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PHASE SHIFTS OF *S*-WAVE SCHRÖDINGER EQUATION FOR MITTAG- LEFFLER FUNCTION

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ABSTRACT

In this paper, we consider a potential function to study the phase shift difference of *s*-wave Schrödinger equation.

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1. Introduction. The phase shifts have great importance in the computation of scattering and experimental work of nuclear and atomic collision. Several mathematical problems are solved to find out a potential function from a observed phase shift difference. Many workers namely Mahajan and Varma [8], Raghuwansi and Sharma [9], Kumar, Chandel and Agrawal [5], Agrawal and Kumar [1], Chandel and Kumar [3] have determined the phase shifts from a given potential function. Recently, Kumar and Singh [6] have obtained an approximation formula for phase-shifts of *s*-wave Schrödinger equation with the application of binomial potential function and study the phase-shifts variation with respect to the parameter involving in binomial potential function.

Here, in our work, we consider a potential function of Mittag- Leffler function and then on applying Tietz method [12], we obtain phase shift difference in series form involving hypergeometric function [8] and then make some studies on shift variation verses the parameters involving in the potential function.

The Mittag- Leffler function (see Erdélyi et al. [4] and Srivastava and Manocha [11]) is defined by

$$E_a(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(an+1)}, a\in C . \qquad \text{...(1.1)}$$

We consider the potential function in the form

$$U(r) = E_a\left(e^{br}\right), \qquad a,b \in C. \qquad ...(1.2)$$

Particularly, on setting $a=0$ in (1.2), we find the potential function of Bhattacharjie and Sudarshan [2] in the form

$$U'(r) = U(r) - 1 = \sum_{n=1}^{\infty} e^{nbr}. \qquad ...(1.3)$$

Also, from (1.3), we get

$$\frac{dU'(r)}{dr} = \frac{dU(r)}{dr} = \sum_{n=1}^{\infty} A_n e^{nbr} \text{ at } a=0. \qquad ...(1.4)$$

The potential function considered in (1.2) have more parameters to that of Bhattacharjie-Sudarshan [2] function and Chandel-Kumar [3] function. In this paper, we determine the phase shift difference formula for this function on making use of the Tietz' s techniques and make some studies on shift variation with respect to the parameters involving in the parameters given in the potential function (1.2).

2. Formulae Used. In our investigation, we need the applications of following formulae :

For the s-wave radial Schrödinger equation

$$\frac{d^2\Psi(r)}{dr^2} + \left[K^2 - U(r) - \frac{L(L+1)}{r^2}\right]\Psi_{(r)} = 0. \qquad ...(2.1)$$

Using Tietz [12] method and the Luke [7] formulae, Chandel and Kumar [3] have given the phase shifts in the form

$$\eta_L - \eta_{L+1} = \frac{\pi K^{2L+1}}{2^{2L+3}\Gamma(L+3/2)\Gamma(L+5/2)}\int_0^{\infty}\frac{dU}{dr} r^{2L+1} {}_1F_2\left[\begin{matrix} L+2; \\ L+5/2, 2L+3; \end{matrix} -K^2r^2\right]dr,$$

provided that $(2L+3)>0$ ...(2.2)

The second phase difference formula is given by

$$\eta_{L-1} - \eta_{L+1} = \frac{\pi K^{2L+1}}{2^{2L+3}\Gamma(L+1/2)\Gamma(L+3/2)}\int_0^{\infty}\frac{dU}{dr} r^{2L+3} {}_1F_2\left[\begin{matrix} L+1; \\ L+3/2, 2L+2; \end{matrix} -K^2r^2\right]dr \qquad (2.3)$$

3. Phase Shift Difference for Mittag-Leffler Function . From (2.1), we find

$$\frac{dU}{dr} = \frac{b}{a}\sum_{n=1}^{\infty}\frac{e^{nbr}}{\Gamma(an)}. \qquad ...(3.1)$$

Now, making an appeal to (3.1) and (2.2), we derive

$$\eta_L - \eta_{L+1} = \frac{\sqrt{\pi}K^{2L+1}\Gamma(L+2)}{ab^{2L+3}\Gamma(L+3/2)}\sum_{n=1}^{\infty}\frac{1}{n^{2L+4}\Gamma(an)}\,{}_2F_1\left[\begin{matrix}L+2, L+2; & \frac{-4K^2}{n^2b^2}\\ 2L+3; & \end{matrix}\right],$$

provided that $\left|(2iK/nb)^2\right| < 1$ . ...(3.2)

Further, Making an appeal to (3.1) and (3.2), we get

$$\eta_{L-1} - \eta_{L+1} = \frac{\sqrt{\pi}K^{2L-1}\Gamma(L+1)}{ab^{2L+1}\Gamma(L+1/2)}\sum_{n=1}^{\infty}\frac{1}{n^{2L+2}\Gamma(an)}\,{}_2F_1\left[\begin{matrix}L+1, L+1; & \frac{-4K^2}{n^2b^2}\\ 2L+2; & \end{matrix}\right],$$

provided that $\left|(2iK/nb)^2\right| < 1$. ...(3.3)

4. Particular Cases . For $L=0$, (3.2), gives

$$\eta_0 - \eta_1 = \frac{\sqrt{\pi}K}{ab^3\Gamma(3/2)}\sum_{n=1}^{\infty}\frac{1}{n^4\Gamma(an)}\,{}_2F_1\left[\begin{matrix}2,2; & \frac{-4K^2}{n^2b^2}\\ 3; & \end{matrix}\right]. \quad ...(4.1)$$

while for $L=1$, (3.2) gives

$$\eta_1 - \eta_2 = \frac{2\sqrt{\pi}K^3}{ab^5\Gamma(5/2)}\sum_{n=1}^{\infty}\frac{1}{n^6\Gamma(an)}\,{}_2F_1\left[\begin{matrix}3,3; & \frac{-4K^2}{n^2b^2}\\ 5; & \end{matrix}\right]. \quad ...(4.2)$$

From (4.2), we may write

$$\eta_1 - \eta_2 = \frac{\sqrt{\pi}K}{ab^3\Gamma(3/2)}\sum_{n=1}^{\infty}\frac{1}{n^4\Gamma(an)}\sum_{s=1}^{\infty}\frac{s(2)_s(2)_s}{3(4)_s}\frac{\left(-4K^2/n^2b^2\right)^5}{s!}. \quad ...(4.3)$$

Now, adding (4.1) and (4.3) we can write

$$\eta_0 - \eta_2 = \frac{\sqrt{\pi}K}{ab^3\Gamma(3/2)}\sum_{n=1}^{\infty}\frac{1}{(n)^4\Gamma(an)}\,{}_2F_1\left[\begin{matrix}2,2; & \frac{-4K^2}{n^2b^2}\\ 4; & \end{matrix}\right]. \quad ...(4.4)$$

If we put $L=1$ in the equation (3.3), then we get directly above result (4.4).

5. Analysis. To analize above results we start with

Table No. 1

a	$\eta_0 - \eta_2$
1	22.9226033
2	11.40945107
3	3.800312649
4	0.950005558

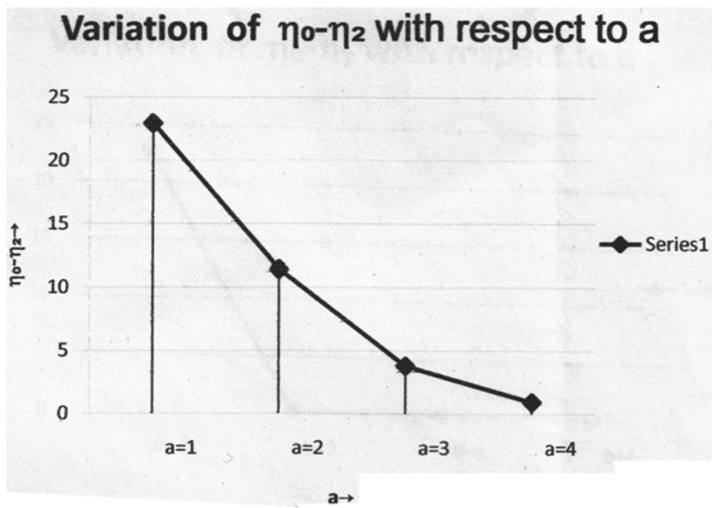

Figure 1.

The above graph shows that when potential function Figune vo.1 woar is reduced on making increment of a so that internal kinetic energy of the particle is increased and by this kinetic energy the particle of s-orbit gains frequency of d-orbit and then phase of s-wave tends to phase of d-wave ($\eta_0 - \eta_2$), thus difference in phase in sharply reducing.

Table No. 2

b	$\eta_0 - \eta_2$
1	22.9226033
2	0.239139705
3	0.085412647
4	0.027040942

Variation η_0–η_2 of with respect to *b*.

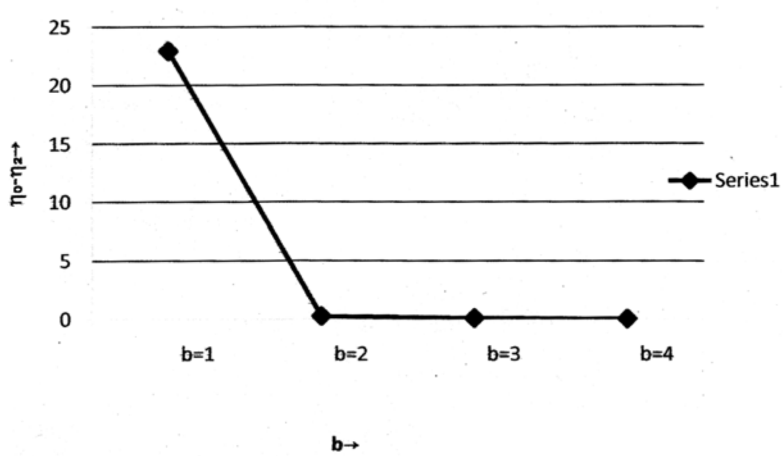

Figure -2

By second graph it is shown that when we make the increment in b, the potential energy of the particle is grown up, as when as the potential energy is changed, the internal energy of the particle is also changed so that the particle is disturbed due to slightly scattering and thus phase shifts is sharply decreased but when potential energy becomes large the difference in phase shifts becomes constant and the particle remains is *s*-orbit.

Table No. 3

b	$\eta_0 - \eta_2$
1/2	3458.01349
1/3	61372.55374
1/4	465550.4809
1/5	2233824.794

Variation of $\eta_0 - \eta_2$ with respect to *b*.

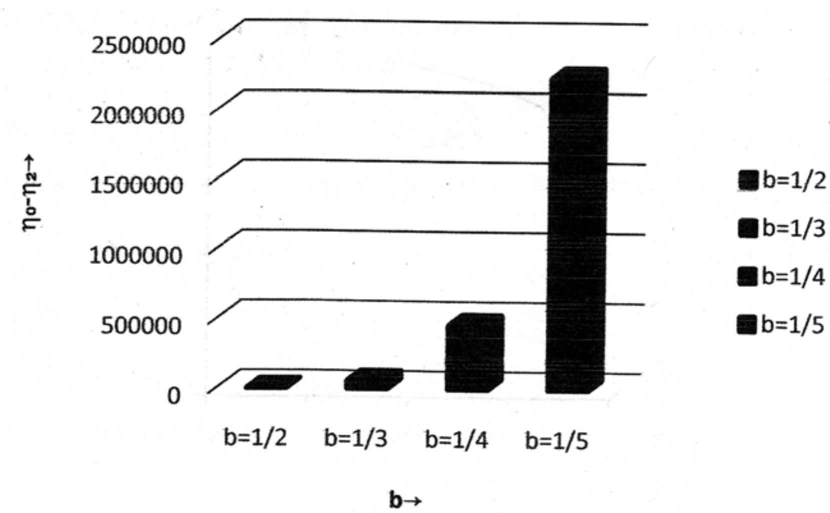

Figure. 3

From third graph, we have that when we reduce the value of b in the formula (1.2), the potential energy of the particle is reduced and thus internal kinetic energy of the particle is increased and by this effect the particle is scattered so that the difference in phase is sharply increasing.

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