Jñānābha, Vol. 41, 2011

SOLUTION OF QUADRATIC DIOPHANTINE EQUATIONS

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(Received : July 10, 2011*)*

ABSTRACT

Our aim is to solve the quadratic Diophantine equations $1161146329226x^2-y^2=\pm 1$. Two starting least positive integer solutions are obtained.

2000 Mathematics Subject Classification : Primary 11GXX, 14GXX; Secondary 11D09.

Keywords: Quadratic Diophantine equation, Quadratic irrational number, Period of Continued fraction, Convergent of continued fraction.

1. Introduction. In Diophantine equations only positive integer solutions are calculated. Solutions of quadratic Diophantine $x^2+19=7^n$ and $x^2+11=3^n$ are discussed by Devi [1]. Unique integer solution of the equation $x^2-13=3^n$ is discussed by Sharma, Singh and Harikishan [3]. Ternary cubic Diophantine equation is solved. Different patterns of non-zero positive integer solutions are obtained by Gopalan and Somnath [2]. Our aim is to find the positive integer solution of quadratic (in two variables) Diophantine equations.

2. Formation of the problem. Consider the quadratic Diophantine equations $Nx^2-y^2=\pm 1$, where N is a positive integer. Let $\sqrt{N}=x_0$ be the quadratic irrational number. The algorithm for $\sqrt{N}$ generates the infinite simple continued fraction of the form $[q_0,q_1,q_2,...,q_n,q_{n+1},...]$ as follows

$$\left.\begin{aligned} q_0&=[x_0],x_1=\frac{1}{x_0-q_0}\\ q_1&=[x_1],x_2=\frac{1}{x_1-q_1}\\ &\dots \qquad \dots.\\ q_n&=[x_n],x_{n+1}=\frac{1}{x_n-q_n}\end{aligned}\right\}$$

We do not have to calculate infinitely many $q_i's$. Since the quadratic irrational number is always periodic. So it is of the form $\left[q_0, q_1, q_2, ..., q_n, \overline{q_{n+1}, ..., q_{n+m}}\right]$. It is also found that $q_{n+m} = 2q_0$. Number of terms *m*, from q_{n+1} to q_{n+m} is called period of continued fraction. If the period is odd both the equations have positive integer solution.

We denote the n^{th} convergent $C_n = P_n / Q_n$, therefore

$$\left.\begin{aligned} C_0 &= \frac{P_0}{Q_0} = \frac{q_0}{1}; \\ C_1 &= \frac{P_1}{Q_1} = \frac{q_0 q_1 + 1}{q_1}; \\ C_n &= \frac{P_n}{Q_n} = \frac{q_n P_{n-1} + P_{n-2}}{q_n Q_{n-1} + Q_{n-2}}, for\ n \geq 2 \end{aligned}\right\} \tag{2.2}$$

The solutions of the equations $Nx^2 - y^2 = \pm 1$ are

$x = Q_1, y = P_i$ for $i = 0,2,4,...$ and

$x = Q_i, y = P_i$ for $i = 1,3,5,...$. respectively.

3. Results and Discussion.

$\sqrt{1161146329226} = \left[1077565, \overline{2155130}\right]$.

In this case, period of continued fraction is one, which is odd. Therefore both the equations can be solved.

From (2.1) and (2.2), we get

$P_0 = 1077565, Q_0 = 1, P_1 = 2322292658451, Q_1 = 2155130, P_2 = 5004842577008581195,$

$Q_2 = 4644585316901, P_3 = 10786086382990825883438801, Q_3 = 10009688515401500 7260$

Solutions of the equations $1161146329226x^2 - y^2 = \pm 1$ are

x	y
1	1077565
4644585316901	5004842577008581195

and

x	y
2155130	2322292658451
10009688515401500 7260	10786086382990825883438801

Only two starting least positive integer solutions are calculated.

4. Conclusion. There will be infinite number of solutions. In this problem only two solutions of each of the equation are calculated. One may find the solution of other Diophantine equation of the form $Nx^2 - y^2 = n,$ where n is a positive integer.

REFERENCES

[1] Usha Devi, On the Diophantine euqation $x^2+19=7^n$ and $x^2+11=3^n$. The *Mathematics Education,* **39**, 2 (2005), 84-86.

[2] M.A. Gopalan, Manju Somnath, Ternary Cubic Diophantine equation $2^{2\alpha-1}\left(x^2+y^2\right)=z^3$, *Acta Ciencia Indica*, **34**, **3** (2008), 1135-1137.

[3] P.K. Sharma, M.P. Singh and Harikishan, On the Diophantine euqation $x^2-13=3^n$. *Acta Ciencia Indica*, **34**, 3 (2008), 1043-1044.