***Jñānābha*, Vol. 50(2) (2020), 236-243**
(Dedicated to Honor Dr. R. C. Singh Chandel on His 75th Birth Anniversary Celebrations)

STUDY OF Alfvén WAVES USING MAGNETO HYDRODYNAMIC EQUATIONS IN SOLAR ATMOSPHERE

By

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(*Received : July* 02, 2020 *; Revised: September* 26, 2020)
DOI: https://doi.org/10.58250/jnanabha.2020.50228

Abstract

This paper presents a three dimensional analysis of the properties of magneto hydrodynamic waves. A quantitative formulation of magneto hydrodynamic Alfvén waves is studied. On considering the motion in ordinary fluid, it is observed that the propagation of low frequency waves is only in the form of pressure waves. We have focused on such modes of propagation of incompressible fluid or plasma in magnetic field for which, the speed does not surpass the speed of sound. The value of conductivity is taken to be infinity and viscosity negligible. In such a case, the sum of the pressures of the field induced and the pressure of the plasma is independent of all the coordinates. The kinetic energy density of wave motion and the energy density of the field induced by the perturbation come out to be equal in both phase and amplitude. The study of the effect of large but finite conductivity for small "Joule losses" reveal that the waves remain periodic, but it is observed that the amplitude of the waves decreases continuously and exponentially at a slow pace.

2010 Mathematics Subject Classifications: 76W05.
Keywords and phrases: *MHD*, Alfvén waves, plasma pressure, magnetic field.

1 Introduction

Most of the physical phenomena that take place in our Sun can be described in terms of waves or oscillations. The behavior of plasma and magnetic fields of Sun is described by solar magneto hydrodynamics *(MHD)*. Solar *MHD* deals with the propagation of *MHD* waves in the solar atmosphere. It is the study of the magnetic properties of electrically conducting fluids. Some of such magneto fluids include plasmas, liquid metals and salt water or electrolytes.It is a macroscopic theory that is valid when the smallest length-scale, namely, the width of the diffusion region, is larger than the mean-free path for collisions([10]). The basic concept behind the theory of *MHD* is that the magnetic fields are able to induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. Hence, the conductive fluids can support magnetic fields. The main concern for a particular conducting fluid is the relative strength of the advecting motions in the fluid, compared to the diffusive effects caused by the electrical resistivity. Different solar activities are due to the interaction of the plasma of both solar interior and atmosphere with the magnetic field of the Sun as well as the convection and differential rotation of the Sun. These interactions are studied by taking into consideration the plasma physics and how it deals with the magnetic field. The interactions play a main role in the physical properties of the medium.

The main aim of the paper is to study and analyze three dimensionally the properties of magneto hydrodynamic waves. A quantitative formulation of magneto hydrodynamic Alfvén waves is studied. An attempt is made to study the Sun from magneto hydrodynamic point of view.

2 Basic *MHD* equations and Alfvén waves

MHD is a combination of the equations of hydrodynamics, electrodynamics and associated equations (see,[8,11,6,3]).The study of *MHD* combines the Eulers equation describing fluid dynamics with Maxwells equations of electromagnetism. These differential equations are resolved together, either in an analytical or numerical manner. From above equations we obtain a set of four equations, referred as ideal or basic *MHD* equations (see,[9]). These consist of two vector and two scalar partial differential equations. The equations are:

$$(2.1)\quad \frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{V}) = 0,$$

$$(2.2)\quad \frac{\partial(\rho\vec{(V)})}{\partial t} + \nabla.(\rho\vec{V}) = -\nabla P + \vec{J}\times\vec{B},$$

(2.3) $$\frac{\partial \rho}{\partial t} + \vec{V}.\nabla P + \gamma P \nabla.\vec{V} = 0,$$

(2.4) $$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = \nabla \times (\vec{V} \times \vec{B}) + \eta \nabla^2 \vec{B},$$

where ρ is the mass density, $\vec{V}$ the fluid velocity, $\vec{(}B)$ the magnetic field or magnetic flux density, γ is the adiabatic coefficient (generally taken as 5/3), $\vec{J}$ is the current density and η is the magnetic diffusivity or the electrical resistivity. Here P is the plasma pressure which is given as,

(2.5) $$P = 2(\rho/m_i)kT,$$

where m_i is the ionic mass, k is the Boltzmann constant and T is the temperature. The magnetic diffusivity or the electrical resistivity is usually taken as zero; hence equation (2.4) takes the form,

(2.6) $$\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{V} \times \vec{B}) = 0.$$

In the above equation, as $\eta = 0$, it indicates the presence of frozen field, i.e., the magnetic field remains tightly coupled to the fluid. Here, the relative strength of resistivity is calculated with the help of a dimensionless number known as the magnetic Reynolds number, R_{me} given as:

(2.7) $$R_{me} = \frac{VL}{\eta},$$

where V is the amplitude of the plasma velocity and L is the global length scale.

In solar magneto hydrodynamics, Reynolds number attains a large value as the length scales are large enough.

The magnetic pressure is given as $\frac{B^2}{2\mu_o}$. The ratio of plasma pressure P to magnetic pressure $\frac{B^2}{2\mu_o}$ is known as the plasma beta β,

(2.8) $$\beta = \frac{P}{B^2/2\mu_o},$$

where $\mu_o$is the magnetic permeability in vacuum.

In the above equation if $\beta = 1$, it implies that gas pressure is equal to the magnetic pressure. If $\beta >> 1$, the magnetic field is weak and it spins along the plasma fluid. On the contrary if $\beta << 1$, the magnetic field dominates and forces the plasma to move along with it. Both Reynolds number and plasma beta are dimensionless numbers.

Alfvén waves are the magneto hydrodynamic waves described by frequencies which are below the gyro frequency and their wavelength is much larger than the inter-particle distance1. The Lorentz force in magnetic field $\vec{B}$ is

(2.9) $$\vec{J} \times \vec{B} = \left(\frac{1}{4\pi}\right)(B.\vec{\nabla})\vec{B} - \nabla(B)^2/8\pi,$$

where $\vec{J}$ is the current density,$(B.\vec{\nabla})$ represents tension due to curvature of field lines and $\nabla(B)^2/8\pi$ pressure transverse to the field lines. This magnetic tension is responsible for the generation of transverse waves propagating along $\vec{B}$ in the x-direction. The equation of motion is given as,

(2.10) $$\left(\frac{\partial^2 \vec{B}}{\partial t^2}\right) = (V_A)^2 \left(\frac{\partial^2 \vec{B}}{\partial x^2}\right),$$

where, V_A is the Alfvén speed given by the equation,

(2.11) $$(V_A) = \frac{B}{\sqrt{4\pi\rho}} 2.2 \times \left(\frac{10^{11} B}{\sqrt{ne}}\right),$$

It represents the speed of propagation of all the magnetic disturbances and controls the growth rate of magnetic instabilities.

When a magnetic disturbance travels at a speed greater than the Alfvén speed, it leads to the production of an *MHD* shock wave. It results in an increase in magnetic field behind the waves. Shock waves are non-linear waves whose character is determined and defined by the conservation laws of mass, total energy and momentum. Such an *MHD* shock exists at the periphery of terrestrial magnetosphere. At this boundary, the solar wind travelling with a velocity of 400 km/s carries charged particles emanating from the Sun. This shock displays a stationary mode with the boundary of the magnetosphere. Alfvén waves provide an exact solution of the non-linear *MHD* equations.

3 Study of Alfvén waves using magneto hydrodynamic equations in solar atmosphere

The properties of Alfvén waves are found to vary in magnetic field. Many studies in the past have been done using magneto hydro dynamical equations (see [7, 15, 5]). Let us consider an incompressible fluid with infinite conductivity, lying in a homogeneous magnetic field (B_o). Any plasma or gas acts as an incompressible fluid till the speed of the mode does not surpass the speed of sound.Quantitatively, on considering the motion in the ordinary fluid, it is observed that the propagation of low frequency waves is only in the form of pressure waves.

Our focus lies on such modes only, and hence the viscosity is considered negligible and conductivity infinite ([1,2]). Initially, let the Alfvén wave propagate in a uniform magnetic field along the z-direction, from $z = -\infty$ to $+\infty$(independent of x and y directions), it satisfies the following differential equations

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{V}) = 0, \tag{3.1}$$

$$\frac{\partial \vec{B}}{\partial t} = Curl(\vec{V} \times \vec{B}) + \nu\nabla^2 B, \tag{3.2}$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V} = \frac{\vec{F}}{\rho} + \frac{1}{4\pi\rho} Curl\vec{B} \times \vec{B} - \frac{1}{\rho}\nabla P. \tag{3.3}$$

From equation (3.2), it follows that $\frac{\partial V_z}{\partial z}$, suggesting V_z to be a constant w.r.t to height. It indicates that the Alfvén wave propagates with constant speed along the z-direction (height). Let us consider a small variation in magnetic field ($\vec{B_o}$) as ($\vec{b}$). On resolving b and V in component form we get equations (3.4) and (3.5).

$$\frac{\partial b_x}{\partial t} = B_o\frac{\partial V_x}{\partial z}, \quad \frac{\partial b_y}{\partial t} = B_o\frac{\partial V_y}{\partial z}, \quad b_z = 0\ . \tag{3.4}$$

$$\frac{\partial V_x}{\partial t} = \frac{B_o}{4\pi\rho}\frac{\partial b_x}{\partial z}, \quad \frac{\partial V_y}{\partial t} = \frac{B_o}{4\pi\rho}\frac{\partial b_y}{\partial z}, \quad \frac{\partial V_z}{\partial t} = 0\ . \tag{3.5}$$

Using equations (3.4) and (3.5), we solve equations (3.2) and (3.3) and get the resultant equation as

$$-\frac{b_y}{4\pi\rho}\frac{\partial b_y}{\partial z} - \frac{b_x}{4\pi\rho}\frac{\partial b_x}{\partial z} - \frac{1}{\rho}\frac{\partial \rho}{\partial z} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{bx^2}{8\pi} + \frac{by^2}{8\pi} + P\right) = 0. \tag{3.6}$$

Since the magnetic field is perpendicular to the direction of propagation, the variation in the field is only along x and y direction and hence the z component of magnetic field is zero (equation 3.5).

From the equation (3.6) it follows that:-

$$\frac{b_x^2}{8\pi} + \frac{b_y^2}{8\pi} + P = Const. \tag{3.7}$$

Equation (3.7) depicts that the sum of the pressures of the field induced (b_x and b_y) and the plasma pressure (P) is independent of the coordinates. From the above equation, it is clear that the sum of plasma pressure and magnetic pressure in solar atmosphere is constant. The pressure is independent of coordinates (***Fig.* 3.1**) indicating that the total plasma pressure is uniformly distributed in solar atmosphere while the plasma pressure is dependent on the magnetic field or magnetic pressure in solar atmosphere. This three dimensional variation of plasma pressure P with magnetic field components (b_x and b_y) can be seen in (***Fig.* 3.2**) Arregui et al. (2004) ([2]), studied the potential magnetic arcades under the condition when the magnetic pressure is dominant over the plasma pressure.

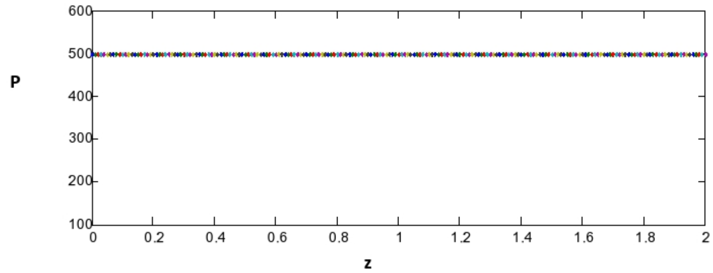

***Figure* 3.1:** Plot depicting that plasma pressure P is independent of coordinates

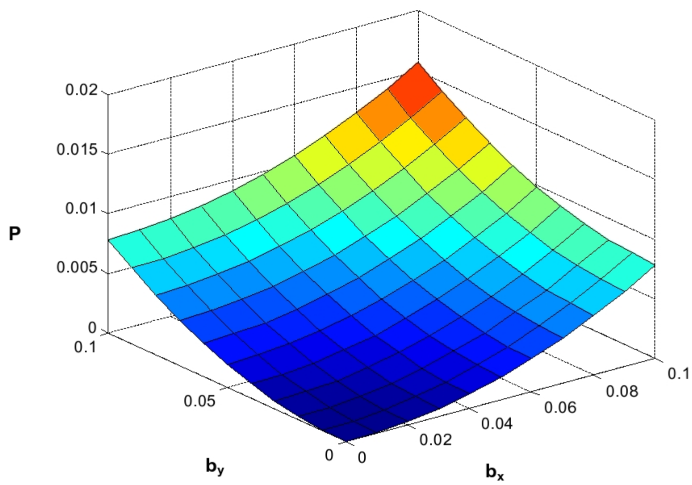

***Figure* 3.2:** Variation of plasma pressure P with magnetic field

The three dimensional variation of plasma pressure with components of magnetic field is parabolic in nature along *x*-axis and *y*-axis. A two dimensional plot of plasma pressure with different components of magnetic field is shown in ***Figs.* 3.3(a)** and **3.3(b)**.

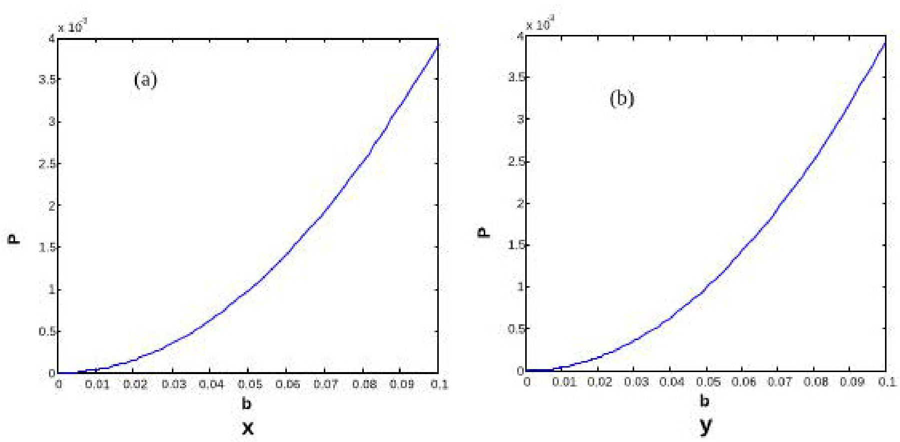

***Figure* 3.3:** Two dimensional plot between plasma pressure and different components of magnetic field (a) Plasma pressure and *x*-component of magnetic field (b) Plasma pressure and *y*-component of magnetic field.

Hence, if we consider a reference point in solar atmosphere (photosphere, chromosphere etc.)the plasma pressure will show an increase with increase of magnetic pressure along a particular direction. ***Fig.* 3.2** and ***Fig.* 3.3** indicate that the propagating Alfvén wavefollows a parabolic trajectory with magnetic field.

On differentiating equations (3.4) and (3.5) further w.r.t. t and z, we obtain two wave equations for b_y and V_y. In order to obtain solutions from equation (3.4), we have eliminated the terms $\frac{\partial^2 V_y}{\partial z \partial t}$ and $\frac{\partial^2 b_y}{\partial z \partial t}$. The resultant equations are

$$\frac{\partial^2 B_y}{\partial t^2} - \frac{B_o^2}{4\pi\rho}\frac{\partial^2 b_y}{\partial z^2}, \quad \frac{\partial^2 V_y}{\partial t^2} - \frac{B_o^2}{4\pi\rho}\frac{\partial^2 V_y}{\partial z^2} = 0. \tag{3.8}$$

Equations (3.8) satisfies functions of argument, $t \pm \frac{z}{V_A}$, where $V_A = \frac{B_o}{\sqrt{4\pi\rho}}$, Since the plasma particles are moving with a velocity V along the direction of magnetic field, hence they vibrate due to the magnetic pressure while the Alfvén wave propagating with velocity $V_A$varies with respect to height z. Therefore, these two velocities are independent of each other.

Due to a small perturbation in magnetic field, Alfvén wave and magnetic plane wave(generated due to the vibration of plasma particles) propagate in solar atmosphere. Hence the solutions of equation(3.8) are sinusoidal periodic waves given by the equation (3.9) and equation (3.10) as below:

$$b_y = a \sin\omega\left(t - \frac{z}{V_A}\right), \tag{3.9}$$

$$V_y = -\frac{a}{\sqrt{4\pi\rho}} \sin\omega\left(t - \frac{z}{V_A}\right). \tag{3.10}$$

Soler et al. [13], studied the three dimensional propagation of *MHD* waves in solar coronal arcades and concluded that the measure of trapped wave energy depends on the wavelength of perturbations in the perpendicular direction. From equation (3.9) and (3.10), amplitude of magnetic plane wave is equal to a. Therefore, plasma pressure = $\frac{1}{8\pi} by^2 = \frac{1}{8\pi} a^2$.

Also, amplitude of particle wave is equal to $\frac{a}{\sqrt{4\pi\rho}}$; hence, kinetic energy of plasma particle

$$= \frac{1}{2}\rho V_y^2 = \frac{1}{2}\rho\frac{a^2}{4\pi\rho} = \frac{1}{8\pi}a^2. \tag{3.11}$$

It indicates, $= \frac{1}{2}\rho V_y^2 = \frac{1}{8\pi} b_y^2$. Hence, we conclude that the kinetic energy density of wave motion and the energy density of the field induced by the perturbation come out to be equal in both phase and amplitude.

Further, on using the equations (3.9) and (3.10), the expressions for the terms P, J_x and E_x are as follow:

$$P = P_o - \frac{b_y^2}{8\pi}, \tag{3.12}$$

$$J_x = \frac{c}{4\pi}\nabla \times \vec{B} = -\frac{c}{4\pi}\frac{\partial B_y}{\partial z} = \frac{ac\omega}{4\pi V_A}\cos\omega\left(t - \frac{z}{V_A}\right), \tag{3.13}$$

$$E_X = -\frac{1}{c}(\vec{V} \times \vec{B}) = \frac{aV_A}{c} = \sin\omega\left(t - \frac{z}{V_A}\right). \tag{3.14}$$

Initially, if the disturbances emerge in a manner such that the plasma in a layer orthogonal to B_o starts moving on its own with the velocity equal to $2\vec{V}$, then it emanates two waves travelling with velocity $\vec{V}$and field $\vec{b}$. Hence, the total energy (comprising of kinetic and magnetic energy) is equal to the energy imparted to the initial perturbation.

On determining V_A, we conclude that $\frac{1}{2}\rho V_A^2 = \frac{1}{8\pi} B_o^2$.

If we compare it with the analogous equality for V and b, we find that $\frac{V}{V_A} = \frac{b}{b_o}$. In case V is sufficiently large, then $b >> B_o$. This clearly indicates that if the Alfvén wave is suitably large, the perturbation in the field surpasses the original field B_o.

We have dealt with three velocities i.e., fluid motion velocity V, Alfvén wave velocity V_A and the velocity of sound c. Alfvén waves are responsible for the perturbation of uniform magnetic field B_o to b and carry it to some good distance. Hence Alfvén waves need not to be strictly transverse. Thus, any state of motion within the acceptable limits of compressibility is found to travel along ($\vec{B_o}$) with velocity (V_A). Moreover, Alfvén waves satisfy the condition of frozen field([3]). This is made clear by the equation (3.15)

$$\frac{dy}{dz} = \frac{By}{Bz} = \frac{a}{B_o}\sin\omega\left(t - \frac{z}{V_A}\right). \tag{3.15}$$

The solution of the above equation is given as

(3.16) $y = y_o + \frac{a}{\omega\sqrt{\pi\rho}} \cos\omega\left(t - \frac{z}{V_A}\right).$

Each point on the line of force progress alongy-axis in direction parallel to velocity:-

(3.17) $\frac{dy}{dt} = -\frac{a}{4\pi\rho} \sin\omega\left(t - \frac{z}{V_A}\right),$

which comes out to be same as (V_y) in equation (3.10).

The waves are undamped so the amplitude of the wave a has been taken as constant with respect to time. Here, the Joule dissipation is ignored taking ($\lambda = \infty$), and viscosity is neglected. Due to this fact, phases of J_x and E_x exhibit a phase difference of $\pi/2$. Therefore, the work done is

(3.18) $\int_0^{2\pi/\omega} J_x E_x dt = 0.$

If we consider large and infinite conductivity and small Joule losses, the average energy density of the waves becomes

(3.19) $U_W = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \omega\left(\frac{1}{8\pi}b^2 + \frac{1}{2}\rho V^2\right)dt = \frac{a^2}{8\pi},$

and the average rate of decrease of the energy density is:-

(3.20) $\frac{dU_W}{dt} = -\frac{\omega}{2\pi} \int_0^{2\pi/\omega} \frac{J^2}{\lambda} dt = \frac{\omega}{2\pi} \frac{a^2}{\lambda} \frac{c^2\omega^2}{B_o^2} \frac{\rho}{4\pi} \int_0^{2\pi/\omega} \cos^2 \omega t \, dt.$

Here, the total time derivative is considered, suggesting that the point at which U_w is calculated, moves with the wave and not with the fluid. Moreover, U_W is the average over a wavelength.

The magnitude of the integral is equal to $\frac{\pi}{\omega}$. Solving with respect to $z(dt = V_A^{-1}dz)$ and replacing $\frac{a^2}{8\pi}$ with U_W, we obtain:

(3.21) $\frac{dlnU_W}{dz} = -2\alpha,$

$$\alpha = \frac{\sqrt{\pi}c^2\omega^2\rho^{8/2}}{B_o^8\lambda} = \frac{c^2\omega^2}{8\pi V_A^8\lambda}.$$

From this we get:

(3.22) $U_W = U_{W_o}e^{-2\alpha z},$

$$b_y = ae^{-2\alpha z} \sin\omega\left(t - \frac{z}{V_A}\right).$$

In the solar atmosphere, plasma pressure rises due to the loss of magnetic energy. As a result, damping in the amplitude of magnetic field takes place with respect to height. This damping is shown by equation (3.23).

(3.23) $b_y = ae^{-2\alpha z} \sin\omega\left(t - \frac{z}{V_A}\right).$

Magnetic field varies as a sinusoidal wave in 3-D with respect to height z and time t which is shown in ***Fig.* 3.4**. It is evident that the waves remain periodic but the amplitude of the waves decreases continuously at a slow pace.

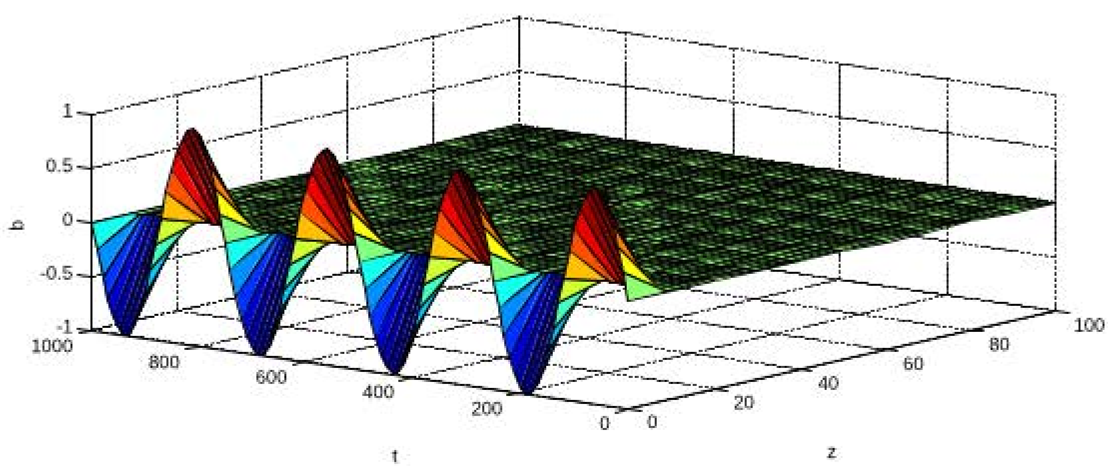

***Figure* 3.4:** Plot showing the variation of magnetic field with height z and time t

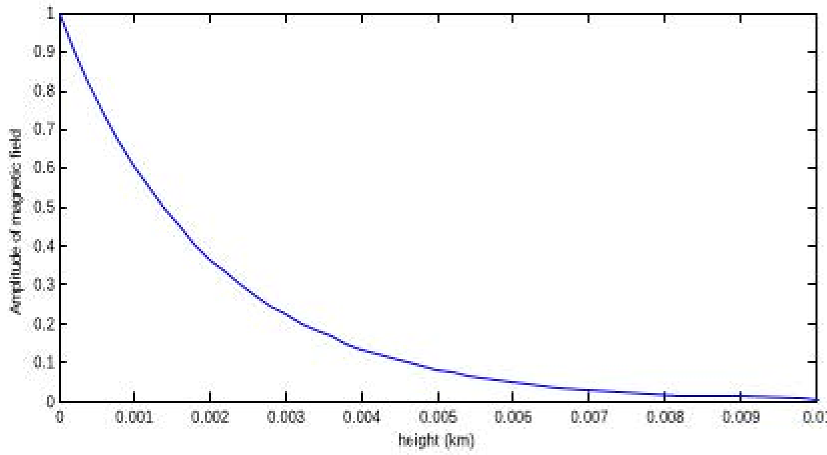

***Figure* 3.5:** Plot showing the variation of amplitude of magnetic field with height z.

Terradas et al. [14], have studied the properties of low-β *MHD* waves and successfully explained the effect of a physical dissipation on Alfvén modes leading to the damping of oscillations. Ryu et al. [12], studied the propagation and damping of Alfvén waves using a 2D *MHD* simulation code and observed effective damping. Carbonell et al. [4], observed time and spatial damping of linear non-adiabatic *MHD* waves in flowing partially ionized plasma.To analyze the variation of amplitude of magnetic field with height, we have plotted ***Fig.* 3.5** which reveals that the amplitude of magnetic field ($ae^{-\alpha z}$ decreases exponentially with height.

From damped amplitude of magnetic field $ae^{-\alpha z}$ we define damping factor α in terms of height.

If

(3.24) $\alpha = 1/z,$

then, $b_y = ae^{\frac{-1\times z}{z}} = a/e.$

Hence α factor is defined as the reciprocal of that height at which the amplitude of magnetic field becomes $\frac{1}{e}$ times its initial value and the time taken in this process is called the characteristic time t_o.

Now as $t_o = \frac{z}{A}$ and $z = \frac{1}{\alpha}$ [equation (3.24)]; therefore,

(3.25) $$t_o = \frac{1}{\alpha V_A}.$$

The characteristic time for change in b and V is given by

(3.26) $$t_o = \frac{1}{\alpha V_A} = \frac{8\pi\lambda V_A^2}{c^2\omega^2} = \frac{4\pi\lambda R_{me^2}}{c^2} = \frac{R_{me^2}}{\nu}.$$

It is evident that for oscillations of higher frequency, the characteristic time is smaller and thus damping is found to be larger.

4 Results and Discussions

The manifestation of different solar activities is due to the interaction of solar plasma with the magnetic field of the Sun as well as its convective motion and differential rotation. These interactions are studied by taking into consideration the dynamics of the plasma and its variation with the magnetic field. For analyzing this, aquantitative formulation of magneto hydrodynamic Alfvén waves has been attempted in this paper. On considering the motion in ordinary fluid, it is observed that the propagation of low frequency waves is only in the form of pressure waves. Here, viscosity is considered negligible, conductivity infinite and the fluid is assumed to be at rest initially. The sum of the pressures of the field induced (b_x b_y) and the plasma pressure (P) is found to be independent of the coordinates.

We have considered plane motion of the fluid wherein the plasma particles travel along ($\vec{B_o}$ with velocity V. It is independent of Alfvén wave velocity ($\vec{V_A}$) as the dependence of ($\vec{V_A}$) on z does not change the fluid velocity V. A small perturbation in magnetic field led to the propagation of Alfvén wave and magnetic plane wave. We conclude that the kinetic energy density of wave motion and the energy density of the field induced by the perturbation come out to be equal in both phase and amplitude. Hence, the total energy (comprising of kinetic and magnetic energy) is equal to the energy imparted to the initial perturbation.

In case V is sufficiently large, then $b >> B_o(B_o$ denotes uniform magnetic field and b is a small change in the value of magnetic field). It clearly indicates that, if the Alfvén wave is suitably large, the perturbation in the field surpasses the original field B_o.

The waves under consideration are undamped so the amplitude of wave "a" has been taken to be constant with respect to time. When we have ignored Joule dissipation and neglected viscosity, a phase difference of $\pi/2$ appeared between J_x and E_x. With theintroduction of large and finite conductivity with small Joule losses , the waves still remainedperiodic; however, the amplitude of the waves decreased exponentially with height. The damping factor α and the characteristic time t_o were obtained as $\alpha = \frac{1}{z}$ and $t_o = \frac{R_{me2}}{\nu}$. Thus, we have concluded that larger damping exists for oscillations of higher frequency.

Acknowledgement. Authors are very much grateful to the Editor and Reviewers for their fruitful suggestions to bring the paper in its present form.

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