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(Dedicated to Honor Dr. R. C. Singh Chandel on His 75^{th} Birth Anniversary Celebrations)

COMBINATORIAL OPTIMIZED TECHNIQUE FOR COMPUTATION OF TRADITIONAL COMBINATIONS

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Abstract

This paper presents a computing method and models for optimizing the combination defined in combinatorics. The optimized combination has been derived from the iterative computation of multiple geometric series and summability by specialized approach. The optimized combinatorial technique has applications in science, engineering and management. In this paper, several properties and consequences on the innovative optimized combination has been introduced that are useful for scientific researchers who are solving scientific problems and meeting today's challenges.

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1 Introduction

Combinatorics is a collection of various counting techniques or methods and models and has many applications in science, technology, and management. In the research paper, optimized combination of combinatorics is introduced that are useful for scientific researchers who are solving scientific problems and meeting today's challenges.

2 Optimized Combination

The growing complexity of mathematical modelling and its application demands the simplicity of numerical equations and combinatorial techniques for solving the scientific problems facing today. In view of this idea, the optimized combination of combinatorics is introduced that is

$$V_r^n = \frac{(r+1)(r+2)(r+3)\cdots(r+n-1)(r+n)}{n!}, (n, r \in N, n \geq 1, r \geq 0)$$

where $N = \{0, 1, 2, 3, 4, 5, ...\}$ is the set of natural numbers including the element 0.

This optimized combination is derived from the iterative computations [1 - 4] of multi-geometric series and summability as follows

(A) $\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1}\cdots\sum_{i_n=i_{n-1}}^{n-1} x^{i_n} = \sum_{i=0}^{n-1} V_i^p x^i, (p \in N,\ 1 \leq p \leq n-1),$

where V_i^p is a binomial coefficient and its mathematical expressions are given below:

$$V_i^p = \frac{(i+1)(i+2)(i+3)\ldots(i+p)}{p!}(1 \leq p \leq n-1).$$

$$V_{i-k}^p = \frac{(i-k+1)(i-k+2)(i-k+3)\ldots(i-k+p)}{p!}.$$

Let us prove the equation (A) using the multiple geometric series.

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1} x^{i_2} = \sum_{i_2=0}^{n-1} x^{i_2} + \sum_{i_2=1}^{n-1} x^{i_2} + \sum_{i_2=2}^{n-1} x^{i_2} + \cdots + \sum_{i_2=n-1}^{n} x^{i_2} = \sum_{i=0}^{n-1}\frac{(i+1)}{1!}x^i = \sum_{i=0}^{n-1} V_i^1 x^i,$$

where

$$\sum_{i_2=0}^{n-1} x^{i_2} + \sum_{i_2=1}^{n-1} x^{i_2} + \sum_{i_2=2}^{n-1} x^{i_2} + \cdots + \sum_{i_2=n-1}^{n} x^{i_2} = 1 + 2x + 3x^2 + \cdots + \frac{n}{1!}x^{n-1}.$$

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} = \sum_{i_2=0}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} + \sum_{i_2=1}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} + \sum_{i_2=2}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} + \cdots + \sum_{i_2=n-1}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3}$$

$$= (1 + 2x + 3x^2 + \cdots + nx^{n-1}) + (x + 2x^2 + 3x^3 \cdots + (n-1)x^{n-1}) + \cdots x^{n-1}$$

$$= 1 + 3x + 6x^2 + 10x^3 + \cdots + \frac{n(n+1)}{2!}x^{n-1} = \sum_{i=0}^{n-1}\frac{(i+1)(i+2)}{2!}x^i = \sum_{i=0}^{n-1} V_i^2 x^i,$$

where

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1} x^{i_3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + 21x^5 + \cdots + \frac{n(n+1)}{2!}x^{n-1}.$$

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1}\sum_{i_4=i_3}^{n-1} x^{i_4} = \sum_{i=0}^{n-1}\frac{(i+1)(i+2)(x+3)}{3!}x^i = \sum_{i=0}^{n-1} V_i^3 x^i$$

where

$$\sum_{i_1=0}^{n-1}\sum_{i_2=i_1}^{n-1}\sum_{i_3=i_2}^{n-1}\sum_{i_4=i_3}^{n-1} x^{i_4} = 1 + 4x + 10x^2 + 20x^3 + 35x^4 + \cdots + \frac{n(n+1)(n+2)}{3!}x^{n-1}.$$

If we continue like this, the binomial coefficient of the multisereis is $V_i^p(1 \leq p \leq n-1)$.

3 To convert combinations

3.1 To convert the combination ${}^nC_r$ into the optimized combination

$$ {}^nC_r = \frac{n!}{r!(n-r)!} = (V_0^r)(V_r^{n-1}) = V_r^{n-r} \text{ where } V_0^r = 1. $$

Let us consider $n - r = k$ for easily understood.
Then,

$$ V_r^{n-r} = V_r^k = \frac{(r+1)(r+2)(r+3)\cdots(r+k)}{k!}. $$

3.2 To convert the combination ${}^nC_n$ into the optimized combination

$$ {}^nC_n = \frac{n!}{n!} = V_0^n = 1. $$

3.3 To convert the combination ${}^{(n+r)}C_r$ into the optimized combination

$$ {}^{(n+r)}C_r = \frac{n!}{r!(n+r-r)!} = \frac{n!}{r!n!} = \frac{1.2.3\cdots r(r+1)(r+2)\cdots(r+n)}{r!n!} = (V_0^r)(V_r^n). $$

$(V_0^r)(V_r^n) = V_r^n$, where $V_0^r = 1$.

Now $V_r^n(n, r \in N, n \geq 1, r \geq 0)$ is considered as optimized combination.

4 Some results with proofs on the optimized combination [5,6]

***Result* 4.1** $V_0^1 = V_0^n = 1$.

Proof.

(4.1) $V_0^1 = \dfrac{(0+1)}{1!} = 1$.

(4.2) $V_0^n = \dfrac{(0+1)(0+2)(0+3)\cdots(0+n)}{n!} = \dfrac{n!}{n!} = 1$.

From (4.1) and (4.2), the ***Result* 4.1** is true.

***Result* 4.2** $V_r^{n+1} - V_r^n = V_{r-1}^n$.

Proof. $V_r^n = \frac{(r+1)(r+2)(r+3)\cdots(r+n)}{n!}$,

$$ V_r^{n+1} = \frac{(r+1)(r+2)(r+3)\cdots(r+n)(r+n+1)}{(n+1)!}, $$

$$ V_r^{n+1} - V_r^n = \frac{(r+1)(r+2)(r+3)\cdots(r+n)}{n!}[\frac{r+n+1}{n+1} - 1], $$

(4.3) $V_r^{n+1} - V_r^n = \dfrac{r(r+1)(r+2(r+3)+\cdots+(r+n)}{n!} = V_{r-1}^n$.

It is understood from (4.3) that the ***Result* 4.2** is true.

***Result* 4.3** $1 + V_1^1 + V_1^2 + V_1^3 + \cdots + V_1^n = V_2^n$.

Proof.

(4.4) $V_2^n = \dfrac{(2+1)(2+2)(2+3)\cdots(2+n-1)(2+n)}{n!} = \dfrac{(n+1)(n+2)}{2!}$,

(4.5) $1 + V_1^1 + V_1^2 + V_1^3 + \cdots + V_1^n = 1 + 2 + 3 + \cdots + n + 1 = \dfrac{(n+1)(n+2)}{2!}$.

From (4.4) and (4.5), the ***Result* 4.3** is true.

***Result* 4.4** $V_r^n = V_n^r(n, r \geq 1n, r \in N)$.

Proof.

$$V_r^n = V_n^r \text{ implies } \frac{(r+1)(r+2)\cdots(r+n)}{n!} = \frac{(n+1)(n+2)\cdots(n+r)}{r!}.$$

Assume that $r = n + m(m \in Nm \geq 1)$. Let us show that $V_{n+m}^n = V_n^{n+m}$.

$$(4.6)\quad V_{n+m}^n = \frac{(n+m+1)(n+m+2)\cdots(n+m+n)}{n!} = \frac{(n+1)(n+2)\cdots(n+m+n)}{(n+m)!}$$

$$(4.7)\quad V_n^{n+m} = \frac{(n+1)(n+2)\cdots(n+n)(n+n+1)(n+n+2)\cdots(n+n+m)}{(n+m)!}$$

From (4.6) and (4.7), $V_{n+m}^n = V_n^{n+m}$ is true.

Assume that $r = n - m(n > m)$. Let us show that $V_{n-m}^n = V_n^{n-m}$.

$$(4.8)\quad V_{n-m}^n = \frac{(n-m+1)(n-m+2)\cdots(n-m+n)}{n!} = \frac{(n+1)(n+2)\cdots(n+n-m)}{(n-m)!}.$$

$$(4.9)\quad V_n^{n-m} = \frac{(n+1)(n+2)\cdots(n+n-m)}{(n-m)!},$$

From (4.8) and (4.9), $V_{n-m}^n = V_n^{n-m}$ is true.

If $r = n$, $V_r^n = V_n^r$ is obivously true for $r = n$.

Hence, the ***Result* 4.4** is true.

***Result* 4.5** $V_n^n = 2V_{n-1}^n$.

Proof.

$$V_n^n = \frac{(n+1)(n+2)\cdots(n+n-1)2n}{(n-1)!n} = \frac{2(n+1)(n+2)\cdots(n+n-1)}{(n-1)!} = 2V_{n-1}^n.$$

Hence, the Result 4.5 is true.

***Result* 4.6** $V_0^n + V_1^n + V_2^n + V_3^n + \cdots + V_{r-1}^n + V_r^n = V_r^{n+1}$.

Proof. This result is proved by mathematical induction. Basis. Let $r = 1.V_0^n + V_1^n = V_1^{n+1}$ implies $n + 2 = n + 2$.

Inductive hypothesis.

Let us assume that $V_0^n + V_1^n + V_2^n + \cdots + V_{k-1}^n = V_{k-1}^{n+1}$ is true for $r = k - 1$.

Inductive step. We must show that the inductive hypothesis is true for $r = k$.

$$V_0^n + V_1^n + \cdots + V_{k-1}^n + V_k^n = V_k^{n+1} \text{ implies } V_0^n + V_1^n + \cdots + V_{k-1}^n = V_k^{n+1} - V_k^n = V_{k-1}^{n+1}.$$

Hence, it is proved.

To convert the combination ${}^{(n+r)}C_r$ into the optimized combination:

$${}^{(n+r)}C_r = \frac{n!}{r!(n+r-r)!} = \frac{n!}{r!n!} = \frac{1.2.3\cdots r(r+1)(r+2)\cdots(r+n)}{r!n!} = (V_0^r)(V_r^n).$$

$(V_0^r)(V_r^n) = V_r^n$ where $V_0^r = 1$.

5 Conclusion

In the research paper, a computing method and models for optimizing the combination defined in combinatorics has been introduced that are useful for scientific researchers who are solving scientific problems and meeting today's challenges.

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