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(Dedicated to Professor D. S. Hooda on His 80^{th} Birth Anniversary Celebrations)

SINGLE FORMULA TO FIND THE AREA OF DIFFERENT FIGURES AND DIFFERENT WAYS TO SOLVE A SINGLE PROBLEM

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Abstract

Mathematics contains magic as a lot of formulas are available to solve a single problem, and a single formula can be used to solve a lot of problems. The logic involved in solving the problem always mesmerizes people by giving them different perspective, vision and outlook of the single problem. Different people may use different approach to solve it depending upon the knowledge and comfort level of the individual. The area of a single trapezium is calculated in 8 different ways, depending on the division of the figure, in this paper. Also a single formula has been used to calculate the area of different figures like square, rectangle, trapezium, parallelogram, and triangle.
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1. Introduction

Many types of figures are discussed in mathematics [3, 4, 5, 6, 7] including linear (a line between two vertices having zero degree angle), triangle (three lines between three vertices having three angles), quadrilateral (four lines between four vertices having four angles), pentagon ((five lines between five vertices having five angles), hexagon (six lines between six vertices having six angles) etc. Different types of calculations are done on these figures using different formulas to calculate perimeter, area, volume etc. Sometimes same formula may be used in different figures and sometimes many formulas may be used in the single figure, to do the same task. [9] has used trapezium to show the movement of a snake robot. In this paper, both of these things are discussed, same formula is used to calculate the area of a triangle, rectangle, square, parallelogram and trapezium; and different ways [1, 2, 8], are used to calculate the area of a single trapezium.

1.1. Trapezium or Trapezoid

It is a convex quadrilateral having exactly two opposite parallel sides out of the four where the parallel sides are known as bases and the non-parallel sides are known as legs. The word trapezium has originated from a Greek word trapeza [3, 5]. The meaning of this word in Greek is table. It is basically of three types:

1. Isosceles trapezium (having equal length legs)
2. Scalene trapezium (all four sides are unequal)
3. Right trapezium (at least two adjacent right angles)

The objective of this paper is to give different visions of calculations so that calculations becomes easier for people having low-confidence for mathematics and also open the door to view different figures in a different perspective.

2. Areas

Different formulas are used in mathematics to calculate area of a triangle, rectangle, square, parallelogram and a trapezium. A triangle has three sides and the formula used to calculate the area of a triangle is

$$Area\ of\ a\ triangle = \frac{1}{2} * base * height. \tag{2.1}$$

A rectangle has four sides with all the angles as 90 degrees [4] and opposite sides are equal, and the formula used to calculate the area of a rectangle is

$$Area\ of\ a\ rectangle = length * width. \tag{2.2}$$

A square has four sides with all the angles as 90 degrees [4] and all sides are equal, and the formula used to calculate the area of a square is

$$Area\ of\ a\ square = length * length. \tag{2.3}$$

A parallelogram has four sides with [4] opposite sides as parallel and equal, and the formula used to calculate the area of a parallelogram is

$$Area\ of\ a\ parallelogram = base * height. \tag{2.4}$$

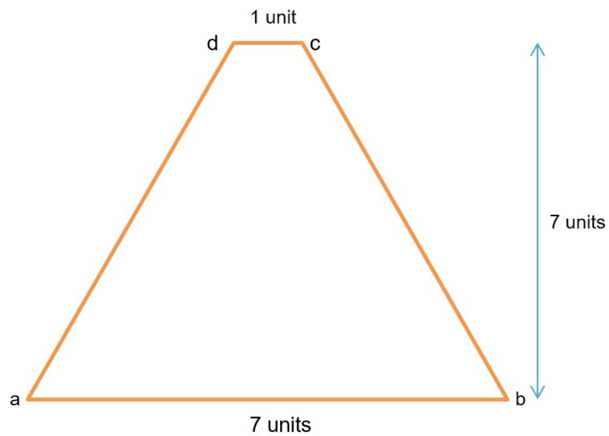

***Figure* 2.1:** The Trapezium

The formula given in (2.5), to calculate the area of a trapezium, can be used to calculate all the above areas (as explained below). So using a single formula, areas of different figures can be calculated. A child need not have to learn different formulas to calculate areas of different figures as a single formula will be able to solve his problem.

2.1. First Method: General Formula

Generally, in mathematics, the formula used to calculate the area of a trapezium in Figure 2.1 is given by (2.5).

$$Area\ of\ a\ trapezium = \frac{sum\ of\ parallel\ lines}{2} * height. \tag{2.5}$$

$$\begin{aligned} Area\ of\ trapezium\ abcd &= \frac{(ab + cd)}{2} * height \\ &= \frac{(7 + 1)}{2} * 7 \\ &= 28\ square\ units. \end{aligned}$$

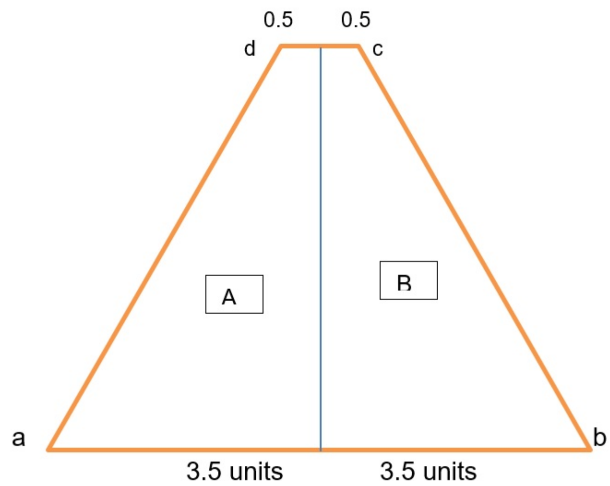

***Figure* 2.2**

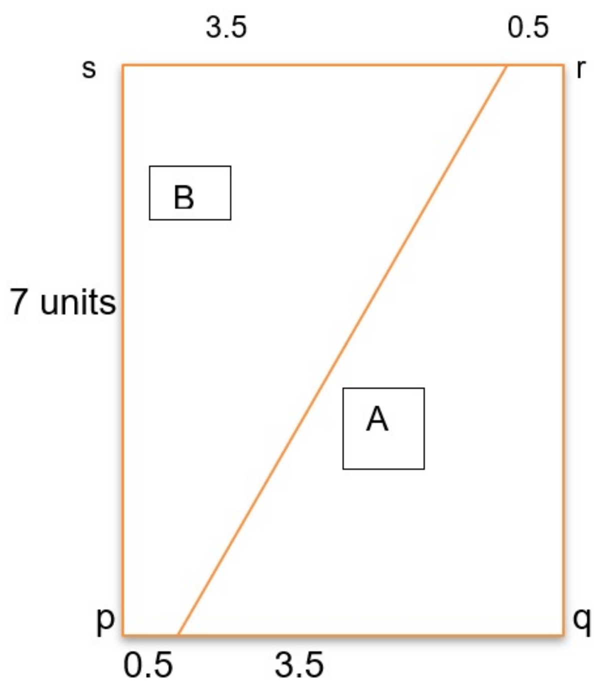

***Figure* 2.3**

2.2. Second Method: Vertical Rectangle

A rectangle is created using a perpendicular mid-division of the trapezium of Figure 2.1 as shown in Figure 2.2. The second half of the trapezium is added with the first half after rotating it at an angle of 180 degree horizontally. The width of this rectangle is more than its length, so it is called a vertical rectangle. Generally, the formula used to calculate the area of a rectangle in Figure 2.3 in mathematics is given by (2.2).

$$Area\ of\ a\ rectangle = length * width.$$

$$Area\ of\ a\ rectangle\ pqrs = pq * qr \qquad \text{[using (2.2)]}$$

$$= 4 * 7$$

$$= 28 \; square \; units.$$

The formula given by (2.5) can also be used to calculate the area of the rectangle as

$$Area \; of \; a \; rectangle \; pqrs = (pq + rs)/2 * height \quad \text{[using (2.5)]}$$

$$= (4 + 4)/2 * 7$$

$$= 28 \; square \; units. \quad \text{[same as above]}$$

Similarly area of a square can also be calculated using (2.5).

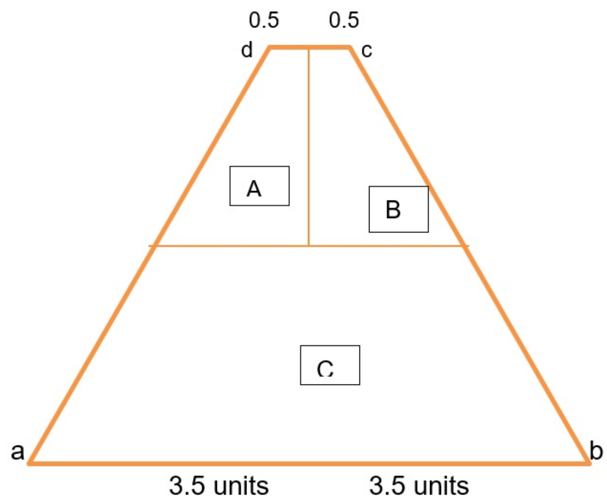

***Figure* 2.4**

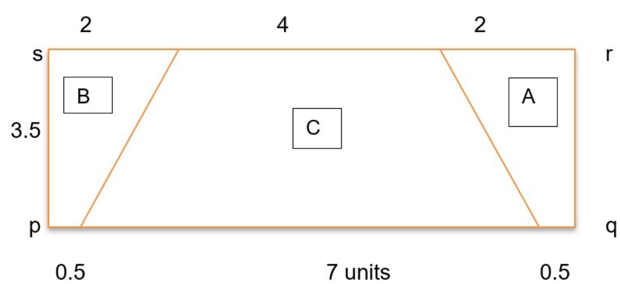

***Figure* 2.5**

2.3. Third Method: Horizontal Rectangle

A rectangle is created using a horizontal mid division of the trapezium of Figure 2.1 as shown in Figure 2.4. Top half of the trapezium is further divided vertically and the two top halves are added with the bottom half at left and right side respectively after rotating them at an angle of 180 degree horizontally. The width of this rectangle is less than its length, so it is called as horizontal rectangle. Generally, the formula used to calculate the area of a rectangle (2.5) in mathematics is given by (2.2).

$$Area \; of \; a \; rectangle = length * width. \quad \text{[using (2.2)]}$$

$$Area \; of \; rectangle \; pqrs = pq * qr$$

$$= 8 * 3.5$$

$$= 28 \; square \; units.$$

The formula given by (2.5) can also be used to calculate the area of the rectangle as

$$Area \; of \; a \; rectangle \; pqrs = (pq + rs)/2 * height \quad \text{([using (2.2)])}$$

$= (8+8)/2 * 3.5$

$= 28\ square\ units.$ [same as above]

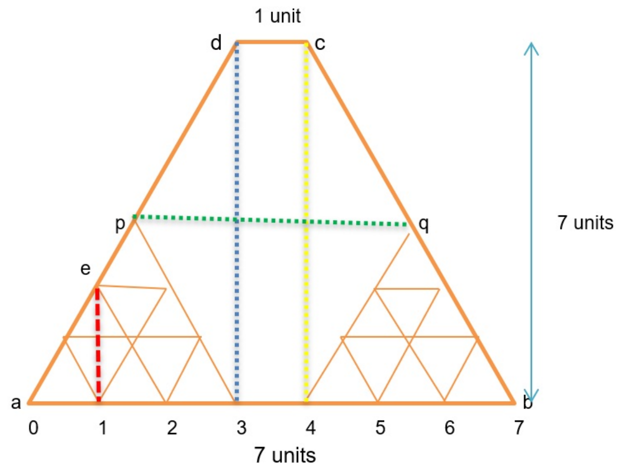

***Figure* 2.6**

2.4. Fourth Method: Triangle Diamond

The base line is divided into 7 equal parts and 9 isosceles triangles are created in the bottom left part and 9 isosceles triangles are created in the bottom right part as shown in Figure 2.6. These 18 triangles are congruent, so area of 18 triangles is given by multiplying the area of 1 triangle by 18. The middle part of Figure 2.6 is divided into two halves. Area of one half (trapezium) is calculated using (2.5) and multiplied by 2 to get the total middle area. Areas of 18 triangles and two trapeziums are added to get the total area of the Figure 2.6.
Height of the triangle is required to calculate its area.
Consider triangle *a3d*, where

$$a3 = 3\ units.$$
$$3d = 7\ units.$$
$$a0 = 0\ units.$$

It means that height is increasing by 7 units from *a*0 to *ad* when there is a movement of 3 units from *a*0 to *a*3. So,

$$\begin{aligned} When\ there\ is\ a\ movement\ of\ 3\ units,\ height\ increase &= 7\ units, \\ When\ there\ is\ a\ movement\ of\ 1\ unit,\ height\ increase &= (7/3) * 1\ units \\ &= 7/3\ units. \end{aligned} \tag{2.6}$$

Red line $1e$ shows the value of (2.6)

$$\begin{aligned} So\ the\ height\ of triangle &= 1/2 * (7/3) \\ &= 7/6\ units. \end{aligned} \tag{2.7}$$

Generally, the formula used to calculate the area of a triangle in mathematics is given in (2.1) as

$$Area\ of\ a\ triangle = \frac{1}{2} * base * height.$$
$$So,\ the\ area\ of\ 1\ triangle = 1/2 * 1 * 7/6$$
$$= 7/12\ square\ units.$$

Further (2.5) can also be used to calculate the area of one triangle as

$$\begin{aligned} Area\ of\ 1\ triangle &= (1+0)/2 * 7/6 \\ &= 7/12\ square\ units.\quad [same\ as\ above] \\ Area\ of\ 18\ triangles &= 18 * 7/12 \\ &= 21/2\ square\ units. \end{aligned} \tag{2.8}$$

Area of trapezium is calculated using (2.5), for that *pq* is required. When there is a movement from *ab* to *cd*, there is a loss of 6 units (from 7 to 1), a loss has to be found at a movement of 3 and 1/2 units to find value of *pq*.

$$\begin{aligned}
When\ there\ is\ a\ movement\ of\ 7\ units,\ loss &= 6\ units,\\
When\ there\ is\ a\ movement\ of\ 7/2\ units,\ loss &= (6/7) * 7/2\ units\\
&= 3\ units.\\
So\ pq &= 7\ units\ -\ 3\ units\\
&= 4\ units.\\
Area\ of\ trapezium\ pqcd &= (pq + cd)/2 * height\\
&= (4 + 1)/2 * 7/2\\
&= 35/4\ square\ units.\\
Area\ of\ p34qcd &= 2 * 35/4\\
&= 35/2\ square\ units. \qquad (2.9)
\end{aligned}$$

Total area of Figure 2.6 is given by adding (2.8) and (2.9) as

$$\begin{aligned}
Total\ area &= 21/2 + 35/2\ square\ units\\
&= 56/2\ square\ units\\
&= 28\ square\ units.
\end{aligned}$$

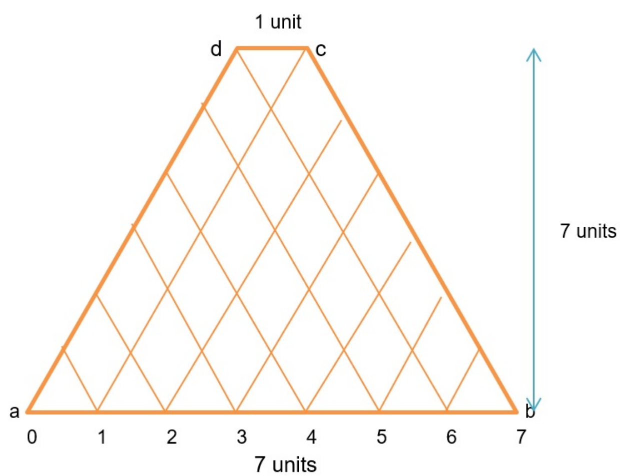

***Figure* 2.7**

2.5. Fifth Method: Diamonds

The base line is divided into 7 equal parts creating 20 diamonds and 8 isosceles triangles as shown in Figure 2.7. These 8 isosceles triangles will create 4 diamonds giving total diamonds as 24. Area of one of the diamond needs to be calculated which will give the total area after multiplying by 24. Area of the diamond can be calculated using (2.5) where horizontal diagonal is used as one parallel side and height is given by (2.6).

$$\begin{aligned}
Area\ of\ 1\ diamond &= (0 + 1)/2 * 7/3\\
&= 7/6\ square\ units,\\
Area\ of\ 24\ diamonds &= 24 * 7/6\\
&= 28\ square\ units.
\end{aligned}$$

2.6. Sixth Method: Sumeru

The Figure 2.1 is divided in 5 parts using 4 horizontal lines at heights 3/4 units, 7/12 units, 43/12 units, 7/12 units respectively leaving height of last trapezium *ijkl* as 3/2 units as shown in Figure 2.8. Area of each of the part is calculated using (2.5) as each one is a trapezium but height and sides of each trapezium need to be calculated. Height of each trapezium is already known as given by each partition above. Loss has to be calculated to find the sides.

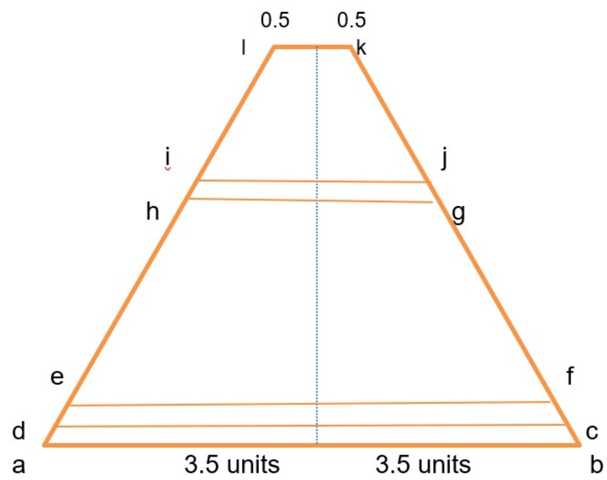

***Figure* 2.8**

$$
\begin{aligned}
ab &= 7\ units. && (2.10)\\
kl &= 1\ unit. && (2.11)\\
For\ a\ movement\ of\ 7\ units,\ the\ loss &= 6\ units, &&\\
For\ a\ movement\ of\ 3/4\ units,\ the\ loss &= 6/7 * 3/4\ units &&\\
&= 9/14\ units. &&\\
cd &= ab\ -\ 9/14 &&\\
&= 7\ -\ 9/14 &&\\
&= 89/14\ units. && (2.12)\\
For\ a\ movement\ of\ 7\ units,\ the\ loss &= 6\ units, &&\\
For\ a\ movement\ of\ 7/12\ units,\ the\ loss &= 6/7 * 7/12\ units &&\\
&= 1/2\ units. &&\\
ef &= cd\ -\ 1/2 &&\\
&= 89/14\ -\ 1/2 &&\\
&= 82/14\ units. && (2.13)\\
For\ a\ movement\ of\ 7\ units,\ the\ loss &= 6\ units, &&\\
For\ a\ movement\ of\ 43/12\ units,\ the\ loss &= 6/7 * 43/12\ units &&\\
&= 43/14\ units. &&\\
gh &= ef\ -\ 43/14 &&\\
&= 82/14\ -\ 43/14 &&\\
&= 39/14\ units. && (2.14)
\end{aligned}
$$

$$
\begin{aligned}
For\ a\ movement\ of\ 7\ units,\ the\ loss &= 6\ units, &&\\
For\ a\ movement\ of\ 7/12\ units,\ the\ loss &= 6/7 * 7/12\ units &&\\
&= 7/14\ units. &&\\
ij &= gh\ -\ 7/14 &&\\
&= 39/14\ -\ 7/14 &&\\
&= 32/14\ units. && (2.15)\\
Area\ of\ trapezium\ abcd &= (7 + 89/14)/2 * 3/4\quad [using\ (2.10), (2.12), (2.5)] &&\\
&= 187/14 * 3/8 &&
\end{aligned}
$$

$$= 561/112\ square\ units. \quad (2.16)$$

$$\begin{aligned}
Area\ of\ trapezium\ cdef &= (89/14 + 82/14)/2 * 7/12 \quad [using\ (2.12), 2.13, 2.5)] \\
&= 171/14 * 7/24 \\
&= 171/48\ square\ units. \quad (2.17) \\
Area\ of\ trapezium\ efgh &= (82/14 + 39/14)/2 * 43/12 \quad [using\ (2.13), 2.14, 2.5)] \\
&= 121/14 * 43/24 \\
&= 5203/336\ square\ units. \quad (2.18)
\end{aligned}$$

$$\begin{aligned}
Area\ of\ trapezium\ ghij &= (39/14 + 32/14)/2 * 7/12 \quad [using\ (2.14), 2.15, 2.5)] \\
&= 71/48\ square\ units. \quad (2.19) \\
Area\ of\ trapezium\ ijkl &= (32/14 + 1)/2 * 3/2 \quad [using\ (2.15), 2.11, 2.5)] \\
&= 69/28\ square\ units. \quad (2.20) \\
Total\ area\ of\ Figure\ 2.8 &= (2.16) + (2.17) + (2.18) + (2.19) + (2.20) \\
&= 561/112 + 171/48 + 5203/336 + 71/48 + 69/28 \\
&= 1683/336 + 1197/336 + 5203/336 + 497/336 + 828/336 \\
&= 9408/336 \\
&= 28\ square\ units.
\end{aligned}$$

2.7. Seventh Method: Tent (Trapezium, Parallelogram, Triangle)

The Figure 2.1 is divided into 7 parts using 6 lines at height 1 unit each as shown in Figure 2.9. Area of each of the part is calculated using (2.5) as first one is a parallelogram, next five are trapezium and last one is a triangle. The base and height of the parallelogram is 1 unit and 7 units respectively, and height of each trapezium is 1 unit. The inner sides of the trapezium have to be calculated.

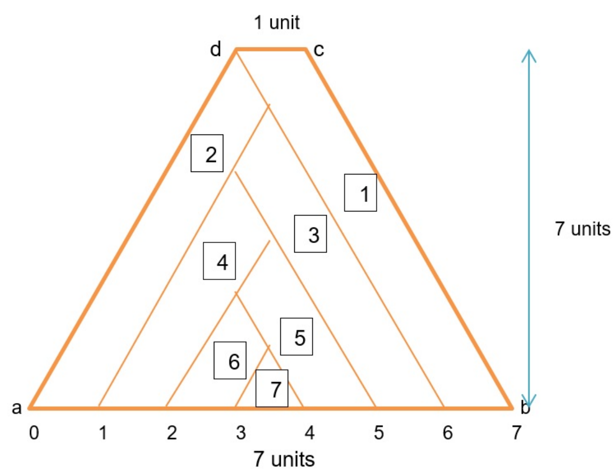

***Figure* 2.9**

$$\begin{aligned}
Height\ of\ part\ 1 &= 7\ units. \\
Base\ of\ part\ 1 &= 1\ unit. \\
Area\ of\ part\ 1\ (parallelogram) &= (1 + 1)/2 * 7 \\
&= 7\ square\ units. \quad (2.21)
\end{aligned}$$

The formula given in (2.4) can also be used to calculate the area of the parallelogram as

$$\begin{aligned}
Area\ of\ part\ 1\ (parallelogram) &= base * height \quad [using\ (2.4)] \\
&= 7 * 1
\end{aligned}$$

$$= 7 \ square\ units. \ [same\ as\ in\ (2.21)]$$

$$Side\ 1\ of\ part\ 2 = 7\ units,$$
$$Side\ 2\ of\ part\ 2 = 7 - loss \quad [using\ (2.7)]$$
$$= 7 - 7/6$$
$$= 35/6\ units,$$

$$Area\ of\ part\ 2\ (trapezium) = (7 + 35/6)/2 * 1$$
$$= 77/12\ square\ units. \tag{2.22}$$

$$Side\ 1\ of\ part\ 3 = 35/6\ units,$$
$$Side\ 2\ of\ part\ 3 = 35/6 - loss\ [using\ (2.7)]$$
$$= 35/6 - 7/6$$
$$= 28/6\ units,$$

$$Area\ of\ part\ 3\ (trapezium) = (35/6 + 28/6)/2 * 1$$
$$= 63/12\ square\ units. \tag{2.23}$$

$$Side\ 1\ of\ part\ 4 = 28/6\ units,$$
$$Side\ 2\ of\ part\ 4 = 28/6 - loss \quad [using\ (2.7)]$$
$$= 28/6 - 7/6$$
$$= 21/6\ units,$$

$$Area\ of\ part\ 4\ (trapezium) = (28/6 + 21/6)/2 * 1$$
$$= 49/12\ square\ units. \tag{2.24}$$

$$Side\ 1\ of\ part\ 5 = 21/6\ units,$$
$$Side\ 2\ of\ part\ 5 = 21/6 - loss\ [using\ (2.7)]$$
$$= 21/6 - 7/6$$
$$= 14/6\ units,$$

$$Area\ of\ part\ 5\ (trapezium) = (21/6 + 14/6)/2 * 1$$
$$= 35/12\ square\ units. \tag{2.25}$$

$$Side\ 1\ of\ part\ 6 = 14/6\ units,$$
$$Side\ 2\ of\ part\ 6 = 14/6 - loss\ [using\ (2.7)]$$
$$= 14/6 - 7/6$$
$$= 7/6\ units,$$

$$Area\ of\ part\ 6\ (trapezium) = (14/6 + 7/6)/2 * 1$$
$$= 21/12\ square\ units. \tag{2.26}$$

$$Side\ 1\ of\ part\ 7 = 1\ unit,$$
$$Side\ 2\ of\ part\ 7 = 0\ unit,$$
$$Height\ of\ part\ 7 = loss \quad [using\ (2.7)]$$
$$= 7/6\ units,$$

$$Area`\ of\ part\ 7\ (triangle) = (0 + 1)/2 * 7/6$$
$$= 7/12\ square\ units. \tag{2.27}$$

$$Total\ area\ of\ Figure\ 2.9 = (2.21) + (2.22) + (2.23) + (2.24) + (2.25) + (2.26) + (2.27)$$
$$= 7 + 77/12 + 63/12 + 49/12 + 35/12 + 21/12 + 7/12$$
$$= 84/12 + 77/12 + 63/12 + 49/12 + 35/12 + 21/12 + 7/12$$
$$= 336/12$$
$$= 28\ square\ units.$$

2.8. Eighth Method: Triangles

The base line is divided into 7 equal parts creating 27 triangles and 21 reverse triangles as shown in Figure 2.10. These 27 triangles are congruent with 21 reverse triangles giving total triangles as 48. Area of one of the triangle needs to be calculated which will give the total area after multiplying by 48. Area of the triangle can be calculated using (2.5) where the length of one parallel side is taken as zero units and height is given by (2.7).

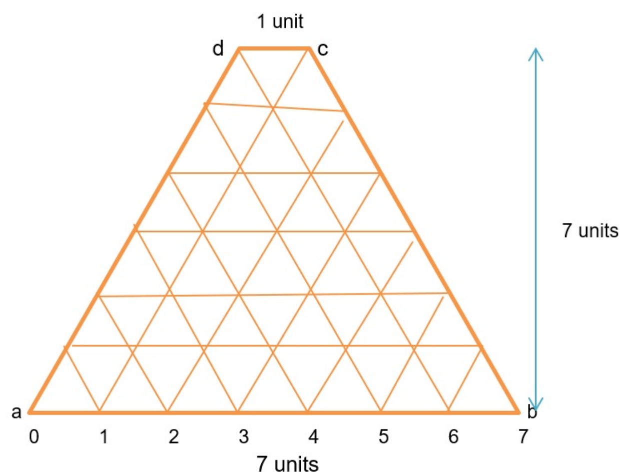

***Figure* 2.10**

$$\begin{aligned} Area\ of\ 1\ triangle &= (0+1)/2 * 7/6 \\ &= 7/12\ square\ units, \\ Area\ of\ 48\ triangles &= 48 * 7/12 \\ &= 28\ square\ units. \end{aligned}$$

3. Conclusions & Future Work

In this paper, the area of a trapezium shown by Figure 2.1 is calculated in 8 different ways. Minimum calculations are done in first, fifth and eighth methods. Different persons may find different methods easier. It opens the doors to visualize a problem in different ways and to explore different solutions.

The area of a triangle, rectangle, parallelogram and trapezium is also calculated using a single formula (2.5) which may help the persons finding it difficult to learn a different formula for a different figure. This formula may further be explored for other quadrilaterals.

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References

[1] Aryika Shri 105 Vishuddhmati Mata Ji, *Trilok Saar Teeka*, **1** (1974), 112-125.

[2] Acharya 108 Shri Nemichandra Siddhant Chakraworty , *TrilokSaar*, 7th Century.

[3] Michael Battista, *Cognition-Based Assessment & Teaching of Fractions*, 2012, ISBN 978-0-325-04345-6 / 0-325-04345-0 / SKU E04345.

[4] W. F. Burger and J. M. Shaughnessy, Characterizing the van Hiele Levels of Development in Geometry, *Journal for Research in Mathematics Education*, **17**(1) (1986), 3148, https://doi.org/10.2307/749317.

[5] M. De Villiers, The role and function of a hierarchical classification of quadrilaterals, *For the learning of mathematics*, **14**(1) (1994), 11-18.

[6] M. Stupel and D. Ben-Chaim, A fascinating application of Steiner's Theorem for Trapezium: Geometric constructions using straightedge alone, *Australian Senior Mathematics Journal*, **27**(2) (2013), 6-24.

[7] E. Trnkl, E. N. AkkaÅ and F. G. Alayl, Mathematics teachers perceptions of quadrilaterals and understanding the inclusion relations, *In Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education*, **2013** (2013), 705-714.
[8] A. N. Upadhyay and Hiralal Jain, *Tiloy Pannati Part 1*, Jain Sanskriti Sangrakshak Sangh, 1956.
[9] Ivan Virgala, Michal Kelemen, Erik Prada, Marek Sukop, TomÅ Kot, Zdenko Bobovsk, Martin Varga and Peter Ferenk, A snake robot for locomotion in a pipe using trapezium-like travelling wave, *Mechanism and Machine Theory*, **158** (2021), ISSN 0094-114X, https://doi.org/10.1016/j.mechmachtheory.2020.104221.