third non-zero element of the indispensable triple

$$\mu=\nabla+3 \text{ and } \rho=\nabla+1. \tag{2.16}$$

Implementing (2.10), (2.11) and these choices of ρ and μ in (2.15), the precise value of ∇ is determined by

$$\nabla=2n^2-n-2.$$

Accordingly,

$$\rho=\nabla+1=2n^2-n-1. \tag{2.17}$$

Retaining (2.11), (2.14) and (2.17), the desirable chance of $F(n)$ in the mandatory triple is triggered by

$$F(n)=8n^4-8n^3-14n^2+8n+1. \tag{2.18}$$

As a result, the arithmetic means of any two elements itemized in the successive triple

$$\left\{24n^2-12n+1, 8n^2-4n+1, 8n^4-8n^3-14n^2+8n+1\right\}$$

is the number with power raised by two.

3. The Python Program

Python Program for conforming the needed triples with numerical values such that arithmetic mean of any two of elements stays a perfect square is described below.

```
import math
Section=int(input(’ENTER THE VALUE OF SECTION’))
if Section == 1:
  n=int(input(’ENTER THE VALUE OF n = ’))
  A=48 * n * n-36 * n+7
  B=24 * n * n-12 * n+1
  C=72 * n * n * n * n-144 * n * n * n+72 * n * n-12 * n+1
  print(’A=’,A,’B=’,B,’C=’,C)
  X=(A+B)/2
  root=math.sqrt(X)
  if int(root+0.5) ** 2==X:
    print(’X=’,X," Arithmetic mean of A and B is a perfect square")
  else:
    print(’X=’,X,"Arithmetic mean of A and B is not a perfect square")
  Y=(B+C)/2
  root=math.sqrt(Y)
  if int(root+0.5) ** 2==Y:
    print(’Y=’,Y,"Arithmetic mean of B and C is a perfect square")
  else:
    print(’Y=’,Y,"Arithmetic mean of B and C is not a perfect square")
  Z=(C+A)/2
  root=math.sqrt(Z)
  if int(root+0.5) ** 2==Z:
    print(’Z=’,Z,"Arithmetic mean of C and A is a perfect square")
  else:
    print(’Z=’,Z,"Arithmetic mean of C and A is not a perfect square")
elif Section == 2:
  n=int(input(’ENTER THE VALUE OF n = ’))
  D=24 * n * n-12 * n+1
  E=8 * n * n-4 * n+1
  F=8 * n * n * n * n-8 * n * n * n-14 * n * n+8 * n+1
  print(’D=’,D,’E=’,E,’F=’,F)
  X=(D+E)/2
  root=math.sqrt(X)
  if int(root+0.5) ** 2==X:
    print(’X=’,X," Arithmetic mean of D and E is a perfect square")
  else:
  print(’X=’,X,"Arithmetic mean of D and E is not a perfect square")
  Y=(E+F)/2
  root=math.sqrt(Y)
  if int(root+0.5) ** 2==Y:
    print(’Y=’,Y,"Arithmetic mean of E and F is a perfect square")
  else:
  print(’Y=’,Y,"Arithmetic mean of E and F is not a perfect square")
  Z=(F+D)/2
  root=math.sqrt(Z)
  if int(root+0.5) ** 2==Z:
    print(’Z=’,Z,"Arithmetic mean of F and D is a perfect square")
  else:
print(’Z=’ Z Arithmetic mean of F and D is not a perfect square’’)
```

Output of Some Examples

```
ENTER THE VALUE OF SECTION 1
ENTER THE VALUE OF n = 1
A= 19 B= 13 C=-11
X= 16.0 Arithmetic mean of A and B is a perfect square
Y= 1.0 Arithmetic mean of B and C is a perfect square
Z= 4.0 Arithmetic mean of C and A is a perfect square

ENTER THE VALUE OF SECTION 1
ENTER THE VALUE OF n = 2
A= 127 B= 73 C= 265
X= 100.0 Arithmetic mean of A and B is a perfect square
Y= 169.0 Arithmetic mean of B and C is a perfect square
Z= 196.0 Arithmetic mean of C and A is a perfect square

ENTER THE VALUE OF SECTION 2
ENTER THE VALUE OF n = 1
D= 13 E= 5 F=-5
X= 9.0 Arithmetic mean of D and E is a perfect square
Y= 0.0 Arithmetic mean of E and F is a perfect square
Z= 4.0 Arithmetic mean of F and D is a perfect square

ENTER THE VALUE OF SECTION 2
ENTER THE VALUE OF n = 2
D= 73 E= 25 F= 25
X= 49.0 Arithmetic mean of D and E is a perfect square
Y= 25.0 Arithmetic mean of E and F is a perfect square
Z= 49.0 Arithmetic mean of F and D is a perfect square
```

Numerical Calculations of the triples in Section 2.1 and Section 2.2 using this Python Program are exemplified in Table 3.1 and Table 3.2 respectively.

***Table* 3.1**

N	$A(n)$	$B(n)$	$C(n)$	$\frac{A(n)+B(n)}{2}$	$\frac{B(n)+C(n)}{2}$	$\frac{A(n)+C(n)}{2}$
1	19	13	-11	$16 = 4^2$	$1 = 1^2$	$4 = 2^2$
2	127	73	265	$100 = 10^2$	$169 = 13^2$	$196 = 14^2$
3	331	181	2557	$256 = 16^2$	$1369 = 37^2$	$1444 = 38^2$
4	631	337	10321	$484 = 22^2$	$5329 = 73^2$	$5476 = 74^2$
5	1027	541	28741	$784 = 28^2$	$14641 = 121^2$	$14884 = 122^2$

***Table* 3.2**

n	$D(n)$	$E(n)$	$F(n)$	$\frac{D(n)+E(n)}{2}$	$\frac{E(n)+F(n)}{2}$	$\frac{F(n)+D(n)}{2}$
1	13	5	-5	$9 = 3^2$		$4 = 2^2$
2	73	25	25	$49 = 7^2$	$25 = 5^2$	$49 = 7^2$
3	181	61	331	$121 = 11^2$	$196 = 14^2$	$256 = 16^2$
4	337	113	1345	$225 = 15^2$	$729 = 27^2$	$841 = 29^2$
5	541	181	3691	$361 = 19^2$	$1936 = 44^2$	$2116 = 46^2$

4. Conclusion

In this paper, the process for generating integer triples comprising some figurate numbers precisely Star number, Centered square number and Centered hexagonal number such that the arithmetic means of any two of quantities leftovers a perfect square is inspected. In this way, one can pursuit triples and quadruples concerning some other numbers sustaining stimulating properties.

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