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ANALYSIS OF A FINITE MARKOVIAN QUEUE WITH SERVER BREAKDOWN, SETUP TIME AND STATE DEPENDENT RATE UNDER *N*-POLICY STRATEGY

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Abstract

This investigation analyzes a breakdown service with setup time under *N*-policy. Only in working condition, the server may fail and dispatch immediately for the repair job which can be performed according to exponential distribution by a repairman. The server goes on vacation if the system does not have job to perform, i.e. the system is empty and later on switches on when the system accumulates *N* jobs. The arrivals at service station of the customers follow Poisson fashion with rate dependent on the server's status which may be idle, busy or brokendown state. By using generating function method, we derive the queue size distribution. The expressions for various performance characteristics including average queue length, probabilities of long-term fraction of time for which the server is idle, busy, broken down and in repair state. The optimal value of threshold parameter *N* which minimizes the total average cost is determined analytically. The parameters' sensitivity on different performance measures is examined by facilitating the numerical results.
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1 Introduction

Queueing problems with server breakdown have found increasing attention of research workers to portray realistic congestion scenarios of routine as well as commercial/industrial systems. Queueing systems subject to server breakdown have many applications in practical problems which ranges from computer networks to high speed distributed networks, call centers to cloud computing centers, Hospitals to malls, mobile computing to Internet of Things (IoT). Several authors have contributed towards server vacation model with threshold-based service policies. In *N*-policy queueing systems, the server starts the service as soon as the customers' count reached to a threshold level (say *N*). In recent past, Gaver [2], Grey et al. [3], Hersh and Brosh [4] and many more authors analyzed *N*- policy for a variant of queueing problems. Mitrans and Wright [10], Jain [5], Wang [12], Wang and Hung [13] analyzed a queueing model for *N*-parallel servers with server breakdowns and repairs.

Jain et al. [6] suggested (*N, F*) policy for the control of service and arrivals in case of an unreliable server queue in which repair is performed in multi-optional phase along with start-up. Kumar and Jain [8] proposed threshold '*N*' based policy for the performance prediction of (*M, m*) degraded machine repair problem with spare provisioning by including the concepts of the multi-heterogeneous servers, standby switching failure and multiple vacations. In recent years, Jiang et al. [6], Lan and Tang [9], Bu et al. [1] and Yen et al. [14] developed some queueing models for congestion problems under *N*-policy strategy. Sethi et al.. [11] developed Markovian model for unreliable single server queue by considering the impatience behavior of the customers and system operating under *N*-policy. They have also proposed the *ANFIS* soft computing approach and cost optimization.

The period for which the server is engaged in service is known as busy period. Due to unreliable server, there may be breakdown period during which the server cannot provide service to the customers. The time taken by the repairman before starting the repairing of the breakdown server is called setup time. A busy cycle is called the amount of idle time, busy period, breakdown period and restored period. In this paper, we investigate a queueing model subject to random breakdowns and repairs by incorporating the state dependent rate and setup time. The method of generating function is used to for establishing the probabilities for various states. The key performance measures are derived by using probabilities generating function (*PGF*). The optimum value of threshold parameter '*N*' is obtained to minimize the average total cost. The rest of the paper is arranged as follows. ***Section* 2** provides the model description and formulation of Chapman-Kolmogorov equations for steady state using appropriate transition rates. ***Section* 3** provides results for *PGF*. In Section 4 we calculate the value of $P_0(0)$ by using the normalizing conditions. In ***Section* 5,** we derive various performance measures in order to establish optimal N-policy. In ***Section*** **6**, cost analysis is suggested so as to evaluate optimal value of threshold parameter based on different parameter values. ***Section*** **7** discusses special cases while ***Section*** **8** discusses sensitivity analysis. Finally, ***Section*** **9** concludes the work done by highlighting the novel features.

2 Model Description

To formulate Markov's finite queueing model with server breakdown, setup time under N-policy and state dependent rates, we denote the server's status any time epoch 't' by

$$J(t) = \begin{cases} 0, & \text{server is idle} \\ 1, & \text{server is turned on and operating} \\ 2, & \text{server is turned on and breakdown} \\ 3, & \text{server is turned on and under repair} \end{cases}$$

The customers arrive independent to each other at the service stations in Poisson fashion. The server performs service in the first come first out *(FIFO)* order. The input rate of the customers is assumed to be state-dependent based on the status of the server. When in working condition, the server is subject to random failure and is returned to its previous state after repair. The repairman needs some time called set up time before beginning repair of the failed server. The repairman's setup process starts as soon as the server fails. The customers' service time as well as server's life, maintenance time and installation time are governed by exponential distributions.

The notations used for mathematical formulation of the model are described as below:

$\lambda, \lambda_1, \lambda_2, \lambda_3$	Customers' arrival rate when $J(t) = 0, 1, 2, 3$, respectively
$\alpha(\beta)$	Failure (repair) rate of the server
$\mu_0(\mu_1)$	Service rate of the server for $1 \le n \le N (N + 1 \le n \le K)$
$\nu_0(\nu_1)$	Setup rate of repairmen for $1 \le n \le N (N + 1 \le n \le K)$
$E[I](E[B])$	Expected idle (busy) time length.
$E[D](E[R])$	Expected breakdown (repair) time length.
$E[C]$	Expected cyclic time length.
$P_I(P_B)$	The long-term portion of the time for which the server is idle (busy)
P_D (P_R)	The long-term portion of the time for which server is in breakdown (under repair) state.
$P_{i,n}$	The steady state probability that the service will have n customers when the server is in state $J(t) = i, (i = 0, 1, 2, 3.)$

To formulate the mathematical model, the governing difference equations are constructed using appropriate transition rates. Chapman-Kolmogorov equations for formulating the mathematical model of concerned system for steady state are framed as follows:

(2.1) $0 = -\lambda p_{0,0} + \mu_0 p_{1,1}$,

(2.2) $0 = -\lambda p_{0,n} + \lambda p_{0,n-1}, \quad 1 \le n \le N - 1$,

(2.3) $0 = -(\lambda_1 + \alpha + \mu_0)P_{1,1} + \mu_0 P_{1,2} + \beta P_{3,1}$,

(2.4) $0 = -(\lambda_1 + \alpha + \mu_0)P_{1,n} + \lambda_1 P_{1,n-1} + \mu_0 P_{1,n+1} + \beta P_{3,n}, \quad 2 \le n \le N - 1$,

(2.5) $0 = -(\lambda_1 + \alpha + \mu_0)P_{1,N} + \lambda_1 P_{1,N-1} + \mu_1 P_{1,N+1} + \beta P_{3,N} + \lambda P_{0,N-1}$,

(2.6) $0 = -(\lambda_1 + \alpha + \mu_1)P_{1,n} + \lambda_1 P_{1,n-1} + \mu_1 P_{1,n+1} + \beta P_{3,n}, \quad N + 1 \le n \le K - 1$,

(2.7) $0 = -(\mu_1 + \alpha)P_{1,K} + \lambda_1 P_{1,K-1} + \beta P_{3,K}$,

(2.8) $0 = -(\lambda_2 + \nu_0)P_{2,1} + \alpha p_{1,1}$,

(2.9) $0 = -(\lambda_2 + \nu_0)P_{2,n} + \lambda_2 P_{2,n-1} + \alpha p_{1,n}, \quad 2 \le n \le N$,

(2.10) $0 = -(\lambda_2 + \nu_1)P_{2,n} + \lambda_2 P_{2,n-1} + \alpha p_{1,n}, \quad N + 1 \le n \le K - 1$,

(2.11) $0 = -\nu_1 P_{2,K} + \alpha P_{1,K} + \lambda_2 P_{2,K-1}$,

(2.12) $0 = -(\lambda_3 + \beta)P_{3,1} + \nu_0 p_{2,1}$,

(2.13) $0 = -(\lambda_3 + \beta)P_{3,n} + \nu_0 p_{2,n} + \lambda_3 P_{3,n-1}, \quad 2 \le n \le N$,

(2.14) $0 = -(\lambda_3 + \beta)P_{3,n} + \nu_1 p_{2,n} + \lambda_3 P_{3,n-1}, \quad N + 1 \le n \le K - 1$,

(2.15) $0 = -\beta P_{3,K} + \nu_1 p_{2,K} + \lambda_3 P_{3,K-1}$.

3 Queue Size Distribution

The generating functions are defined as:

(3.1) $G_0(z) = \sum_{n=0}^{N-1} P_{0,n} z^n$,

(3.2) $G_i(z) = \sum_{n=1}^{K} P_{i,n} z^n$, $\quad i = 1, 2, 3..$

Using (2.1)-(2.15) and (3.1)-(3.2), we get

(3.3) $G_0(z) = \dfrac{1-z^N}{1-z} P_{0,0}$,

(3.4) $$G_1(z) = \frac{\left[\begin{array}{l} [\lambda z(1-z^N)P_{0,0} + (z-1)a(z)]L_2(z)L_3(z) + \lambda_2\beta z(z-1)b(z) - \lambda_1(1-z) \\ z^{K+1}L_2(z)L_3(z)P_{1,K} - \beta z(1-z)\lambda_2\nu_1 z^{K+1}P_{2,K} + \lambda_3\beta z z^{K+1}(1-z)L_2(z)P_{3,K} \end{array}\right]}{[\alpha\beta\nu_1 z + L_1(z)L_2(z)L_3(z)]},$$

(3.5) $$G_2(z) = \frac{\left[\begin{array}{l} [\alpha\lambda z(z^N-1)P_{0,0} + \alpha(1-z)a(z)]L_3(z) - [\alpha\beta z - L_1(z)L_3(z)]b(z) + \alpha\lambda_1(1-z) \\ z^{K+1}L_3(z)P_{1,K} - \lambda_2 L_1(z)L_3(z)z^{K+1}(1-z)P_{2,K} - \alpha\beta z\lambda_3 z^{K+1}(1-z)P_{3,K} \end{array}\right]}{[\alpha\beta\nu_1 z + L_1(z)L_2(z)L_3(z)]},$$

(3.6) $$G_3(z) = \frac{\left[\begin{array}{l} \alpha\nu_1[\lambda z(1-z^N)P_{0,0} + (z-1)a(z)] - \lambda_2(z-1)L_1(z)b(z) - \alpha\nu_1\lambda_1(1-z)z^{K+1}P_{1,K} \\ +\lambda_2\nu_1(1-z)L_1(z)z^{K+1}P_{2,K} - \lambda_3 L_1(z)L_2(z)(1-z)z^{K+1}P_{3,K} \end{array}\right]}{[\alpha\beta\nu_1 z + L_1(z)L_2(z)L_3(z)]},$$

where,

$L_1(z) = \left[\lambda_1 z^2 - (\lambda_1 + \alpha + \mu_1)z + \mu_1\right]$,

$L_2(z) = (\lambda_2 z - \lambda_2 - \nu_1)$,

$L_3(z) = (\lambda_3 z - \lambda_3 - \beta)$,

$a(z) = (\mu_0 - \mu_1)\sum_{n=1}^{N} P_{1,n} z^n$,

$b(z) = (\nu_0 - \nu_1)\sum_{n=1}^{N} P_{2,n} z^n$.

4 Computation of $P_{0,0}$

Using normalizing condition stated as

$G(1) = \sum_{i=1}^{3} G_i(1) = 1$,

we get,

(4.1) $$P_{0,0} = \frac{\left[\begin{array}{l} \gamma + [\alpha\beta + \nu_1(\alpha+\beta)]a(1) + [\alpha(\lambda_2 - \lambda_3) + \beta(\lambda_2 - \lambda_1 + \mu_1)]b(1) \\ \qquad +[\alpha\beta + \nu_1(\alpha+\beta)]\sum_{i=1}^{3}\lambda_i P_{i,K} \end{array}\right]}{N[\gamma + \lambda\{\alpha\beta + \nu_1(\alpha+\beta)\}]}.$$

5 Performance Characteristics

Various performance characteristics are obtained explicitly as follows:
The long-term portion of the time for which the server is idle, busy, in breakdown and repair states respectively are

(5.1) $P_I = G_0(1) = NP_{0,0}$.

(5.2) $$P_B = G_1(1) = \frac{\beta\lambda\nu_1 - \beta\nu_1 a(1) - \beta(\lambda - \lambda_2)b(1) - \beta\nu_1\sum_{i=1}^{3}\lambda_i P_{i,K}}{[\gamma + \lambda\{\alpha\beta + \nu_1(\alpha+\beta)\}]}.$$

(5.3) $$P_D = G_2(1) = \frac{\alpha\lambda\beta - \alpha\beta a(1) - [\alpha(\lambda_3 - \lambda) + \beta(\lambda_1 - \lambda - \mu_1)]b(1) - \alpha\beta\sum_{i=1}^{3}\lambda_i P_{i,K}}{[\gamma + \lambda\{\alpha\beta + \nu_1(\alpha+\beta)\}]}.$$

(5.4) $$P_R = G_3(1) = \frac{\alpha\lambda\nu_1 - \alpha\nu_1 a(1) - \alpha(\lambda - \lambda_2)b(1) - \alpha\nu_1\sum_{i=1}^{3}\lambda_i P_{i,K}}{[\gamma + \lambda\{\alpha\beta + \nu_1(\alpha+\beta)\}]}.$$

We derive the formulae for the average number of customers in various states namely when idle [$E(N_0)$] busy [$E(N_1)$], broken down [$E(N_2)$], and under repair [$E(N_3)$] as follows:

(5.5) $E(N_0) = \dfrac{N(N-1)}{2} P_{0,0}$, (5.6) $\mathrm{E(N_i)} = \lim_{z\to 1} G_i'(z) = \lim_{z\to 1} \frac{\mathrm{N_i''(1)D'(1)-D''(1)N_i'(1)}}{2(\mathrm{D'(1)})^2}$, $\quad i = 1, 2, 3$

where

$$D'(1) = (\mu_1 - \lambda_1)\beta\nu_1 - \alpha\beta\lambda_2 - \alpha\nu_1\lambda_3 = \gamma,$$

$$D''(1) = 2[\lambda_1\beta\nu_1 - (\lambda_1 - \alpha - \mu_1)(\nu_1\lambda_3 + \beta\lambda_2) - \alpha\lambda_2\lambda_3] = 2\delta,$$

$$N_1'(1) = [-\beta\lambda\nu_1 NP_{0,0} + \beta\nu_1 a(1) + \beta\lambda_2 b(1) + \beta\nu_1 \textstyle\sum_{i=1}^{3} \lambda_i P_{i,K}],$$

$$\begin{aligned} N_1''(1) = {} & [-\beta\lambda\nu_1 N(N+1)P_{0,0} + 2\beta\nu_1 a'(1) + 2(\beta\lambda_2 + \lambda_3\nu_1)\{\lambda NP_{0,0} - a(1)\} \\ & +2\beta\lambda_2\{b(1) + b'(1)\} + 2\lambda_1\{(K+1)\beta\nu_1 - \nu_1\lambda_3 - \beta\lambda_2\}P_{1,K} \\ & +2\lambda_2\beta\nu_1(K+2)P_{2,K} + 2\lambda_3\beta\{(K+2)\nu_1 - \lambda_2\}P_{3,K}], \end{aligned}$$

$$\begin{aligned} N_2''(1) = {} & [\alpha\lambda NP_{0,0}\{2\lambda_3 - \beta(N+1)\} - 2\alpha\lambda_3 a(1) + 2\alpha\beta a'(1) - 2(\lambda_1\beta - \lambda_1\lambda_3 \\ & +\alpha\lambda_3 + \mu_1\lambda_3)b(1) + 2(\mu_1\beta - \lambda_1\beta - \alpha\lambda_3)b'(1) + 2\alpha\lambda_1\{(K+1)\beta - \lambda_3\}P_{1,K} \\ & +\lambda_2\{2(K+1)\alpha\beta - 2\alpha\lambda_3 - (\lambda_1 - \alpha - \mu_1)\}P_{2,K} + 2\alpha\beta\lambda_3(K+2)P_{3,K}], \end{aligned}$$

$$N_3'(1) = [-\alpha\lambda\nu_1 NP_{0,0} + \alpha\nu_1 a(1) + \alpha\lambda_2 b(1) + \alpha\nu_1 \textstyle\sum_{i=1}^{3} \lambda_i P_{i,K}],$$

$$\begin{aligned} N_3''(1) = {} & [-\alpha\lambda\nu_1 N(N+1)P_{0,0} + 2\alpha\nu_1 a'(1) - 2\lambda_2(\lambda_1 - \alpha - \mu_1)b(1) \\ & +2\alpha\lambda_2 b'(1) + 2\alpha\lambda_1\nu_1(K+1)P_{1,K} + \lambda_2\nu_1\{2(K+1)\alpha - 2(\lambda_1 - \alpha - \mu_1)\} \\ & P_{2,K} - 2\lambda_3\{-\alpha\nu_1(K+1) + \alpha\lambda_2 + \nu_1(\lambda_1 - \alpha - \mu_1)\}P_{3,K}]. \end{aligned}$$

Using (5.5)-(5.6) the average number of customers in the system obtained

(5.7) $E(N_S) = E(N_0) + \sum_{i=1}^{3} E(N_i)$.

Some More important Results are obtained as follows:

(5.8) $E[I] = \dfrac{N}{\lambda}$.

(5.9) $E[C] = E[I] + E[D] + E[B] + E[R]$,

(5.10) $P_I = \dfrac{\mathrm{E\,[I]}}{\mathrm{E[C]}}, P_D = \dfrac{\mathrm{E\,[D]}}{\mathrm{E[C]}}, P_B = \dfrac{\mathrm{E\,[B]}}{\mathrm{E[C]}}, P_R = \dfrac{\mathrm{E\,[R]}}{\mathrm{E[C]}}$.

Expected cyclic time length is now obtained as

(5.11) $$E[C] = \frac{E[I]}{P_I} = \frac{N[\gamma + \lambda\{\alpha\beta + \nu_1(\alpha+\beta)\}]}{\lambda\left[\begin{array}{c} \gamma + [\alpha\beta + \nu_1(\alpha+\beta)]a(1) + [\alpha(\lambda_2 - \lambda_3) + \beta(\lambda_2 - \lambda_1 + \mu_1)]b(1) \\ +[\alpha\beta + \nu_1(\alpha+\beta)] \sum_{i=1}^{3} \lambda_i P_{i,K} \end{array}\right]}.$$

Expected busy time length is

(5.12) $$\mathrm{E\,[B]} = \mathrm{E[C]P_B} = \frac{N\left[\beta\lambda\nu_1 - \beta\nu_1 a(1) - \beta(\lambda - \lambda_2)b(1) - \beta\nu_1 \sum_{i=1}^{3} \lambda_i P_{i,K}\right]}{\lambda\left[\begin{array}{c} \gamma + [\alpha\beta + \nu_1(\alpha+\beta)]a(1) + [\alpha(\lambda_2 - \lambda_3) + \beta(\lambda_2 - \lambda_1 + \mu_1)]b(1) \\ +[\alpha\beta + \nu_1(\alpha+\beta)] \sum_{i=1}^{3} \lambda_i P_{i,K} \end{array}\right]}.$$

Also, expected bkeakdown time length is given by

(5.13) $$\mathrm{E\,[D]} = \mathrm{E[C]P_D} = \frac{N\left[\alpha\lambda\beta - \alpha\beta a(1) - [\alpha(\lambda_3 - \lambda) + \beta(\lambda_1 - \lambda - \mu_1)]b(1) - \alpha\beta \sum_{i=1}^{3} \lambda_i P_{i,K}\right]}{\lambda\left[\begin{array}{c} \gamma + [\alpha\beta + \nu_1(\alpha+\beta)]a(1) + [\alpha(\lambda_2 - \lambda_3) + \beta(\lambda_2 - \lambda_1 + \mu_1)]b(1) \\ +[\alpha\beta + \nu_1(\alpha+\beta)] \sum_{i=1}^{3} \lambda_i P_{i,K} \end{array}\right]}.$$

Expected repair time length is obtained as

(5.14) $$\mathrm{E\,[R]} = \mathrm{E[C]P_R} = \frac{N\left[\alpha\lambda\nu_1 - \alpha\nu_1 a(1) - \alpha(\lambda - \lambda_2)b(1) - \alpha\nu_1 \sum_{i=1}^{3} \lambda_i P_{i,K}\right]}{\lambda\left[\begin{array}{c} \gamma + [\alpha\beta + \nu_1(\alpha+\beta)]a(1) + [\alpha(\lambda_2 - \lambda_3) + \beta(\lambda_2 - \lambda_1 + \mu_1)]b(1) \\ +[\alpha\beta + \nu_1(\alpha+\beta)] \sum_{i=1}^{3} \lambda_i P_{i,K} \end{array}\right]}.$$

6 Cost Analysis

In order to determine the optimal value of control parameter N, total expected cost per unit time is evaluated using different cost components.

The total cost per unit time is estimated

$$E\{TC(N)\} = (C_1 + C_2)\frac{1}{E[C]} + C_3P_B + C_4P_I + C_5P_D + C_6P_R + C_7E(N_s)$$

where

C_1 Start-up costs when the system is up and running,
C_2 Shutdown costs when the server is turned off,
C_3 Cost per unit time for server servicing,
C_4 Cost per unit time to shut down the server,
C_5 Price for a broken down server per unit time,
C_6 Price per system failure server repair time,
C_7 Holding costs per unit time per system customer.

We observe that P_B, P_I, P_D, P_R are not the function of decision variable N, hence the effective expected total cost per unit time is given by

$$(6.1)\quad E\{C(N)\} = (C_1 + C_2)\frac{1}{E[C]} + C_7E(N_s).$$

The aim is to minimize $E\{C(N)\}$ to determine the optimal value (say $N*$) of the N factor for decision. A heuristic approach based on a discrete distribution can be used to measure $N*$.

7 Special Cases

Now we deduce results for some special cases by setting appropriate parameter as follows:

I. When $\mu_0 = \mu_1, v_0 = v_1,\ \lambda = \lambda_1 = \lambda_2 = \lambda_3$ and $K \to \infty$ then our results matches with Wang [12].
II. If $\lambda = \lambda_1 = \lambda_2 = \lambda_3$ and service station is perfect i.e. $\beta = \alpha = 0$, then the results coincide with Wang and Huang [13].
III. If we take service rate constant, then the model is without setup time. It is noticed that $G_3(z) \to G_2(z), K \to \infty$ and $\lambda_3 \to \lambda_2$ so that our model matches with Jain [5].

8 Sensitivity Analysis

We conduct statistical experiments using MATLAB to show the effect of different parameters on the average queue size. ***Figures*** **8.1-8.6** depict the comparison of total queue size for homogenous arrival rate ($\lambda = \lambda_1 = \lambda_2 = \lambda_3 = 0.5$) with heterogeneous arrival rates ($\lambda_1 = 0.9\lambda, \lambda_2 = 0.5\lambda, \lambda_3 = 0.3\lambda$) and ($\lambda_1 = 0.7\lambda, \lambda_2 = 0.4\lambda, \lambda_3 = 0.3\lambda$) by varying other parameters. From ***Table*** **8.1** which displayed the effects of parameters $\lambda, (\alpha,\beta)$ and N on cost function, we note that the cost function decreases first and then increases by increasing the value of N. The minimum costs corresponding to different values of $\lambda, (\alpha,\beta)$ are shown by bold letters. ***Table*** **8.2** exhibits the effect of parameters $(\lambda, \mu, \alpha, \beta, v)$ on queue length in the idle state, busy state, breakdown state, under repair state and in the cycle for homogenous and heterogeneous cases.

Table **8.1:** The threeshold value of *N* and corresponding expected cost by varying different parameters

λ	*N*	*E{C(N)}*			
		α=.1	α=.1	α=.5	α=.5
		β=2	β=4	β=2	β=4
0.5	5	14.36	14.44	13.13	13.52
	6	13.74	13.81	12.71	13.04
	7	13.44	13.49	**12.55**	12.84
	8	**13.33**	**13.39**	12.56	**12.81**
	9	13.37	13.41	12.68	12.90
0.6	5	15.44	15.54	13.85	14.33
	6	14.78	14.87	13.45	13.85
	7	14.46	14.53	**13.30**	13.64
	8	**14.34**	**14.40**	13.31	**13.61**
	9	14.35	14.41	13.43	13.70
0.7	5	16.25	16.38	14.38	14.91
	6	15.61	15.71	14.04	14.48
	7	15.29	15.38	**13.94**	14.30
	8	**15.18**	**15.25**	13.98	**14.30**
	9	15.20	15.27	14.12	14.40
0.8	5	16.85	16.99	14.87	15.37
	6	16.27	16.38	14.64	15.03
	7	15.99	16.09	**14.61**	**14.92**
	8	**15.91**	**15.99**	14.71	14.96
	9	15.96	16.03	14.89	15.10

***Table* 8.2:** The expected number of customers for homogenous and heterogeneous arrival rates

	Homogenous arrival rate					Heterogeneous arrival rate				
λ	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*
0.1	20.51	2.44	0.81	0.61	24.37	20.62	2.30	0.77	0.57	24.25
0.2	13.07	3.83	1.28	0.96	19.13	13.38	3.36	1.12	0.84	18.69
0.3	9.35	5.35	1.78	1.34	17.82	9.95	4.29	1.43	1.07	16.75
0.4	6.77	7.39	2.46	1.85	18.46	7.78	5.24	1.75	1.31	16.08
0.5	4.56	10.95	3.65	2.74	21.91	6.19	6.29	2.10	1.57	16.16
μ	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*
1.0	4.56	10.95	3.65	2.74	21.91	6.19	6.29	2.10	1.57	16.16
1.5	6.87	4.85	1.62	1.21	14.55	7.46	3.76	1.25	0.94	13.41
2.0	7.77	3.22	1.07	0.80	12.87	8.08	2.71	0.90	0.68	12.37
2.5	8.27	2.42	0.81	0.60	12.10	8.46	2.12	0.71	0.53	11.82
3.0	8.58	1.94	0.65	0.49	11.66	8.71	1.75	0.58	0.44	11.48
α	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*
0.4	5.16	9.68	2.58	1.94	19.36	6.42	6.25	1.67	1.25	15.58
0.5	4.56	10.95	3.65	2.74	21.91	6.19	6.29	2.10	1.57	16.16
0.6	3.87	12.91	5.16	3.87	25.82	5.97	6.35	2.54	1.90	16.76
0.7	3.03	16.51	7.71	5.78	33.03	5.75	6.41	2.99	2.24	17.40
0.8	1.83	27.39	14.61	10.95	54.77	5.53	6.48	3.46	2.59	18.07
β	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*
2.0	4.56	10.95	3.65	2.74	21.91	6.19	6.29	2.10	1.57	16.16
3.0	5.00	10.00	3.33	1.67	20.00	6.32	6.32	2.11	1.05	15.81
4.0	5.20	9.61	3.20	1.20	19.22	6.39	6.34	2.11	0.79	15.64
5.0	5.32	9.39	3.13	0.94	18.79	6.44	6.35	2.12	0.64	15.54
6.0	5.40	9.26	3.09	0.77	18.52	6.46	6.36	2.12	0.53	15.47
ν	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*	*E[I]*	*E[B]*	*E[D]*	*E[R]*	*E[C]*
1.5	4.56	10.95	3.65	2.74	21.91	6.19	6.29	2.10	1.57	16.16
2.0	5.00	10.00	2.50	2.50	20.00	6.37	6.22	1.55	1.55	15.70
2.5	5.24	9.53	1.91	2.38	19.07	6.48	6.17	1.23	1.54	15.43
3.0	5.40	9.26	1.54	2.31	18.52	6.55	6.14	1.02	1.54	15.26
3.5	5.51	9.07	1.30	2.27	18.15	6.61	6.12	0.87	1.53	15.14
4.0	5.59	8.94	1.12	2.24	17.89	6.65	6.11	0.76	1.53	15.05

In ***Fig.*** **8.1** we display the effect of arrival rate λ on the average queue length $E(N_S)$ for fixed values $N = 5$, $\alpha = .1$, $\beta = 5$, $\nu = 2.5$, $\mu = 1$. It is observed that when λ increases, the average queue length increases, the effect is more prevalent for higher λ value. In ***Fig.*** **8.2**, we fix $N = 5$, $\lambda = .5$, $\nu = 2.5$, $\beta = 2$, $\alpha = .1$ and plot the graph for average queue length by varying μ from 0.7 to 2.8. From the graph we examine that $E(N_S)$ decreases with the increase in value of μ. As we expect, $E(N_S)$ decreases sharply for lower value of μ.

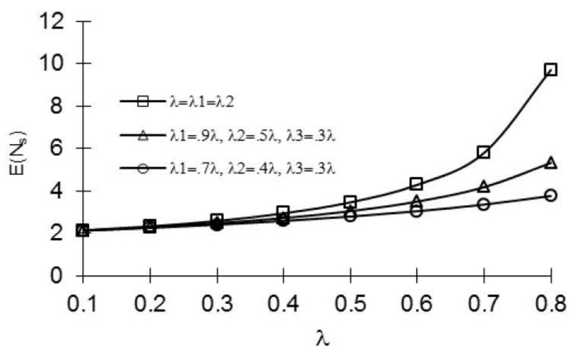

Figure **8.1:** Average Queue Length vs. λ
($N = 5, \mu = 1, \nu = 2.5, \beta = 2, \alpha = 1$)

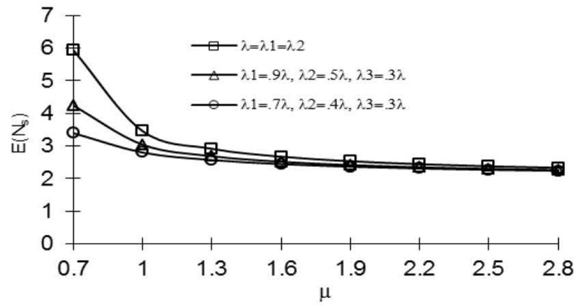

Figure **8.2:** Average Queue Length vs. μ
($N = 5, \lambda = .5, \nu = 2.5, \beta = 2, \alpha = 1$)

In ***Fig.*** **8.3** and **8.4**, we vary $\alpha(\beta)$ for fix values $N = 5$, $\lambda = .5$, $\nu = 2.5$, $\mu = .1$ and find that the average queue length increases (decreases) as the value of $\alpha(\beta)$ increases. By fixing $\lambda = 0.5$, $\alpha = 0.1$, $\beta = 5$, $\mu = 1$, we plot graphs for the average queue length $E(N_S)$ vs. ν and N in ***Figures*** **8.5** and **8.6**, respectively. It is seen that $E(N_S)$ decreases asymptotically with the increase in ν. Also, $E(N_S)$ increases linearly with N.

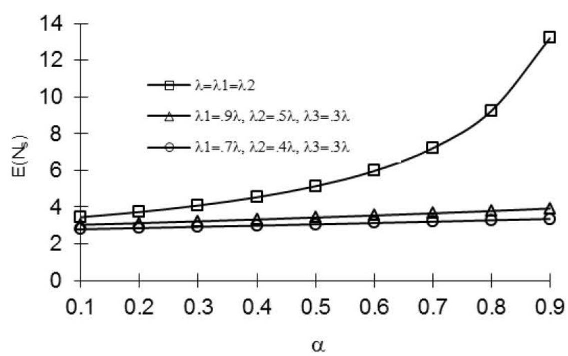

Figure **8.3:** Average Queue Length vs. α
($N = 5, \lambda = .5, \nu = 2.5, \beta = 2, \mu = 1$)

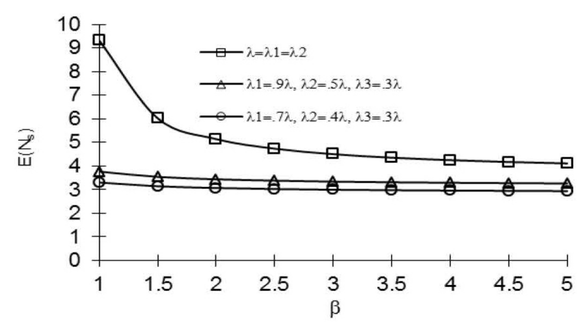

Figure **8.4:** Average Queue Length vs. β
($N = 5, \lambda = .5, \nu = 2.5, \alpha = .5, \mu = 1$)

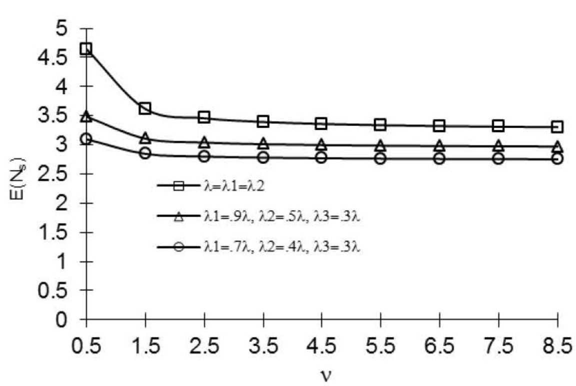

Figure **8.5:** Average Queue Length vs. ν
($N = 5, \lambda = .5, \beta = 2, \alpha = .1, \mu = 1$)

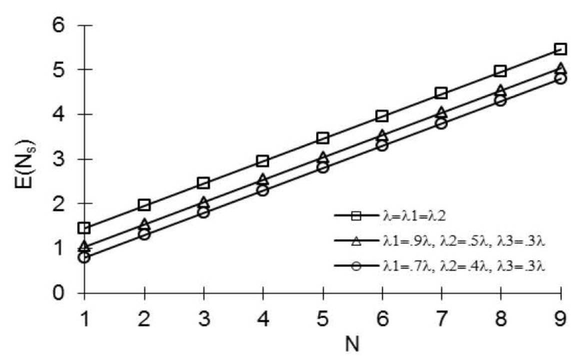

Figure **8.6:** Average Queue Length vs. N
($\lambda = 5, \beta = 2, \alpha = .1, \mu = 1, \nu = 2.5$)

We conclude numerical results based on the sensitivity analysis carried as follows:

- that minimum expected length increases for both homogenous and heterogeneous arrival rate.
- As we expect, average queue lengths, indicates increasing (decreasing) trend with (λ, μ) and (α, β), but for threshold parameter N, $E(N_S)$ linearly increase.
- By improving the rate of service station to some specific threshold level, the average queue length can be reduced to some extend only.
- For constant input rate system, all results seem to be better in terms of reduced average queue length.

9 Concluding Remarks

In this investigation, we have analyzed a finite queue operating under N-policy by including some realistic features namely setup time, server breakdown and state dependent rates. The incorporation of setup time, which can be realized in many real time systems makes our model more versatile than previous existing models in the literature. The explicit expressions provided for various staging measures in term of steady state probabilities can be easily computed as illustrated by taking numerical examples. The sensitivity analysis facilitated may be helpful in exploring the effect of different parameters on key indices describing the system dynamic.

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