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IMPACT OF SEXUAL MATURATION ON THE TRANSMISSION DYNAMICS OF *HIV* INFECTION IN HETEROGENEOUS COMMUNITY : A MODEL AND ITS QUALITATIVE ANALYSIS

By

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ABSTRACT

In this paper, we develop a model to study the spread of *HIV* infection, which can cause Acquired Immunodificiency Syndrome (*AIDS)*, through Vertical and horizontal transmissions and introduce the concept of sexually immature and mature individuals by considering maturation rates in heterosexual community and partially analyzed. We obtain equilibrium points of the system at two states (Disease-free and Endemic). We investigate the criteria for existence of endemic steady state of the system. We determine local and global dynamics of these steady states of the system and conclude.

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1. Introduction. Today's, Acquired Immunodeficiency Syndrome (*AIDS*), has shown a very high degree of prevalence in populations all over the world, which is caused by Human Immunodeficiency Virus (*HIV*). The nature of human interactions, the uncertainties in the current estimates of epidemiological parameters and the lack of enough reliable data make it extremely difficult to understand the dynamics of the virus transmission without the frame works provided by mathematical modelling. The study of mathematical modelling is also helpful in determining the demographic and economic impact of the epidemic which is turn help us to develop reasonable scientifically and socially sound investigation plans in order to reduce the spread of the infection.

In recent decades, several mathematical modelling studies have been conducted to describe the transmission dynamics of *HIV* infection for homogeneous and heterogeneous populations, Anderson et al. ([1][2]), Bailey [5], Knox [16], Pickering et al. [21], May and Anderson ([17][20]), Grant et al. [12], Hethcote [13], Anderson et al. [3], May [18] Castillo-Chavez, Cooke, Huang and Levin ([7],[8],[9]), Sun [23], Chen [10] and Hethcote [13].

In particular, Anderson et al. [1] described some preliminary attempts to use mathematical models for transmission of *HIV* in a homosexual community. May and Anderson [17] presented simple *HIV* transmission models to help clarify the effects of various factors on the overall pattern of *AIDS* epidemic. Blythe and Anderson (1988) considered *HIV* transmission models with four forms for the distribution of incubation period by assuming that the infectious period is equal to the incubation period. Castillo-Chavez et al. [7] analyzed a model where the mean rate of acquisition of new partners depends on the size of the sexually active population. Most of the above mentioned models consider only one population but *HIV* transmission takes place in the population that are heterogeneous in a variety of ways and this aspect should be taken in modelling *HIV.* Knox [16], Colgete et al. [11], Jacquez et al. [14], Koopman et al. [15].

2. Model Formulation. Let us consider a heterosexual community of size P with uniform promiscuous behaviour and taking only heterosexual encounters and assume that the birth and death rates are same, making community size to be a constant. We have assumed that infection passes in the considering population through the member of one male or female class to the other female or male class respectivley. The infection can also be transmitted vertically to the offspring of infected mother.

Let any instant of time t this considering community be subdivided into six classes of $S_1(t)$ mature male susceptibles, $I_1(t)$ mature male infective having *HIV* infection, $S_2(t)$ mature female susceptibles, $I_2(t)$ mature female infective having *HIV* infection, $X(t)$ immature susceptibles and $Y(t)$ immature infectives having *HIV* infection. The susceptibles become infected with transmssion efficiency k and immature susceptibles and infectives being sexually matured at rates m and m' respectively.

This leads to the following system of ordinary differential equations

$$\frac{dS_1}{dt} = -kS_1I_2 + \alpha mX - bS_1, \; \frac{DI_1}{dt} = kS_1I_2 + \alpha m'Y - b'I_1,$$

$$\frac{DS_2}{dt} = -kS_2I_1 + (1-\alpha)mX - bS_2, \; \frac{dI_2}{dt} = kS_2I_1 + (1-\alpha)m'Y - b'I_2,$$

$$\frac{dX}{dt} = bS + pb'I - \alpha mX - (1-\alpha)mX, \; \frac{dY}{dt} = qb'I - \alpha m'Y - (1-\alpha)m'Y, \tag{2.1}$$

with initial data

$$S_1(0) = S_{10} > 0, S_2(0) = S_{20} > 0, I_1(0) = I_{10} > 0, I_2(0) = I_{20} > 0, \; X(0) = X_0 > 0,$$

$$Y(0) = Y_0 > 0, P = S + I + X + Y, S = S_1 + S_2, I = I_1 + I_2, 1 = p + q, 0 < \alpha < 1 \tag{2.2}$$

where

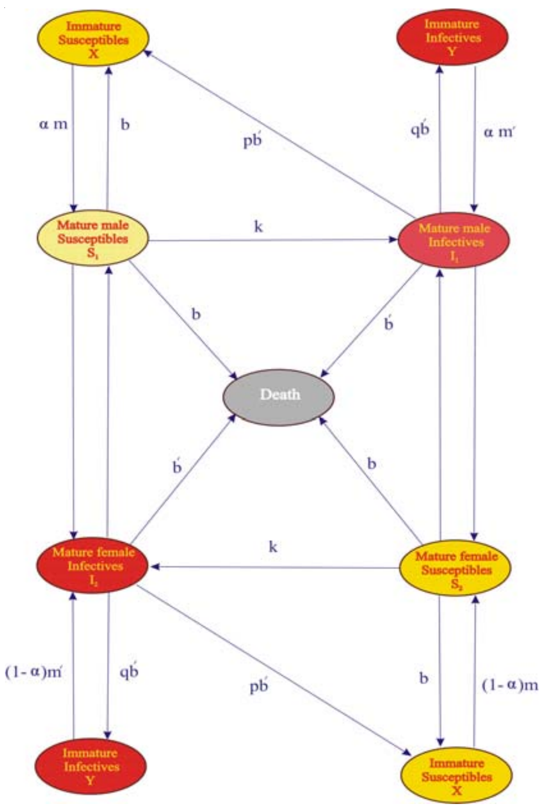
α and $(1-\alpha)$= The proportions of male and female hosts, who converts from immature class to mature class respectively.

b and b' are Birth rtes of immature susceptible and infectives respectively.

P= The fraction of newborn offspring of infective parents, who are susceptible at birth.

q= The fraction of newborn offsprings of infective parents, who are infective at birth.

b and b' are also Death rate of mature susceptibles and infectives respectively. The above mathematical modelling can be understood by Fig.1.



3. Equilibrium Points of Model. The equilibrium points of the model can be derived from the following set of equations

$$-kS_1^*I_2^* + \alpha mX^* - bS_1^* = 0,$$

$$kS_1^*I_2^* + \alpha m'Y^* - b'I_1^* = 0,$$

$$-kS_2^*I_1^* + (1-\alpha)mX^* - bS_2^* = 0,$$

$$kS_2^*I_1^* + (1-\alpha)m'Y^* - b'I_2^* = 0,$$

$$b\left(S_1^* + S_2^*\right) + pb'\left(I_1^* + I_2^*\right) - \alpha m X^* - (1-\alpha) m X^* = 0 \,,$$

$$qb'\left(I_1^* + I_2^*\right) - \alpha m' Y^* - (1-\alpha) m' Y^* = 0 \,,$$

$$S_1^* + S_2^* + I_1^* + I_2^* + X^* + Y^* = p \,. \tag{3.1}$$

Disease Free Equilibrium Point $E_0\left(S_1^*, I_1^*, S_2^*, I_2^*, X^*, Y^*\right)$**.**

$$I_1^* = I_2^* = Y^* = 0,$$

$$S_1^* = \frac{\alpha m P}{m+b}, S_2^* = \frac{(1-\alpha) m P}{m+b}, X = \frac{bP}{m+b},$$

Endemic Equilibrium Point $E_1\left(S_1^*, I_1^*, S_2^*, I_2^*, X^*, Y^*\right)$**.**

$$S_1^* = \frac{a_2 + \sqrt{a_2^2 + 4a_1a_3}}{2a_1},$$

$$S_1^* > 0; \text{ if } ka\left(mX^* + m'Y^*\right) > bb' + \frac{km'}{q} Y^*,$$

$$S_2^* = \frac{a_4 + \sqrt{a_4^2 + 4a_1a_5}}{2a_1}$$

$$S_2^* > 0; \text{ if } k(1-\alpha)\left(mX^* + m'Y^*\right) > bb' + \frac{km'}{q} Y^*,$$

$$I_1^* = \frac{\alpha a_8 - bS_1^*}{2a_1},$$

$$I_1^* > 0, \text{ if } a\left(mX^* + m'Y^*\right) > bS_1^*,$$

$$I_2^* = \frac{(1-\alpha) a_8 - bS_2^*}{b'},$$

$$I_2^* > 0, \text{ if } (1-\alpha)\left(mX^* + m'Y^*\right) > bS_2^*.$$

Now we investigate a criteria for endemic steady state E_1 to exist. Here we need that the system of equations (3.1) should have a positive solution.
From the set of equation (3.1), we get

$$X^* = b_1 - b_2 Y^*, \tag{3.2}$$

which implies

$X^* \to b_1$ when $Y^* \to 0$ and $Y^* \to b_1/b_2$ when $X^* \to 0$.

We can also obtain $\dfrac{dX^*}{dY^*} = -b_2,$

$\dfrac{dX^*}{dY^*} < 0,$ if $(m' + qb')b > pb'm'$.

Hence, we find that X^* is decreasing function of Y^*

Also, form the set of equations (3.1) we get

$$a_{10}X^* + \left[2ka_{12} - (a_{14} + a_{13})\right]Y^* + 2a_{15} = \left[\left\{\alpha a_{10}X^* + (\alpha a_{14} - ka_{12})Y^* - a_{15}\right\}^2 + b'a_{11}a_{10}X^*\right]^{1/2}$$

$$+\left[\left\{(1-\alpha)a_{10}X^* + ((1-\alpha)a_{14} - ka_{12})Y^* - a_{15}\right\}^2 + b'a_{10}a_{16}X^*\right], \tag{3.3}$$

which implies

$$a_{22}\left(X^*\right)^4 + a_{23}\left(X^*\right)^3 + a_{24}\left(X^*\right)^2 + a_{25}X^* + a_{26} = 0, \text{ when } Y^* = 0. \tag{3.4}$$

By Discarte's rule of sign the equation (3.4) has at least one positive root under condition $2bb' > 1$. Let us denote that positive root by Q_1.

Hence, $X^* \to Q_1$ when $Y^* \to 0$.

Further, when $X^* \to 0$, we get

$$a_{27}\left(Y^*\right)^4 + a_{28}\left(Y^*\right)^3 + a_{29}\left(Y^*\right)^2 + a_{30}Y^* + a_{31} = 0. \tag{3.5}$$

By Discarte's rules of sign, the equation (3.5) also has at least one positive root under condition $2bb'>1$. Let us denote that positive root by Q_2.

Hence $Y^* \to Q_2$ when $X^* \to 0$.

We can also obtain

$$\frac{dX^*}{dY^*} = \frac{\left(\dfrac{\alpha a_{33}}{a_{32}^{1/2}} + \dfrac{(1-\alpha)a_{35}}{a_{34}^{1/2}}\right)a_{14} - \left\{\left(\dfrac{a_{33}}{a_{32}^{1/2}} + \dfrac{a_{35}}{a_{34}^{1/2}}\right)ka_{12} + 2ka_{12} - (a_{14} + a_{13})\right\}}{a_{10} - \left\{\left(\dfrac{\alpha a_{33}}{a_{32}^{1/2}} + \dfrac{(1-\alpha)a_{35}}{a_{34}^{1/2}}\right)a_{10} + \dfrac{1}{2}\left(\dfrac{a_{11}}{a_{32}^{1/2}} + \dfrac{a_{16}}{a_{34}^{1/2}}\right)b'a_1\right.}.$$

$$\frac{dX^*}{dY^*} < 0 \text{, if } \left(\frac{\alpha a_{33}}{a_{32}^{1/2}} + \frac{(1-\alpha)a_{35}}{a_{34}^{1/2}}\right)a_{14} < \left\{\left(\frac{a_{33}}{a_{32}^{1/2}} + \frac{a_{35}}{a_{34}^{1/2}} + 2\right)ka_{12} - (a_{14} + a_{13})\right\},$$

$$\left(\frac{a_{33}}{a_{32}^{1/2}} + \frac{a_{35}}{a_{34}^{1/2}} + 2\right)ka_{12} > (a_{14} + a_{13})$$

and

$$0 < \left[\frac{\alpha a_{33}}{a_{32}^{1/2}} + \frac{(1-\alpha)a_{35}}{a_{34}^{1/2}} + \frac{1}{2}\left(\frac{a_{11}}{a_{32}^{1/2}} + \frac{a_{16}}{a_{34}^{1/2}}\right)b'\right] < 1$$

or if

$$\left(\frac{\alpha a_{33}}{a_{32}^{1/2}} + \frac{(1-\alpha)a_{35}}{a_{34}^{1/2}}\right)a_{14} > \left\{\left(\frac{a_{33}}{a_{32}^{1/2}} + \frac{a_{35}}{a_{34}^{1/2}} + 2\right)ka_{12} - (a_{14} + a_{13})\right\},$$

$$\left(\frac{a_{33}}{a_{32}^{1/2}} + \frac{a_{35}}{a_{34}^{1/2}} + 2\right)ka_{12} > (a_{14} + a_{13})$$

and

$$\left[\frac{\alpha a_{33}}{a_{32}^{1/2}} + \frac{(1-\alpha)a_{35}}{a_{34}^{1/2}} + \frac{1}{2}\left(\frac{a_{11}}{a_{32}^{1/2}} + \frac{a_{16}}{a_{34}^{1/2}}\right)b'\right] > 1.$$

Hence, we find that X^* is decreasing function of Y^*. Form above, we see that the two isoclines given by (3.2) and (3.3) will intersect provided that $Q_1 > b_1$ and $\frac{b_1}{b_2} > Q_2$.

Hence, with the above considerations, equation (3.2) represents X^* as decreasing from b_1 and equation (3.3) represents X^* as decreasing from Q_1.

Therefore, two isoclines must intersect provided that intersection value $0 < X^* \leq Q_1$ and $0 < Y^* \leq b_1 / b_2$, then endemic equilibrium point E_1

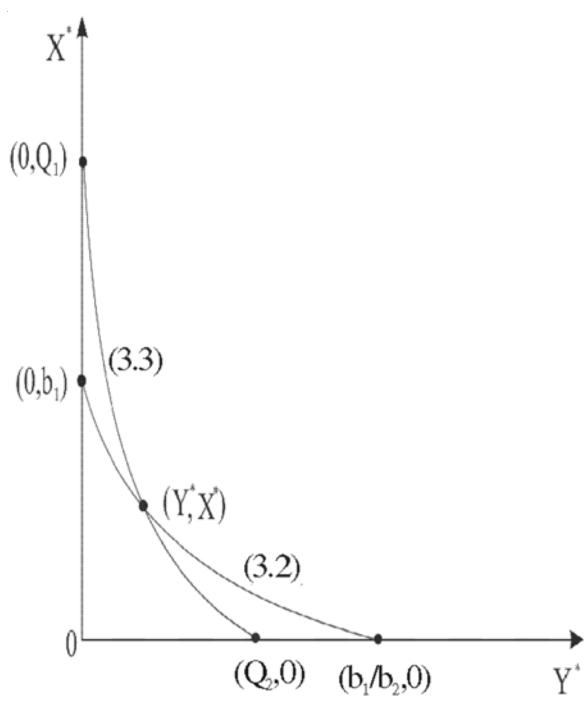


Fig-2

exists in the positive Y^*–X^* plane, shown in Fig. 2.

Note : The values of constants a_i (i=1,2,...,35) and b_i (i=1,2) are given in the Appendix.

4. Qualitative Analysis. Now to determine local and global dynamics of steady states E_0 and E_1 of system (2.1) we use, Lyapunov's Second Method. We analyze local dynamic of above steady states (E_0 and E_1) and therefore find sufficient conditions under which E_0 and E_1 are locally asymptotically stable in the form of the following **Theorems** 4.1 and 4.2 respectively:

Theorem 4.1. Let the following inequalities hold

$$b' > \left(kS_1^* + \alpha m'\right)/2 \tag{4.1a}$$

$$b > \left\{kS_2^* + (1-\alpha)m\right\} \tag{4.1b}$$

$$b' > \left(kS_2^* + (1-\alpha)m'\right)/2 \tag{4.1c}$$

$$m > \left(b + pb'\right) \tag{4.1d}$$

$$m' > qb'. \tag{4.1e}$$

Then E_0 is locally asymptotically stable.

Proof. Using the following positive definite function in the linearized form of system (2.1),

$$V_1 = \left[n_1^2 + A_1 n_2^2 + A_2 n_3^2 + A_3 n_4^2 + A_4 n_5^2 + A_5 n_6^2\right]/2, \tag{4.2}$$

Where, A_i are arbitrary positive constants (i=1,2,...,6), a it can be checked that the derivative of V_1 with respect to t under the conditions (4.1) is negative definite. Hence in view of theory of stability, E_0 is locally asympotically stable.

Theorem 4.2. Steady state E_1 of the system (2.1) is locally asymptotically stable if

$$b' > \left(kS_1^* + kI_2^* + \alpha m'\right)/2, \tag{4.3a}$$

$$\left(kI_1^* + b\right) > \left[kS_2^* + (1-\alpha)m\right]/2, \tag{4.3b}$$

$$b' > \left(kS_2^* + kI_1^* + (1-\alpha)m'\right)/2, \tag{4.3c}$$

$$m > \left(b + pb'\right), \tag{4.3d}$$

$$m' > qb', \tag{4.3e}$$

and satisfied

Proof. Similar to Theorem 4.1.

Now to show that steady states E_0 and E_1 of system (2.1) are globally asymptotically stable, we consider a region of attraction for the system (2.1) in the form of following

Lemma 1. Consider the set

$$R_1 = \{(S_1, I_1, S_2, I_2, X, Y) : 0 < S_{1m} \le S_1 \le P, 0 < I_{1m} \le I_1 \le P, 0 < S_{2m} \le S_2 \le P,$$

$$0 < I_{2m} \le I_2 \le P, 0 \le x, 0 \le Y$$

Which is a region of attraction for all solutions initially in the positive orthant, where $S_{1m} = S_{2m} = I_{1m} = I_{2m}$ are positive constants.

Now we obtain here the conditions for asymptotic stabiity of positive steady states E_0 and E_1 of system (2.1) in non-linear (global) case in the form of following

Theorem 4.3 . The steady state E_0 of system (2.1) is non-linearly (globally) asmptotically stable in a region R_1 given by Lemma 1 where the following conditions are satisfied :

$$0 < B_4 < 2, \tag{4.5a}$$

$$b' > (kP + \alpha m')/2, \tag{4.5b}$$

$$b > \{kS_{2m} + (1-\alpha)m\}/2, \tag{4.5c}$$

$$b' > \{kP + (1-\alpha)m'\}/2, \tag{4.5d}$$

$$m > (b + pb'), \tag{4.5e}$$

$$m' > qb', \tag{4.5f}$$

where B_4 is arbitrary positive constant.

Proof. Consider the following positive definite function about E_0

$$V_2 = [u_1^2 + B_1u_2^2 + B_2u_3^2 + B_3u_4^2 + B_4u_5^2 + B_5u_6^2]/2. \tag{4.6}$$

Here

B_i are arbitrary positive constants (i=1,2,...,6).

Take perturbations in $E_0(S_1^*, I_1^*, S_2^*, I_2^*, X^*, Y^*)$ as $u_1(t), u_2(t), u_3(t), u_4(t), u_5(t)$ and $u_6(t)$ respectively and putting

$$S_1 = S_1^* + u_1(t), I_1 = I_1^* + u_2(t),$$

$$S_2 = S_2^* + u_3(t), I_2 = I_2^* + u_4(t),$$

$X = X^* + u_5(t), Y = Y^* + u_6(t)$,

in the system (2.1), we get

$$\frac{du_1}{dt} = -bu_1 - k\left(S_1^* + u_1\right)u_4 + \alpha m u_5\,, \tag{4.7a}$$

$$\frac{du_2}{dt} = -b'u_2 + k\left(S_1^* + u_1\right)u_4 + \alpha m' u_6\,, \tag{4.7b}$$

$$\frac{du_3}{dt} = -k\left(S_2^* + u_3\right)u_2 - bu_3 + (1-\alpha) m u_5\,, \tag{4.7c}$$

$$\frac{du_4}{dt} = k\left(S_2^* + u_3\right)u_2 - b'u_4 + (1-\alpha) m' u_6\,, \tag{4.7d}$$

$$\frac{du_5}{dt} = bu_1 + pb'u_2 + bu_3 + pb'u_4 - mu_5\,, \tag{4.7e}$$

$$\frac{du_6}{dt} = qb'u_2 + qb'u_4 - m'u_6\,. \tag{4.7f}$$

Differentiating V_2 with respect to t along the solution of (2.1) and using Lemma 1, we get

$$\begin{aligned}\frac{dV_2}{dt} = & -bu_1^2 - b'B_1u_1^2 - bB_2u_3^2 - b'B_3u_4^2 - mB_4u_5^2 - m'B_5u_6^2 - kS_{1m}u_1u_4 + \left(\alpha m + bB_4\right)u_1u_5 \\ & + kPB_1u_2u_4 + \alpha m'B_1u_2u_6 - kS_{2m}B_2u_2u_3 + \left[(1-\alpha)mB_2 + bB_4\right]u_3u_5 + kPB_3u_2u_4 \\ & + (1-\alpha)m'B_3u_4u_6 + pb'B_4u_2u_5 + pb'B_4u_4u_5 + qb'B_5u_2u_6 + qb'B_5u_4u_6\ .\end{aligned} \tag{4.8}$$

Applying the inequality $\pm ab \le \left(a^2 + b^2\right)/2$ and making algebraic manipulation, we derive

$$\frac{dV_2}{dt} \le -\left[B_{11}u_1^2 + B_{12}u_2^2 + B_{13}u_3^2 + B_{14}u_4^2 + B_{15}u_5^2 + B_{16}u_6^2\right], \tag{4.9}$$

where

$B_{11} = b - \left(kS_{1m} + \alpha m + bB_4\right)/2,$

$B_{12} = b'B_1 - \left(kPB_1 + kS_{2m}B_2 + \alpha m'B_1 + kPB_3 + pb'B_4 + qb'B_5\right)/2\,,$

$B_{13} = bB_2 - \left[kS_{2m}B_2 + (1-\alpha)mB_2 + bB_4\right]/2\,,$

$$B_{14} = b'B_3 - \left[kS_{1m} + kPB_1 + kPB_3 + (1-\alpha)m'B_3 + qb'B_5 + pb'B_4\right]/2,$$

$$B_{15} = mB_4 - \left\{\alpha m + (1-\alpha)mB_2 + 2(b+pb')B_4\right\}/2,$$

$$B_{16} = m'B_5 - \left[\alpha m'B_1 + (1-\alpha)m'B_3 + 2qb'B_5\right]/2.$$

From (4.9), it can be shown that, $\frac{dV_2}{dt}$ is negative definite under following conditions

$$b > \left(kS_{1m} + \alpha m + bB_4\right)/2, \tag{4.10a}$$

$$b'B_1 > \left[kPB_1 + kS_{2m}B_2 + kPB_3 + \alpha m'B_1 + qb'B_5 + pb'B\right], \tag{4.10b}$$

$$bB_2 > \left[kS_{2m}B_2 + (1-\alpha)mB_2 + bB_4\right]/2, \tag{4.10c}$$

$$b'B_3 > \left[kS_{1m} + kPB_1 + kPB_3 + (1-\alpha)m'B_3 + qb'B_5 + pb'B_4\right]/2, \tag{4.10d}$$

$$mB_4 > \left[\alpha m + 2bB_4 + 2pb'B_4 + (1-\alpha)mB_2\right]/2, \tag{4.10e}$$

$$m'B_5 > \left\{\alpha m'B_1 + (1-\alpha)m'B_3 + 2qb'B_5\right\}/2. \tag{4.10f}$$

The above sufficient conditions for $\frac{dV_2}{dt}$ to be negative dfinite, may be further manipulated to get the following simplified conditions:

In (4.10a) choosing B_4 as

$$0 < B_4 < 2, \qquad \text{...(4.11a)}$$

and in (4.10b) choosing B_1 as

$$B_1 > \frac{kS_{2m}B_2 + kPB_3 + pb'B_4 + qb'B_5}{b' - (kP + \alpha m')/2},$$

the condition (4.10b) reduces to

$$b' > (kP + \alpha m')/2. \tag{4.11b}$$

In (4.10c) choosing B_2 as

$$B_2 > \frac{(bB_4)/2}{b - \left\{kS_{2m} + (1-\alpha)m\right\}/2},$$

the condition (4.10c) reduces to

$$b > \left\{kS_{2m} + (1-\alpha)m\right\}/2. \tag{4.11c}$$

In (4.10d) choosing B_3 as

$$B_3 > \frac{\left(kS_{1m} + kPB_1 + pb'B_4 + qb'B_5\right)}{b' - \left(kP + (1-\alpha)m'\right)/2},$$

the condition (4.10d) reduces to

$$b' > \left(kP + (1-\alpha)m'\right)/2 \ . \tag{4.11d}$$

In (4.10e) choosing B_4 as

$$B_4 > \frac{\left(\alpha m + (1-\alpha)mB_2\right)/2}{m - \left(b + pb'\right)} \ ,$$

the condition (4.10e) reduces to

$$m > \left(b + pb'\right). \tag{4.11e}$$

In (4.10f) choosing B_5 as

$$B_5 > \frac{\left(\alpha m' B_1 + (1-\alpha)B_3\right)/2}{2m' - 2qb'},$$

the condition (4.10f) reduces to

$$m' > qb'. \tag{4.11f}$$

Hence, disease free steady state E_0 of system (2.1) is non-linearly (Globally) asymptotically stable in the region R_1 under the conditions given by (4.11), provig Theorem 4.3.

Similarly we also determine that steady state E_1 of system (2.1) is also globally asymptotically stable in a region R_1 given by Lemma 1 under the following conditions :

$$kI_{2m} + b > \left(\alpha m + kPD_1 + kS_1^* + bD_4\right)/2\,, \tag{4.12a}$$

$$b' > \left(kS_2^* + kP + \alpha m'\right)/2\,, \tag{4.12b}$$

$$\left(kI_1^* + b\right) > \left(kS_{2m} + (1-\alpha)m\right)/2\,, \tag{4.12c}$$

$$b' > \left\{kI_1^* + kp + (1-\alpha)m'\right\}/2\,, \tag{4.12d}$$

$$m > \left(b + pb'\right), \tag{4.12e}$$

$$m' > qb', \tag{4.12f}$$

where D_1 and D_4 are arbitrary positive constants.

5. Conclusion. In this model, disease free and endemic steady states, have been obtained, which are shown to be both linearly (locally) and non-linearly (globally) asymptotically stable under the conditions involving disease related parameters.

From the qualitative analysis of the disease free steady state E_0, it may be concluded that the infection will die out eventually in the underlying population and only mature and immature population of susceptible males and females will exist. From the qualitative analysis of the endemic steady state E_1, it may be concluded that the infection will remain always in the population provided the following disease related parameters satisfy

$\alpha\left(mX^* + m'Y^*\right) > bS_1^*, (1-\alpha)\left(mX^* + m'Y^*\right) > bS_2^*, 2bb' > 1, m > (b + pb')$ and $m' > qb'$.

APPENDIX

$$a_1 = kb,\ a_2 = \alpha a_{10}X^* + (\alpha a_{14} - ka_{12})Y^* - a_{15},\ a_3 = \alpha\left(a_6X^* + a_7Y^*\right),$$

$$a_4 = (1-\alpha)a_{10}X^* + \left[(1-\alpha)a_{14} - ka_{12}\right]Y^* - a_{15},\ a_5 = (1-\alpha)\left(a_6X^* + a_7Y^*\right)$$

$$a_6 = mb', a_7 = m', a_8 = mX^* + m'Y^*, a_{10} = km, a_{11} = 4b\alpha, a_{12} = \frac{m'}{q}, a_{13} = 4kpb', a_{14}km',$$

$$a_{15} = bb', a_{16} = 4b(1-\alpha), a_{17} = a_{14} + a_{13} - 2ka_{12}, a_{18} = (1-\alpha)a_{14} - ka_{12}, a_{19} = \alpha a_{14} - ka_{12},$$

$$a_{20} = a_{14}a_{16} - 2(1-\alpha)a_{14}a_{15} + 2ka_{12}a_{15}, a_{21} = a_{14}a_{11} - 2\alpha a_{14}a_{15} + 2ka_{12}a_{15},$$

$$a_{22} = \left[\left\{\alpha^4 + (1-\alpha)^4 - 2\alpha^2(1-\alpha)^2\right\}a_{10}^4 - 2\left\{\alpha^2 + \left(1-\alpha^2\right)\right\}a_{10}^3 + a_{10}^2\right],$$

$$a_{23} = \left[\alpha^3 + (1-\alpha)^3 - \left\{\alpha(1-\alpha) + 5/2\right\}\right]4a_{10}^3a_{15} + \left\{\alpha^2 + (1-\alpha)^2\right\}a_{10}^2 + 8a_{10}a_{15},$$

$$a_{24} = \left\{\alpha^4 + (1-\alpha)^4\right\}a_{10}^4 - 4\left[2\left\{\alpha^2 + (1-\alpha)^2\right\} - 5\right]a_{10}^2a_{15},$$

$$a_{25} = 2\left\{\alpha^2 + (1-\alpha)^2\right\}a_{10}^2a_{15}^2 - 60a_{10}a_{15}^3, a_{26} = 4a_{15}^2\left(1 - 4a_{15}^2\right),$$

$$a_{27} = a_{17}^4 + a_{18}^4 + a_{19}^4 + 2a_{18}^2a_{19}^2 - 2a_{17}^4\left(a_{18}^2 + a_{19}^2\right),$$

$$a_{28} = 2a_{18}^2a_{20} + 2a_{19}^2a_{21} - 8a_{17}^2a_{15} - 2\left[a_{19}^2a_{20} + a_{18}^2a_{21} + a_{17}^2\left(a_{20} + a_{21}\right) - 4a_{17}a_{15}\left(a_{18}^2 + a_{19}^2\right)\right]$$

$$a_{29} = a_{18}^4 + a_{19}^4 + 2a_{18}^2a_{15}^2 + 2a_{19}^2a_{15}^2 + 24a_{17}^2a_{15}^2 - 2\left[a_{19}^2a_{15}^2 + a_{20}a_{21} + a_{15}^2a_{18}^2 + 4a_{15}^2\left(a_{18}^2 + a_{19}^2\right)\right.$$

$$\left. -4a_{17}a_{15}\left(a_{20} + a_{21}\right) - 2a_{17}^2a_{15}^2\right],$$

$$a_{30} = 2a_{18}^2 a_{15}^2 + 2a_{19}^2 a_{15}^2 - 32a_{17}a_{15}^3 - 2\left[a_{21}a_{15}^2 + a_{20}a_{15}^2 + 4a_{15}^2\left(a_{20} + a_{21}\right) - 8a_{17}a_{15}^3\right],$$

$$a_{31} = 4a_{15}^2\left(1 - 4a_{15}^2\right), a_{32} = a_{33}^2 + b' a_{10}a_{11}X^*, \quad a_{33} = \alpha a_{10}X^* + \left(\alpha a_{14} - ka_{12}\right)Y^* - a_{15},$$

$$a_{34} = a_{35}^2 + b' a_{10}a_{16}X^*, a_{35} = \left(1-\alpha\right)a_{10}X^* + \left\{\left(1-\alpha\right)a_{14} - ka_{12}\right\}Y^* - a_{15},$$

$$b_1 = \frac{Pb}{m+b}, b_2 = \frac{\left(1 + qb'/m' - pb'/b\right)m'}{\left(m+b\right)qb'/b}.$$

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