APPLICATION OF INTERVAL VALUED FUZZY SET AND SOFT SET

By

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Abstract

Molodtsov was a father of soft set approach. We can't easily settle the membership degree in some practical application. So it must be much better to describe interval-valued data instead of explaining membership degree. In this paper, we introduce the latest approach of the interval-valued fuzzy soft set by combining the interval-valued fuzzy set and soft set models. This approach successfully follows distributive, associative and DeMorgans laws as well. In the end, a decision problem is solved by this approach.

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1 Introduction

In 1999 a great researcher Molodtsov was born in Russia who gave an approach which is known as a soft set approach. There are many approach which deal with uncertainty for example, probability, the theory of fuzzy set etc. But this approach is different from all these approach. Molodtsov applied this approach and got positive results.

Many researchers are working on it, because of this approach has many application in several directions. Many operations applied on soft sets by Maji et al. [21]. This approach has also solved many judgment constructing issue by adopting fuzzy mathematics [20]. The characterization devaluation of soft sets analyzed by Chen at al. [10]. The main objective of Kong at al. [16, 17, 18] is to show a devaluation of soft sets and fuzzy soft sets. They also explained algorithm of normal parameter reduction.

By above analysis, we observed that classical soft set [26, 28] approach is the root of all above work. We link this model with any mathematical model. Aktas et al. [1] obtained elementary properties of soft groups by this approach. Feng et al. [11] suggested approach of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings, soft semiring homomorphisms and their characteristics. Approach of soft BCK/BCI-algebras and soft subalgebras are presented by Jun [13, 14, 15]. Maji et al. [20] suggested this approach. It is a amalgam of fuzzy set and soft set models. Yange et al. [29] applied some process on soft sets, these were negation, triangular norm etc. Zou et al. [31] presented soft set, fuzzy soft set into the insufficient situation.

In fuzzy mathematics, The utility of fuzzy set theory in handling uncertainty, arising from deficiencies of information available from a situation in pattern recognition problems have been proposed by many researchers. Controversy has surrounded the concept of fuzziness since its inception. Some maintains that probability theory can handle any kind of uncertainty; other think that fuzziness is probability in disguise or that probability is only sensible way to describe any kind of uncertainty. This theory provide an approximate, effective and more flexible means of classifying the patterns which are too complex or have to ill-defined features to be handled by the classical approach. Both fuzzy logic and probability theory are closely related, the key difference is their meaning. Probability is associated with events and not facts, and those events will either occur or not occur. There is nothing fuzzy about it. Fuzzy Logic is all about degree of truth.

The merging of interval-valued fuzzy set and soft set are shown here. So we get latest soft set model which is Interval-valued Fuzzy Soft Set. To make this work easy, Section 2 having detail about standard soft set and fuzzy soft set. Section 3 having approach about interval-valued fuzzy soft set. We then apply the concept of complement, *AND* and *OR* on interval-valued fuzzy soft sets. Section 4, show that interval-valued fuzzy soft set successfully works on judgment constructing issue. At last we then give summary and direction for new research.We observed these hotels by net and saw their websites. First we saw these attributes and got the rating of all these attributes from their websites.

Triangular norms and conorms are operations which generalize the logical conjunction and logical disjunction to fuzzy logic. They are a natural interpretation of the conjunction and disjunction in the semantics of mathematical fuzzy logics and they are used to combine criteria in multi-criteria decision making.

2 Preliminaries

A fuzzy soft relation is defined as soft set over the fuzzy power set of the Cartesian product of two crisp sets.

By this process Molodtsov [22] explained soft set. Suppose M and N indicate universal set, parameter set respectively. We show soft set by this pair (F, N). In this case F indicates a function of N towards all subsets of set M.

Now make all subsets of M and indicated by $P(M)$, F mapping shown below [10].

$$F : N \rightarrow P(M),$$

$\forall n \in N$, $F(n)$ indicates the family of n-approximate members of soft sets (F, N) [2, 7, 23].

Suppose assemblage of every subsets of M can denote by $P(M)$. So fuzzy soft set for $\mathcal{P}(M)$ as reported by $(\tilde{F}, N)$. Here $\tilde{F}$ transformation represented.

$$\tilde{F} : N \rightarrow \mathcal{P}(M).$$

It indicates the parameterized group of fuzzy subsets of M. It is mapping from parameters to a universe, so this indicates the particular process of soft set.

In general, fuzzy value set for n is indicated by $\tilde{F}(n)$ and it is the subset of M. We can differentiate it from the classic soft set, suppose $(\mu_{\tilde{F}(n)}(HO))$ indicate the membership degree of element HO, which holds the parameter n here. $HO \in M, n \in N$. $\tilde{F}$ given below.

$$\tilde{F}(n) = \left\{ (HO, \mu_{\tilde{F}(n)}(HO)) \colon HO \in M \right\}.$$

Definition 2.1 *The AND operation on the two interval-valued fuzzy soft sets* $(\tilde{F}, A)$, $(\tilde{G}, B)$ *is defined by*

$$(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B),$$

where $\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \cap \tilde{G}(\beta), \forall (\alpha, \beta) \in (A \times B)$.

Definition 2.2 *The OR operation on the two interval-valued fuzzy soft sets* $(\tilde{F}, A)$, $(\tilde{G}, B)$ *is defined by*

$$(\tilde{F}, A) \vee (\tilde{G}, B) = (\tilde{H}, A \times B),$$

where $\tilde{H}(\alpha, \beta) = \tilde{F}(\alpha) \cup \tilde{G}(\beta), \forall (\alpha, \beta) \in (A \times B)$.

3 Interval-valued fuzzy soft set

3.1 Approach of interval-valued fuzzy soft set

It can obtained with the help of fuzzy set and soft set theory definitely. Note that we can't easily settle the membership degree in some practical application. So it must be much better to describe interval-valued data instead of explaining membership degree. Due to this approach, Zadeh described this approach. Interval-valued fuzzy soft set model can be described by joining interval-valued fuzzy set and soft set. Now, we shortly suggest this theory. Mapping of interval-valued fuzzy set [8, 9, 30] $\hat{S}$ on the universe M.

$$\hat{S} : M \rightarrow Int([0, 1]),$$

Here, collection of every closest subintervals of [0, 1] is indicated by $Int([0, 1])$. Group of all interval-valued fuzzy sets on M is denoted by $\mathcal{P}(M)$.

Let $\hat{S} \in \tilde{P}(M), \forall m \in M$.

The degree of membership is given by $\mu_{\hat{S}}(HO) = [\mu_{\hat{S}}^-(HO), \mu_{\hat{S}}^+(HO)]$ of element HO to $\hat{S}$.

$\mu_{\hat{S}}^-(HO), \mu_{\hat{S}}^+(HO)$ Indicate the lower and upper membership degree of HO to $\hat{S}$. Here

$$\mu_{\hat{S}}^-(HO), \mu_{\hat{S}}^+(HO).$$

We can describe complement, intersection and other operation of the interval-valued fuzzy set here.

Suppose $\hat{S}, \hat{T} \in \tilde{\mathcal{P}}(M)$

1. $\hat{S}^C$ indicates the complement of $\hat{S}$
   $\mu_{\hat{S}^C}(HO) = 1 - \mu_{\hat{S}}(HO) = [1 - \mu_{\hat{S}}^+(HO), 1 - \mu_{\hat{S}}^-(HO)]$;
2. $\hat{S} \cap \hat{T}$ Indicates the intersection of $\hat{S}$ and $\hat{T}$
   $\mu_{\hat{S} \cap \hat{T}}(HO) = inf[\mu_{\hat{S}}(HO), \mu_{\hat{T}}(HO)]$
   $= [inf(\mu_{\hat{S}}^-(HO), \mu_{\hat{T}}^-(HO)), inf(\mu_{\hat{S}}^+(HO), \mu_{\hat{T}}^+(HO))]$;

Table 3.1: $(I\tilde{v}f_1, \Omega_1)$ serve as interval valued fuzzy soft set.

M	n_1	n_2	n_3	n_4
HO_1	$[0.6 - 0.8]$	$[0.4 - 0.6]$	$[0.6 - 0.8]$	$[0.7 - 0.9]$
HO_2	$[0.6 - 0.8]$	$[0.5 - 0.7]$	$[0.6 - 0.8]$	$[0.4 - 0.6]$
HO_3	$[0.8 - 1.0]$	$[0.7 - 0.9]$	$[0.8 - 1.0]$	$[0.8 - 1.0]$
HO_4	$[0.8 - 1.0]$	$[0.6 - 0.8]$	$[0.8 - 1.0]$	$[0.8 - 1.0]$
HO_5	$[0.6 - 0.8]$	$[0.5 - 0.7]$	$[0.7 - 0.9]$	$[0.7 - 0.9]$
HO_6	$[0.6 - 0.8]$	$[0.5 - 0.7]$	$[0.6 - 0.8]$	$[0.7 - 0.9]$

Table 3.2: $(I\tilde{v}f_2, \Omega_2)$ serve as interval valued fuzzy soft set.

M	σ_1	σ_2	σ_3
HO_1	$[0.6 - 0.8]$	$[0.4 - 0.6]$	$[0.6 - 0.8]$
HO_2	$[0.6 - 0.8]$	$[0.4 - 0.6]$	$[0.6 - 0.8]$
HO_3	$[0.8 - 1.0]$	$[0.7 - 0.9]$	$[0.8 - 1.0]$
HO_4	$[0.8 - 1.0]$	$[0.7 - 0.9]$	$[0.8 - 1.0]$
HO_5	$[0.7 - 0.9]$	$[0.6 - 0.8]$	$[0.7 - 0.9]$
HO_6	$[0.6 - 0.8]$	$[0.4 - 0.6]$	$[0.6 - 0.8]$

3. $\hat{S} \cup \hat{T}$ indicates the union of $\hat{S}$ and $\hat{T}$.

$\mu_{\hat{S} \cup \hat{T}}(HO) = sup[\mu_{\hat{S}}(HO), \mu_{\hat{T}}(HO)] = [sup(\mu_{\hat{S}}^{-}(HO), \mu_{\hat{T}}^{-}(HO)), sup(\mu_{\hat{S}}^{+}(HO), \mu_{\hat{T}}^{+}(HO))].$

Suppose M and N indicate the universal set, parameters set respectively, an ordered pair $(I\tilde{v}f_1, N)$ indicates interval-valued fuzzy soft set for $\tilde{\mathcal{P}}(M)$, here $I\tilde{v}f_1$ defined below [3, 4, 5].

$I\tilde{v}f_1 : N \to \tilde{\mathcal{P}}(M).$

By parameterized family of subsets of M we prepare interval-valued fuzzy soft set. Hence, its universe is the set of all interval-valued fuzzy sets of M, i.e. $\tilde{\mathcal{P}}(M)$.

$\forall n \in N$, the interval fuzzy value set of guideline represented by $I\tilde{v}f_1(n)$, which indicates interval-valued fuzzy set of M where $HO \in M$ and $n \in N$, we write in the form:

$I\tilde{v}f_1(n) = \{\langle HO, \mu_{I\tilde{v}f_1(n)}(HO)\rangle : HO \in M\},$

here, $I\tilde{v}f_1(n)$ is the interval-valued fuzzy membership degree of object HO which holds on guideline n $\forall$ $n \in N$, $\forall$ $HO \in M$, $\mu_{I\tilde{v}f_1(n)}^{-}(HO) = \mu_{I\tilde{v}f_1(n)}^{+}(HO)$, then $I\tilde{v}f_1(n)$ will degenerated to be a standard fuzzy set and then $(I\tilde{v}f_1, N)$ will be degenerated to be a traditional fuzzy soft set. Let us consider,

- M is the collection of hotel according to choice and $M=\{HO_1, HO_2, HO_3, HO_4, HO_5, HO_6\}$
- Ω_1 is the collection of attributes of hotel,
  $\Omega_1=\{n_1, n_2, n_3, n_4\}=\{$Staff, Value for Money, Facilities, Location$\}$

An interval valued fuzzy soft set $(I\tilde{v}f_1, \Omega_1)$ has given in **Table 3.1**.

3.2 Source of Numerical Data

The lower and upper limits of an evaluation is given. For example, we cannot present the precise degree of how beautiful hotel HO_1 is, yet, hotel HO_1 is at least beautiful on the degree of 0.6 and it is at most beautiful on the degree of 0.8. According to these attributes. By questionnaire to 150 visiters who visites these hotels, we collected membership value of these attributes. For example: By net we obtained the staff rating of hotel Le Meridien is 8.7. So 90 percent visiters assign this [0.6-0.8] and 10 percent [0.7-0.9] then we give [0.6-0.8] vale to the staff attribute. likewise, we make interval valued fuzzy soft sets for all attributes of all hotels.

Let $(I\tilde{v}f_1, N)$ indicates interval-valued fuzzy soft set for $\tilde{\mathcal{P}}(M)$, $I\tilde{v}f_1(n)$ indicates interval fuzzy value set for n, then all interval fuzzy value sets in interval-valued fuzzy soft set $(I\tilde{v}f_1, N)$ are specify to as the interval fuzzy value class of $(I\tilde{v}f_1, N)$, $G_{(I\tilde{v}f_1, N)}$ symbolic representation of it and presented as follows [19, 24, 25].

$G_{(I\tilde{v}f_1, N)} = \{I\tilde{v}f_1(n): n \in N\}.$

Table 3.1, we got $G_{(I\tilde{v}f_1,\Omega_1)} = \{I\tilde{v}f_1(n_1), I\tilde{v}f_1(n_2), I\tilde{v}f_1(n_3), I\tilde{v}f_1(n_4)\}$ here:

$I\tilde{v}f_1(n_1) = \{(HO_1, [0.6-0.8]), (HO_2, [0.6-0.8]), (HO_3, [0.8-1.0]), (HO_4, [0.8-1.0]), (HO_5, [0.6-0.8]), (HO_6, [0.6-0.8])\}$

$I\tilde{v}f_1(n_2) = \{(HO_1, [0.4-0.6]), (HO_2, [0.5-0.7]), (HO_3, [0.7-0.9]), (HO_4, [0.6-0.8]), (HO_5, [0.5-0.7]), (HO_6, [0.5-0.7])\}$

$I\tilde{v}f_1(n_3) = \{(HO_1, [0.6-0.8]), (HO_2, [0.6-0.8]), (HO_3, [0.8-1.0]), (HO_4, [0.8-1.0]), (HO_5, [0.7-0.9]), (HO_6, [0.6-0.8])\}$

$I\tilde{v}f_1(n_4) = \{(HO_1, [0.7-0.9]), (HO_2, [0.9-0.6]), (HO_3, [0.8-1.0]), (HO_4, [0.8-1.0]), (HO_5, [0.7-0.9]), (HO_6, [0.7-0.9])\}$

Suppose M and N indicates universal set and parameter set respectively, let Ω_1, $\Omega_2 \subset N$, $(I\tilde{v}f_1,\Omega_1)$ and $(I\tilde{v}f_2,\Omega_2)$ indicates interval-valued fuzzy soft sets, $(I\tilde{v}f_1,\Omega_1)$ is an interval-valued fuzzy soft subset of $(I\tilde{v}f_2,\Omega_2)$ if

1. $\Omega_1 \subset \Omega_2$

2. $\forall\ n \in \Omega_1, I\tilde{v}f_1(n)$

shows interval-valued fuzzy subset of $I\tilde{v}f_2(n)$; shown as follows $(I\tilde{v}f_1,\Omega_1)\tilde{\subset}(I\tilde{v}f_2,\Omega_2)$.

$(I\tilde{v}f_1,\Omega_1)$ and $(I\tilde{v}f_2,\Omega_2)$ indicates two interval-valued fuzzy soft sets, $M = \{HO_1, HO_2, HO_3, HO_4,$

$HO_5, HO_6\}$. M indicates the collection of hotels, $\Omega_1 = \{n_1, n_2\} =$

$\{Staff, ValueforMoney\}$, $\Omega_2=\{n_1,n_2,n_3\}=\{Staff, ValueforMoney, Facilities\}$, and

$I\tilde{v}f_1(n_1) = \{(HO_1, [0.6-0.8]), (HO_2, [0.6-0.8]), (HO_3, [0.8-1.0]), (HO_4, [0.8-1.0]), (HO_5, [0.6-0.8]), (HO_6, [0.6-0.8])\}$

$I\tilde{v}f_1(n_2) = \{(HO_1, [0.4-0.6]), (HO_2, [0.5-0.7]), (HO_3, [0.7-0.9]), (HO_4, [0.6-0.8]), (HO_5, [0.5-0.7]), (HO_6, [0.5-0.7])\}$

$I\tilde{v}f_2(n_1) = \{(HO_1, [0.8-1.0]), (HO_2, [0.6-0.8]), (HO_3, [0.9-1.0]), (HO_4, [0.8-1.0]), (HO_5, [0.8-1.0]), (HO_6, [0.8-1.0])\}$

$I\tilde{v}f_2(n_2) = \{(HO_1, [0.6-0.7]), (HO_2, [0.9-1.0]), (HO_3, [0.8-0.9]), (HO_4, [0.7-0.8]), (HO_5, [0.6-0.8]), (HO_6, [0.9-1.0])\}$

$I\tilde{v}f_2(n_3) = \{(HO_1, [0.6-0.8]), (HO_2, [0.6-0.8]), (HO_3, [0.8-1.0]), (HO_4, [0.8-1.0]), (HO_5, [0.6-0.8]), (HO_6, [0.6-0.8])\}$

Obviously, we can see $(I\tilde{v}f_1,\Omega_1) \tilde{\subset} (I\tilde{v}f_2,\Omega_2)$. Suppose $(I\tilde{v}f_1,\Omega_1)$ and $(I\tilde{v}f_2,\Omega_2)$ indicates two interval-valued fuzzy soft sets, These two sets will equal if

1. $(I\tilde{v}f_1,\Omega_1)$ is a subset of $(I\tilde{v}f_2,\Omega_2)$,

2. $(I\tilde{v}f_2,\Omega_2)$ is a subset of $(I\tilde{v}f_1,\Omega_1)$.

It is represented as $(I\tilde{v}f_1,\Omega_1) = (I\tilde{v}f_2,\Omega_2)$.

Operation on interval-valued fuzzy soft sets $(I\tilde{v}f_1,\Omega_1)^C$ is the complement of $(I\tilde{v}f_1,\Omega_1)$ and it is explained as $(I\tilde{v}f_1,\Omega_1)^C=(I\tilde{v}f_1{}^C, \neg\Omega_1)$,

Here $\forall\ \theta_1 \in \Omega_1$, $\neg\ \theta_1$=not θ_1, not belongs to the parameter θ_1, it means it is opposite of θ_1;

$$I\tilde{v}f_1{}^C: \neg\Omega_1 \to \tilde{\mathcal{P}}(M),$$

is the function given by $I\tilde{v}f_1{}^C(\theta_2) = (I\tilde{v}f_1(\neg\theta_2))^C, \forall\theta_2 \in \neg\Omega_1$.

Another interval-valued fuzzy soft set taken into consideration $(I\tilde{v}f_2,\Omega_2)$ and given in **Table** 3.2. Universal set M is same in both table, i.e $M = \{HO_1, HO_2, HO_3, HO_4, HO_5, HO_6\}$ it is the collection of hotels; $\Omega_2= \{\sigma_1, \sigma_2, \sigma_3\} =$ {Cleanliness, Free WiFi, Comfort} it is the collection of parameters.

So, with the help **1** of **Section** 3.1, we get

$I\tilde{v}f_2{}^C(\neg\sigma_1) = \{(HO_1, [0.2, 0.4]), (HO_2, [0.2, 0.4]), (HO_3, [0.0, 0.2]), (HO_4, [0.0, 0.2]), (HO_5, [0.1, 0.3]), (HO_6, [0.2, 0.4])\}$

$I\tilde{v}f_2{}^C(\neg\sigma_2) = \{(HO_1, [0.4, 0.6]), (HO_2, [0.4, 0.6]), (HO_3, [0.1, 0.3]), (HO_4, [0.1, 0.3]), (HO_5, [0.2, 0.4]), (HO_6, [0.4, 0.6])\}$

$I\tilde{v}f_2{}^C(\neg\sigma_3) = \{(HO_1, [0.2, 0.4]), (HO_2, [0.2, 0.4]), (HO_3, [0.0, 0.2]), (HO_4, [0.0, 0.2]), (HO_5, [0.1, 0.3]), (HO_6, [0.2, 0.4])\}$

The "*AND*" procedure on $(I\tilde{v}f_1,\Omega_1)$ $(I\tilde{v}f_2,\Omega_2)$ is shown below

$$(I\tilde{v}f_1,\Omega_1) \wedge (I\tilde{v}f_2,\Omega_2) = (I\tilde{v}f_3,\Omega_1 \times \Omega_2).$$

Here

$$I\tilde{v}f_3(\theta_1,\theta_2) = I\tilde{v}f_1(\theta_1) \cap I\tilde{v}f_2(\theta_2), \forall(\theta_1,\theta_2) \in \Omega_1 \times \Omega_2.$$

We can apply "AND" procedure on $(I\tilde{v}f_1,\Omega_1)$ and $(I\tilde{v}f_2,\Omega_2)$ and can show in tabular form below.

Table 3.3: "AND" procedure on **Tables 3.1** and **3.2**.

M	$n_1\sigma_1$	$n_1\sigma_2$	$n_1\sigma_3$	$n_2\sigma_1$	$n_2\sigma_2$	$n_2\sigma_3$
HO_1	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]	[0.4, 0.6]	[0.4, 0.6]	[0.4, 0.6]
HO_2	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]	[0.5, 0.7]	[0.4, 0.6]	[0.5, 0.7]
HO_3	[0.8, 1.0]	[0.7, 0.9]	[0.8, 1.0]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]
HO_4	[0.8, 1.0]	[0.7, 0.9]	[0.8, 1.0]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]
HO_5	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.5, 0.7]	[0.5, 0.7]	[0.5, 0.7]
HO_6	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]	[0.5, 0.7]	[0.4, 0.6]	[0.5, 0.7]

M	$n_3\sigma_1$	$n_3\sigma_2$	$n_3\sigma_3$	$n_4\sigma_1$	$n_4\sigma_2$	$n_4\sigma_3$
HO_1	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]
HO_2	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]	[0.4, 0.6]	[0.4, 0.6]	[0.4, 0.6]
HO_3	[0.8, 1.0]	[0.7, 0.9]	[0.8, 1.0]	[0.8, 1.0]	[0.7, 0.9]	[0.8, 1.0]
HO_4	[0.8, 1.0]	[0.7, 0.9]	[0.8, 1.0]	[0.8, 1.0]	[0.7, 0.9]	[0.8, 1.0]
HO_5	[0.7, 0.9]	[0.6, 0.8]	[0.7, 0.9]	[0.7, 0.9]	[0.6, 0.8]	[0.7, 0.9]
HO_6	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]

Table 3.4: "OR" procedure on **Tables 3.1** and **3.2**.

M	$n_1\sigma_1$	$n_1\sigma_2$	$n_1\sigma_3$	$n_2\sigma_1$	$n_2\sigma_2$	$n_2\sigma_3$
HO_1	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]
HO_2	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.5, 0.7]	[0.6, 0.8]
HO_3	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.7, 0.9]	[0.8, 1.0]
HO_4	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.7, 0.9]	[0.8, 1.0]
HO_5	[0.7, 0.9]	[0.6, 0.8]	[0.7, 0.9]	[0.7, 0.9]	[0.6, 0.8]	[0.7, 0.9]
HO_6	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.5, 0.7]	[0.6, 0.8]

M	$n_3\sigma_1$	$n_3\sigma_2$	$n_3\sigma_3$	$n_4\sigma_1$	$n_4\sigma_2$	$n_4\sigma_3$
HO_1	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]
HO_2	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.4, 0.6]	[0.6, 0.8]
HO_3	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]
HO_4	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]	[0.8, 1.0]
HO_5	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]
HO_6	[0.6, 0.8]	[0.6, 0.8]	[0.6, 0.8]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]

By **Definition 2.1**, we obtain

$I\tilde{v}f_3(n_1, \sigma_2) = I\tilde{v}f_1(n_1) \cap I\tilde{v}f_2(\sigma_2)$

$=\{(HO_1, [0.4, 0.6]), (HO_2, [0.4, 0.6]), (HO_3, [0.7, 0.9]), (HO_4, [0.7, 0.9]), (HO_5, [0.6, 0.8]), (HO_6, [0.4, 0.6])\}$

By **Table 3.3** we obtain the value of $(I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2)$

The "OR" procedure on $(I\tilde{v}f_1, \Omega_1)$ $(I\tilde{v}f_2, \Omega_2)$ is shown below [6, 12].

$(I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_2, \Omega_2) = (I\tilde{v}f_3, \Omega_1 \times \Omega_2),$

Here $I\tilde{v}f_3(\theta_1, \theta_2) = I\tilde{v}f_1(\theta_1) \cup I\tilde{v}f_2(\theta_2), \forall (\theta_1, \theta_2) \in \Omega_1 \times \Omega_2$.

The output of the "OR" procedure on $(I\tilde{v}f_1, \Omega_1)$ and $(I\tilde{v}f_2, \Omega_2)$ in **Tables 3.1** and **3.2** is given in **Table 3.4**.

Let us, we apply DeMorgan's Laws on two interval-valued fuzzy soft sets $(I\tilde{v}f_1, \Omega_1)$, $(I\tilde{v}f_2, \Omega_2)$ as shown below.

$((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2))^C = (I\tilde{v}f_1, \Omega_1)^C \vee (I\tilde{v}f_2, \Omega_2)^C$

$((I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_2, \Omega_2))^C = (I\tilde{v}f_1, \Omega_1)^C \wedge (I\tilde{v}f_2, \Omega_2)^C$

Proof. $(I\tilde{v}f_1, \Omega_1)^C \vee (I\tilde{v}f_2, \Omega_2)^C = (I\tilde{v}f_1^C, \neg\Omega_1) \vee (I\tilde{v}f_2^C, \neg\Omega_2) = (I\tilde{v}f_4, \neg\Omega_1 \times \neg\Omega_2)$

$where, I\tilde{v}f_4(\neg\theta_1, \neg\theta_2) = I\tilde{v}f_1^C(\neg\theta_1) \cup I\tilde{v}f_2^C(\neg\theta_2)$

$= (I\tilde{v}f_4, \neg(\Omega_1 \times \Omega_2)).$

Assume $(I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2) = (I\tilde{v}f_3, \Omega_1 \times \Omega_2)$, then we have $((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2))^C = (I\tilde{v}f_3, \Omega_1 \times \Omega_2)^C = (I\tilde{v}f_3^C, \neg(\Omega_1 \times \Omega_2)).$

M	$n_1\sigma_2$	$n_2\sigma_1$	$n_3\sigma_2$	$n_3\sigma_3$	$n_4\sigma_1$
HO_1	$[0.4 - 0.6]$	$[0.4 - 0.6]$	$[0.4 - 0.6]$	$[0.6 - 0.8]$	$[0.6 - 0.8]$
HO_2	$[0.4 - 0.6]$	$[0.5 - 0.7]$	$[0.4 - 0.6]$	$[0.6 - 0.8]$	$[0.4 - 0.6]$
HO_3	$[0.7 - 0.9]$	$[0.7 - 0.9]$	$[0.7 - 0.9]$	$[0.8 - 1.0]$	$[0.8 - 1.0]$
HO_4	$[0.7 - 0.9]$	$[0.6 - 0.8]$	$[0.7 - 0.9]$	$[0.8 - 1.0]$	$[0.8 - 1.0]$
HO_5	$[0.6 - 0.8]$	$[0.5 - 0.7]$	$[0.6 - 0.8]$	$[0.7 - 0.9]$	$[0.7 - 0.9]$
HO_6	$[0.4 - 0.6]$	$[0.5 - 0.7]$	$[0.4 - 0.6]$	$[0.6 - 0.8]$	$[0.6 - 0.8]$

$\forall (\theta_1, \theta_2) \in \Omega_1 \times \Omega_2$, we know
$$I\tilde{v}f_3^C(\neg\theta_1, \neg\theta_2) = (I\tilde{v}f_3(\theta_1, \theta_2))^C = (I\tilde{v}f_1(\theta_1) \cap I\tilde{v}f_2(\theta_2))^C = (I\tilde{v}f_1(\theta_1))^C \cup (I\tilde{v}f_2(\theta_2))^C = (I\tilde{v}f_1^C(\neg\theta_1)) \cup (I\tilde{v}f_2^C(\neg\theta_2))$$
By above analysis,we got $((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2))^C = (I\tilde{v}f_1, \Omega_1)^C \vee (I\tilde{v}f_2, \Omega_2)^C$.
In the same way, we can prove that $((I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_2, \Omega_2))^C = (I\tilde{v}f_1, \Omega_1)^C \wedge (I\tilde{v}f_2, \Omega_2)^C$.
Suppose we have interval-valued fuzzy soft sets, $(I\tilde{v}f_1, \Omega_1), (I\tilde{v}f_2, \Omega_2) and (I\tilde{v}f_3, \Omega_3)$.
[**Associative law**]
$(I\tilde{v}f_1, \Omega_1) \wedge ((I\tilde{v}f_2, \Omega_2) \wedge (I\tilde{v}f_3, \Omega_3)) = ((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2)) \wedge (I\tilde{v}f_3, \Omega_3)$.
$(I\tilde{v}f_1, \Omega_1) \vee ((I\tilde{v}f_2, \Omega_2) \vee (I\tilde{v}f_3, \Omega_3)) = ((I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_2, \Omega_2)) \vee (I\tilde{v}f_3, \Omega_3)$.
[**Distributive law**]
$(I\tilde{v}f_1, \Omega_1) \wedge ((I\tilde{v}f_2, \Omega_2) \vee (I\tilde{v}f_3, \Omega_3)) = ((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2)) \vee ((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_3, \Omega_3))$.
$(I\tilde{v}f_1, \Omega_1) \vee ((I\tilde{v}f_2, \Omega_2) \wedge (I\tilde{v}f_3, \Omega_3)) = ((I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_2, \Omega_2)) \wedge ((I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_3, \Omega_3))$.
Proof. $\forall \theta_1 \in \Omega_1, \theta_2 \in \Omega_2$ and $\forall \theta_3 \in \Omega_3$, Now
$$I\tilde{v}f_1(\theta_1) \cap (I\tilde{v}f_2(\theta_2) \cap I\tilde{v}f_3(\theta_3)) = (I\tilde{v}f_1(\theta_1) \cap I\tilde{v}f_2(\theta_2)) \cap I\tilde{v}f_3(\theta_3)$$
from it, we can say $(I\tilde{v}f_1, \Omega_1) \wedge ((I\tilde{v}f_2, \Omega_2) \wedge (I\tilde{v}f_3, \Omega_3)) = ((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2)) \wedge (I\tilde{v}f_3, \Omega_3)$.
In the same way, we can say $(I\tilde{v}f_1, \Omega_1) \vee ((I\tilde{v}f_2, \Omega_2) \vee (I\tilde{v}f_3, \Omega_3)) = ((I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_2, \Omega_2)) \vee (I\tilde{v}f_3, \Omega_3)$.
$\forall \theta_1 \in \Omega_1, \theta_2 \in \Omega_2$ and $\forall \theta_3 \in \Omega_3$, we have $I\tilde{v}f_1(\theta_1) \cap (I\tilde{v}f_2(\theta_2) \cup I\tilde{v}f_3(\theta_3)) = (I\tilde{v}f_1(\theta_1) \cap I\tilde{v}f_2(\theta_2)) \cup (I\tilde{v}f_1(\theta_1) \cap I\tilde{v}f_3(\theta_3))$
In the same way, we can say $(I\tilde{v}f_1, \Omega_1) \wedge ((I\tilde{v}f_2, \Omega_2) \vee (I\tilde{v}f_3, \Omega_3)) = ((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_2, \Omega_2)) \vee ((I\tilde{v}f_1, \Omega_1) \wedge (I\tilde{v}f_3, \Omega_3))$.
Similarity, we also have $(I\tilde{v}f_1, \Omega_1) \vee ((I\tilde{v}f_2, \Omega_2) \wedge (I\tilde{v}f_3, \Omega_3)) = ((I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_2, \Omega_2)) \wedge ((I\tilde{v}f_1, \Omega_1) \vee (I\tilde{v}f_3, \Omega_3))$.

4 Application of interval-valued fuzzy soft set
Object can be identified by using algorithm, given by Roy et al. [27], different objects are compared in it. Kong et al. [18] said that Roy's innovation has error, Kong gave correct innovation, in it they compare choice values of different objects.
***Rule* 4.1**:Take the set of interval-valued fuzzy soft sets.
$(I\tilde{v}f_1, \Omega_1)$ and $(I\tilde{v}f_2, \Omega_2)$ demonstrate in ***Tables* 3.1** and **3.2** are under deliberation.
Here, we obtain the value of sets $(I\tilde{v}f_1, \Omega_1)$ and $(I\tilde{v}f_2, \Omega_2)$ from ***Tables* 3.1** and **3.2**.
***Rule* 4.2**:Input the guideline set X as detected by the viewer.
We take these two sets $(I\tilde{v}f_1, \Omega_1)$ and $(I\tilde{v}f_2, \Omega_2)$,***Table* 3.3** obtained by applying "*AND*" procedure on these sets.
Suppose we are taking these values of parameters X such that $X=\{\{n_1\sigma_2\},\{n_2\sigma_1\},$
$\{n_3\sigma_2\},\{n_3\sigma_3\},\{n_4\sigma_1\}\}$.
***Rule* 4.3**:We obtain resultant $(I\tilde{v}f_3, \Omega_3)$ set from $(I\tilde{v}f_1, \Omega_1)$ and $(I\tilde{v}f_2, \Omega_2)$.
***Table* 3.5** the resultant interval-valued fuzzy soft set is obtained from ***Table* 3.3** according to the set of parameters.
***Rule* 4.4**: $\forall HO_i \in M$, In this way, we calculate the choice value d for every hotel HO.

$$d_i = [d_i^-, d_i^+] = \left[\sum_{r \in R} \mu_{I\tilde{v}f_3(r)}^-(HO_i), \sum_{r \in R} \mu_{I\tilde{v}f_3(r)}^+(HO_i) \right].$$

The result is shown in ***Table* 4.1**.

Table 4.1: Choice value.

M	$n_1\sigma_2$	$n_2\sigma_1$	$n_3\sigma_2$	$n_3\sigma_3$	$n_4\sigma_1$	d_i
HO_1	[0.4, 0.6]	[0.4, 0.6]	[0.4, 0.6]	[0.6, 0.8]	[0.6, 0.8]	$d_1 = [2.4, 3.4]$
HO_2	[0.4, 0.6]	[0.5, 0.7]	[0.4, 0.6]	[0.6, 0.8]	[0.4, 0.6]	$d_2 = [2.3, 3.3]$
HO_3	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]	[0.8, 1.0]	[0.8, 1.0]	$d_3 = [3.7, 4.7]$
HO_4	[0.7, 0.9]	[0.6, 0.8]	[0.7, 0.9]	[0.8, 1.0]	[0.8, 1.0]	$d_4 = [3.6, 4.6]$
HO_5	[0.6, 0.8]	[0.5, 0.7]	[0.6, 0.8]	[0.7, 0.9]	[0.7, 0.9]	$d_5 = [3.1, 4.1]$
HO_6	[0.4, 0.6]	[0.5, 0.7]	[0.4, 0.6]	[0.6, 0.8]	[0.6, 0.8]	$d_6 = [2.5, 3.5]$

Rule 4.5:$\forall HO_i \in M$,calculate the value of t_i for each hotel HO_i in this way.

$$t_i = \sum_{HO_i \in M} ((d_i^- - d_j^-) + (d_i^+ - d_j^+)).$$

By calculation, we obtained $t_1 = -6.4, t_2 = -7.4, t_3 = 9.2, t_4 = 8.0, t_5 = 2.0, t_6 = -5.2$.

Rule 4.6:The solution is any one of the elements in Z here $Z = max_{HO_i \in M}(t_i)$

In this problem, hotel HO_3 is the best option because $max_{HO_i \in M}(t_i) = (HO_3)$. This result is reasonable because we can see that $d_3 > d_i$ here i=1,2,3,4,5,6 i.e. HO_3 has the highest choice value.

5 Conclusion

Soft set theory solves problems which contain uncertainty, fuzziness or vagueness. At last, an example shows that interval-valued fuzzy soft set works properly in judgment constructing issue. In previous work Yang at al. [29] took house problem which can be solved by Fuzzy Soft Maximum Minimum Decision Making Method. In the present work we took very important daily life problem which can only solved by this method. There are parameterizations reduction of interval-valued fuzzy soft set which is new topic for further research. In the end, this work also having the utilization in the field of automobile for buying car, buying laptop according to our desire parameter.

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