NUMERICAL STUDY OF MHD BOUNDARY LAYER FLOW OF WILLIAMSON FLUID WITH VARIABLE FLUID PROPERTIES

By
Praveen Kumar
Department of Mathematics, University of Rajasthan, Jaipur-302004, India
Email: praveenydvuor79@gmail.com

R. S. Yadav
Department of Mathematics, University of Rajasthan, Jaipur-302004, India
Corresponding Author- R. S. Yadav
Email: rajendrauor@gmail.com

(Received: September 16, 2020; Revised: September 24, 2020)

DOI: https://doi.org/10.58250/jnanabha.2020.50227

Abstract

This numerical study investigates the MHD boundary layer flow and heat transfer of the Williamson fluid over a permeable nonlinearly stretching sheet. The partial differential equations corresponding to the momentum and energy are converted into ordinary differential equations with the help of similarity transformations. The numerical solution is computed by Runge-Kutta fourth order method with shooting technique. The effects of various parameters such as viscosity variation parameter, thermal conductivity variation parameter, magnetic field parameter, suction parameter, Williamson fluid parameter, radiation parameter and Eckert number on velocity and temperature profiles are discussed through graphs.

2010 Mathematics Subject Classifications: 76D05, 76D10, 76W05, 80A05.

Keywords and phrases: Williamson fluid, MHD, Viscous dissipation, Thermal radiation, Non-linearly permeable stretching sheet.

1 Introduction

In this paper we use the Williamson fluid flow model which was given by Williamson [21]. The objective of present paper is to analyze the MHD Williamson fluid flow over a nonlinearly permeable stretching sheet with thermal radiation, viscous dissipation and variable fluid properties.

2 Formulation of the Problem

We assume moving fluid as a Williamson fluid with a time constant \( \Gamma \). The fluid is flowing on nonlinearly permeable stretching sheet in the presence of radiation, viscous dissipation phenomena and magnetic field. Here \( x \)-axis is along the sheet whereas \( y \)-axis is taken perpendicular to stretching sheet. The stretching may create the velocity \( U_w = cx^m \) for the fluid, where \( c \) is a constant and \( m \) is an exponent. Here it is assumed that both thermal conductivity and the fluid viscosity are varying with temperature while remaining properties are constant.

The governing equations of present fluid flow can be introduced in the following form [13]:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{1}{\rho_\infty} \frac{\partial}{\partial y} \left( \mu(T) \frac{\partial u}{\partial y} + \mu(T) \frac{\Gamma}{\sqrt{2}} \left( \frac{\partial u}{\partial y} \right)^2 \right) - \frac{\sigma B_0^2}{\rho_\infty} u, \\
\frac{\partial u}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{1}{\rho_\infty c_p} \frac{\partial}{\partial y} \left( \kappa(T) \frac{\partial T}{\partial y} \right) + \frac{\mu(T)}{\rho_\infty c_p} \left( 1 + \frac{\Gamma}{\sqrt{2}} \frac{\partial u}{\partial y} \right) \left( \frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho_\infty c_p} \frac{\partial q_r}{\partial y},
\end{align*}
\]

with boundary conditions

\[
\begin{align*}
(2.4) \quad u &= cx^m, \quad v = -V_w, \quad T_w(x) = T_\infty + Ax \quad \text{at} \quad y = 0, \\
(2.5) \quad u \to 0, \quad T_w(x) \to T_\infty \quad \text{at} \quad y \to 0,
\end{align*}
\]

where \( u \) and \( v \) are the components of velocity in the \( x \) and \( y \) directions, respectively. Also, \( T \) is the temperature for the Williamson fluid, \( \rho_\infty \) refers to the fluid density at the ambient, \( q_r \) is the radiation heat flux and \( c_p \) is the specific heat at constant pressure. \( T_\infty \) is the constant ambient temperature, \( A \) and \( r \) are constants.

By taking Rosseland approximation [15], \( q_r \) can be described in the following form:

\[
(2.6) \quad q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y},
\]

where the constant \( \sigma^* \) is the Stefan-Boltzmann and \( k^* \) is the absorption coefficient. The term \( T^4 \) can be simplified by using Taylor expansion about the constant value \( T_\infty \) as \( T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \), after neglecting all higher order terms [16].

Using the similarity transformation

\[
(2.7) \quad \eta = \left( \frac{cx^m}{\nu_\infty} \right)^{1/2}, \quad \psi(x,y) = \left( \frac{cx^m+1}{\nu_\infty} \right)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},
\]

Figure 2.1: Sketch of problem.
where $v_\infty$ is the kinematic viscosity at the ambient and $\psi(x, y)$ is the stream function.

The important assumptions in this research is that the viscosity is altering exponentially with the temperature and the thermal conductivity is changing linearly with temperature for these equations [7]:

\begin{equation}
\mu = \mu_\infty e^{-\alpha \theta}, \quad \kappa = \kappa_\infty (1 + \epsilon \theta),
\end{equation}

where $\mu_\infty$ is the viscosity at the ambient, $\kappa_\infty$ represents the thermal conductivity at the ambient, $\alpha$ is the viscosity variation parameter and $\epsilon$ is the thermal conductivity variation parameter.

Equations (2.2)-(2.3) converted into

\begin{equation}
e^{-\alpha \theta} \left(1 + \delta f''\right) f''' - \alpha \theta f'' \left(1 + \delta \frac{\theta}{2}\right) - M f' + \frac{m+1}{2} f f'' - mf'^{2} = 0,
\end{equation}

\begin{equation}
\frac{1}{Pr} \left(\epsilon \theta^{2} + (1 + R + \epsilon \theta) \theta'\right) + \frac{m+1}{2} f' \theta' - rf' \theta + Ec \left(1 + \delta \frac{\theta}{2}\right) f'' e^{-\alpha \theta} = 0,
\end{equation}

and reduced boundary conditions are

\begin{equation}
f(0) = S, \quad f'(0) = 1, \quad \theta(0) = 1,
\end{equation}

\begin{equation}
f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \text{at} \quad \eta \rightarrow \infty,
\end{equation}

where $\delta = \left(\frac{\sqrt{\pi} \sqrt{\frac{\nu_{\infty}}{\nu_{\infty}}} \delta}{\nu_{\infty}}\right) \Gamma$ is the local Williamson fluid parameter, $M$ is magnetic field parameter, $S$ is the suction parameter, $R$ is the radiation parame ter, $Ec = \frac{u_{\infty}^{2}}{\epsilon \theta_{w}(r_{f_{1}} - r_{f_{3}})} = \frac{c_{2}^{\sqrt{\theta_{w} - \delta}}}{A_{c}}$ is the Eckert number, $Pr = \frac{\mu_{\infty} c_{p} m}{\kappa_{\infty}}$ is the Prandtl number. Here we take $r = 2m = \frac{3}{2}$, so, these parameters take the form $\delta = \left(\frac{\sqrt{\pi} \sqrt{\frac{\nu_{\infty}}{\nu_{\infty}}} \delta}{\nu_{\infty}}\right) \Gamma$ and $Ec = \frac{c_{2}}{A_{c}}$.

The local skin - friction coefficient $C_{f_{1}}$ and the local Nusselt number $Nu_{x}$ are given as

\begin{equation}
C_{f_{1}} = -2Re_{x}^{1/2} \left(1 + \frac{\theta}{2} f''(0)\right) f'''(0) e^{-\alpha \theta(0)},
\end{equation}

\begin{equation}
Nu_{x} = -Re_{x}^{1/2} (1 + R + \epsilon \theta(0)) \theta'(0),
\end{equation}

where $Re_{x} = \frac{\nu_{\infty} \eta}{\nu_{\infty}}$ is the local Reynolds number.

### 3 Method of Solution

Numerical shooting technique with Runge-Kutta fourth order method was adopted to solve the problem. Equations (2.10)-(2.11) subject to the boundary conditions (2.12)-(2.13) are transformed into the following system of first-order differential equations:

\begin{equation}
f'_{1} = f_{2},
\end{equation}

\begin{equation}
f'_{2} = f_{3},
\end{equation}

\begin{equation}
f'_{3} = \frac{e^{\alpha f_{4}}}{(1 + \delta f_{3})} \left(m f_{2}^{2} - \left(1 + \frac{m+1}{2}\right) f_{1} f_{3} + M f_{2}\right) + \frac{\alpha f_{3} f_{5}}{(1 + \delta f_{3})} \left(1 + \frac{\delta f_{3}}{2}\right),
\end{equation}

\begin{equation}
f'_{4} = f_{5},
\end{equation}

\begin{equation}
f'_{5} = \frac{1}{(1 + R + \epsilon f_{4})} \left(Pr \left(r f_{2} f_{4} - \left(1 + \frac{m+1}{2}\right) f_{1} f_{5} - Ec \left(1 + \frac{\delta f_{3}}{2}\right) f_{2}^{2} e^{-\alpha \theta}\right) - \epsilon f_{2}^{2}\right),
\end{equation}

where $f = f_{1}, \quad f' = f_{2}, \quad f'' = f_{3}, \quad \theta = f_{4} \quad \text{and} \quad \theta' = f_{5}$

and initial conditions are $f_{1}(0) = S, \quad f_{2}(0) = 1, \quad f_{3}(0) = 1$.

This system can not be solved with the infinite conditions which appear in eq.(2.13). So, these conditions are replaced by appropriate finite guessing values $f''(0)$ and $\theta'(0)$.
4 Results and Discussion
The influence of viscosity variation parameter \( \alpha \) on velocity and temperature profile is shown in Fig. 4.1 and Fig. 4.2. From Fig. 4.1 and Fig. 4.2, it is clear that velocity decreases with respect to increasing value of \( \alpha \) and temperature profile increases with respect to increasing value of \( \alpha \).

![Figure 4.1: Velocity profiles for different values of viscosity variation parameter \( \alpha \).](image1)

![Figure 4.2: Temperature profiles for different values of viscosity variation parameter \( \alpha \).](image2)

The variation of local Williamson fluid parameter \( \delta \) on velocity and temperature distributions is shown in Fig. 4.3 and Fig. 4.4. From Fig. 4.3, it is noticed that increase in \( \delta \) leads to decrease in velocity. From Fig. 4.4, it is observed that increase in \( \delta \) tends to increase in temperature.

![Figure 4.3: Velocity profiles for different values of Williamson fluid parameter \( \delta \).](image3)

![Figure 4.4: Temperature profiles for different values of Williamson fluid parameter \( \delta \).](image4)

The influence of magnetic field parameter \( M \) on both velocity and temperature profiles are shown in Fig. 4.5 and Fig. 4.6. From Fig. 4.5, it is observed that velocity of fluid decreases with increase in \( M \). The magnetic field causes a amount of resistance to its motion due to Lorentz force, which reduces the fluid velocity. From Fig. 4.6, it is clear that increase in \( M \) leads to increase in temperature. Fig. 4.7 and Fig. 4.8 depict the variation of suction parameter \( S \) on velocity and temperature profile. From Fig. 4.7 and Fig. 4.8, it is observed that velocity and temperature decrease with increase in \( S \).

![Figure 4.5: Velocity profiles for different values of magnetic field parameter \( M \).](image5)

![Figure 4.6: Temperature profiles for different values of magnetic field parameter \( M \).](image6)
Figure 4.7: Velocity profiles for different values of suction parameter $S$.

Figure 4.8: Temperature profiles for different values of suction parameter $S$.

The effect of thermal conductivity parameter $\epsilon$ and radiation parameter $R$ on temperature profile are given in Fig. 4.9 and Fig. 4.10 respectively, it is observed that increase in $\epsilon$ and $R$, leads to increase in temperature. Because increase in radiation parameter $R$ provides more heat to fluid that leads increase in temperature profile.

Figure 4.9: Temperature profiles for different values of thermal conductivity parameter $\epsilon$.

Figure 4.10: Temperature profiles for different values of radiation parameter $R$.

The effect of Eckert number $Ec$ on temperature profile is shown in Fig. 4.11. From Fig. 4.11, it is clear that increase in $Ec$ leads to increase in temperature profile because viscosity of fluid converts the energy from motion into the internal energy of fluid which results in increasing of temperature.

Figure 4.11: Temperature profiles for different values of Eckert number $Ec$. 

233
Table 4.1: Comparison of Nusselt number $Re_1^{1/2}Nu_x$ for various values of $Pr$ when $\delta = \alpha = \epsilon = R = Ec = M = S = r = 0$ and $m = 1$

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.91142</td>
<td>0.911358</td>
<td>0.911355</td>
</tr>
<tr>
<td>7.0</td>
<td>1.89046</td>
<td>1.895453</td>
<td>1.894460</td>
</tr>
<tr>
<td>20.0</td>
<td>3.35391</td>
<td>3.353902</td>
<td>3.353904</td>
</tr>
</tbody>
</table>

The comparison of present problem with previous research works done by Megahed[9] and Gorla and Sidawi[2] are shown in Table 4.1. The present problem can be transformed into the previous published work when $\delta = \alpha = \epsilon = R = Ec = M = S = r = 0$ and $m = 1$.

Table 4.2 presents the numerical values of both the local Nusselt number and the local skin friction parameter for various values of Eckert number $Ec$, magnetic field parameter $M$, suction parameter $S$, Williamson fluid parameter $\delta$, radiation parameter $R$, viscosity variation parameter $\alpha$ and thermal conductivity parameter $\epsilon$. Analysis of table shows that the Eckert number, the Williamson fluid parameter, the viscosity parameter lead to decreasing behaviour for both the local Nusselt number and the local skin friction coefficient and the suction parameter leads to an increasing behaviour for both. Magnetic field parameter leads a diminishing behaviour for the local Nusselt number and increasing behaviour for the the local skin friction.

Table 4.2: Values of $1/2Re_1^{1/2}C_{f_x}$ and $Re_1^{1/2}Nu_x$ for various values of $\alpha, \delta, \epsilon, M, S, R, Ec$ with $m = 1/3, r = 2/3, Pr = 2.0$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\epsilon$</th>
<th>$M$</th>
<th>$S$</th>
<th>$R$</th>
<th>Ec</th>
<th>$\frac{1}{2}Re_1^{1/2}C_{f_x}$</th>
<th>$Re_1^{1/2}Nu_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.8541</td>
<td>1.3875</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7168</td>
<td>1.3122</td>
</tr>
<tr>
<td>1.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.5862</td>
<td>1.2173</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7491</td>
<td>1.3298</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7168</td>
<td>1.3122</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6744</td>
<td>1.2789</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7201</td>
<td>1.2639</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7168</td>
<td>1.3122</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7120</td>
<td>1.2615</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6195</td>
<td>1.3881</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7168</td>
<td>1.3122</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7168</td>
<td>1.3122</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7120</td>
<td>1.2615</td>
</tr>
</tbody>
</table>

5 Conclusions

In this paper we have studied the boundary layer flow and heat transfer of Williamson fluid over a permeable nonlinearly stretching sheet in the presence of magnetic field. We draw the following conclusions from our study:

(1) Both the thermal radiation parameter and Eckert number enhance the temperature distribution, thicken thermal region.
(2) An increase in both the viscosity parameter and the Williamson parameter results in a rise in the temperature distribution.
(3) Thermal conductivity leads to an increasing behaviour for the local Nusselt number.
(4) Magnetic field parameter leads to decreasing behaviour for the local Nusselt number and increasing behaviour for the local skin-friction coefficient.
(5) Velocity and temperature profiles are decreasing as the value of suction parameter is increasing

Acknowledgements. The authors are grateful to the Editor and Reviewer for the suggestions which led to the paper in the present form.

References