Jñānābha, Vol. 11, 1981

THERMOHALINE CONVECTION THROUGH POROUS MEDIUM OF COMPRESSIBLE FLUIDS

by

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(*Received : December 15, 1980; Revised : May 20, 1981*)

ABSTRACT

The problem of thermohaline convection through porous Medium in a layer of viscous and compressible fluid heated from below and subjected to a stable solute gradient is studied. The instability problem in the presence of rotation and magnetic field is also studied. The system is found to be stable for $(C_p\, \beta/g) < 1$; C_p, β and g being specific heat at constant pressure, uniform adverse temperature gradient and acceleration due to gravity. For $(C_p\, \beta/g) > 1$, the compressibility, rotation, magnetic field and stable solute gradient are found to have stabilizing effects on the system. The oscillatory modes are introduced due to the presence of rotation, magnetic field and solute gradient which were not allowed for $(C_p\, \beta/g) > 1$ in their absence. The effect of medium permeability is destabilizing in the presence or absence of magnetic field whereas its effect is both to stabilize and destabilize the layer in the presence of rotation. the sufficient conditions for the nonexistence of overstability, in the presence or absence of magnetic field and rotation on the problem, are also investigated.

1. Introduction

The theoretical and experimental results on the stability of a fluid layer heated from below, in the absence and presence of rotation and magnetic field, have been given in a treatise by Chandrasekhar [1]. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been investigated by Veronis [6]. Nield [4] has studied the thermohaline convection in a horizontal layer of viscous fluid heated from below and salted from above. In all the above studies the Boussinesq approximation, which is well justified in the case of incompressible fluids, has been used and the medium has been considered to be nonporous.

When the fluids are compressible. the equations governing the system become quite complicated. Jeffreys [2] suggested a formal justification for the applicability of the Boussinesq–Rayleigh equations for compressible fluids but restricted his discussion to the study of infinitesimal, steady motions. Spiegel and Veronis [5], under certain vaild assumptions, have shown that the equations governing convection in a perfect gas are formally equivalent to those for an incompressible fluid if the static temperature gradient is replaced by its excess over the adiabatic and C_v is replaced by C_p. The stability of flow of a single component fluid through porous medium taking into account the Darcy resistance has been studied by Lapwood [3] and Wooding [7]. The problem of stability of two–component fluid layer (or fluid layer in the presence of solute gradient) through porous medium subjected to a temperature gradient is of importance in geophysics, soil sciences, porous bearing system and ground water–hydrology etc. The effects of magnetic field and rotation on the stability of such a flow are of importance in geophysics particularly in the study of earth's core and in the extraction of energy in the geothermal regions. The conditions under which convective motions through porous medium are important in geophysics are usuallyfor removed from the consideration of single component fluid and rigid bounderies and therefore it is desirable to consider two–component fluid and free boundaries. In the present paper the stability of steady motions in a layer of compressible fluid through porous medium, which is heated from below and subjected to a stable solute gradient, is investigated. The thermohaline instability of a layer of compressible fluid through porous medium in the presence of rotation and magnetic field is also studied.

2. Formulation of the Problem and Perturbation Equations

Consider an infinite horizontal layer of compressible and viscous fluid of depth d flowing through a porous medium, which is heated from below and subjected to a stable solute gradient. Consider the cartesian coordinates (x, y, z) and the origin to be on the lower boundary $z=0$ with the axis of z perpendicular to it along the vertical. The gravity force $\mathbf{g}$ (0, 0, − g) pervades the system. This layer of fluid is heated from below so that the temperatures and solute concentrations at bottom surface z=0 are T_0 and C_0 and at the upper surface $z=d$ are T_d and C_d respectively. Spiegel and Veronis [5] represented f as any one of the state variables : density (ρ) , temperature (T) or pressure (p) and expressed these in the form

$$(1) \quad f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t),$$

where f_m is the constant space average of f ; f_0 is the variation in the absence of motion and f' is the fluctuation resulting from motion.

Let $\delta\rho$, δp, $\mathbf{v}$ (u, v, w), θ and γ denote respectively the perturbations in density ρ, pressure p, velocity(which is zero initially),temperature T and solute mass concentration C; μ, ν ($= \mu/\rho_m$), k, κ ($= k/\rho_m C_p$), κ', k_1, α, α' and g/C_p stand for viscosity, kinematic viscosity, thermal conductivity, thermal diffusivity, analogous solute diffusivity, permeability of the medium (which has the dimension of length squared), thermal coefficient of expansion, analogous solvent coefficient of expansion and adiabatic gradient respectively.

β [$= (T_d - T_0)/d$] and β' [$= (C_d - C_0)/d$] stand for the magnitudes of uniform adverse temperature and concentration gradients. Then the linearized perturbation equations of motion, continuity, heat conduction and solute, following spiegel and Veronis [5], when both Darcy as well as viscous resistances are present, are

(2) $\dfrac{\partial \mathbf{v}}{\partial t} = -\left(\dfrac{1}{\rho_m}\right)\nabla\, \delta p + \nu\, \nabla^2 \mathbf{v} - \left(\dfrac{\nu}{k_1}\right)\mathbf{v} + \mathbf{g}\left(\dfrac{\delta\rho}{\rho_m}\right),$

(3) $\nabla . \mathbf{v} = 0\,,$

(4) $\dfrac{\partial \theta}{\partial t} = \left(\beta - \dfrac{g}{C_p}\right) w + \varkappa\, \nabla^2\, \theta\,,$

(5) $\dfrac{\partial \gamma}{\partial t} = \beta'\, w + \varkappa'\, \nabla^2\, \gamma\,,$

where

(6) $\rho = \rho_m\, [\, 1 - \alpha\, (T - T_m) + \alpha'\, (C - C_m)\,]$

The suffix m refers to values at the reference level $z = 0$ and the change in density $\delta\rho$, caused by the perturbations θ and γ in temperature and concentration, is given by

(7) $\delta\rho = -\rho_m\, (\alpha\theta = \alpha'\gamma).$

Consider the case in which both the boundaries are free as well as perfect conductors of both heat and solute. Then the boundary conditions appropriate to the problem are (Chandrasekhar [1], Lapwood [3], Veronis [6]) :

(8) $w = \dfrac{\partial^2 w}{\partial z^2} = \theta = \gamma = 0 \quad$ at $z = 0$ and $z = d.$

Equations (2) – (5) with the help of equation (7) give

(9) $\left(\dfrac{\partial}{\partial t} + \dfrac{\nu}{k_1} - \nu\nabla^2\right)\nabla^2 w = g\alpha\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)\theta - g\alpha'\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)\gamma,$

$$(10)\quad \left(\frac{\partial}{\partial t}-\varkappa\nabla^2\right)\theta=\left(\beta-\frac{g}{C_p}\right)w,$$

$$(11)\quad \left(\frac{\partial}{\partial t}-\varkappa'\nabla^2\right)\gamma=\beta' w.$$

3. Dispersion Relation and Discussion

Analyzing the disturbances in terms of normal modes, We assume that the perturbation quantities are of the form

$$(12)\quad [w, \theta, \gamma, \zeta, h_z]=[W(z),\Theta(z),\Gamma(z),Z(z),K(z)]\, exp\,(ik_x x+ik_y y+nt),$$

where n is the growth rate, k_x and k_y are the wave numbers in the x and y directions respectively and $k=(k_x^2+k_y^2)^{1/2}$ is the resultant wave number. $\zeta\left(=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$ denotes the z— component of vorticity and h_z stands for the component of the perturbation in magnetic field. Using expression (12), equations (9) – (11) in non - dimensional form become

$$(13)\quad (D^2-a^2)\,(D^2-a^2-\sigma-\frac{d^2}{k_1})w-\frac{gd^2}{\nu}\,a^2(\alpha\Theta-\alpha'\Gamma)=0,$$

$$(14)\quad (D^2-a^2-p_1\sigma)\Theta=-\frac{d^2}{\varkappa}\;\frac{g}{C_p}\;(G-1)\,w,$$

$$(15)\quad (D^2-a^2-q\,\sigma)\,\Gamma=-\frac{\beta' d^2}{\varkappa'}\;w,$$

where $p_1=\nu/\varkappa$ is the prandtl number, $q=\nu/\varkappa'$ is the schmidt number, $a=kd$, $\sigma=nd^2/\nu$ and $G=C_p\,\beta/g$. We have also put the coordinates x, y, z in the new unit of length d and $D=d/d_z$. The boundary conditions transform to

(16) $W = D^2W = \text{Ⓗ} = \Gamma = 0$ at $z=0$ and 1.

Using the boundary conditions (16), it can be shown with the help of equations (13)–(15) that all the even order derivatives of W vanish at the boundaries. Hence the proper solution of equation in W, obtained by eliminating Ⓗ, Γ from equations (13) – (15), characterizing the lowest mode is

(17) $W = A \sin \pi z$,

where A is constant. Substituting (17) in the resulting equation in W, we obtain

$$(18)\quad R_1 = \left(\frac{G}{G-1}\right)\left\{\frac{(1+x)\left(1+x+\frac{\sigma}{\pi^2}+\frac{1}{p}\right)\left(1+x+\flat_1\frac{\sigma}{\pi^2}\right)}{x} + S_1\frac{\left(1+x+\flat_1\frac{\sigma}{\pi^2}\right)}{\left(1+x+q\frac{\sigma}{\pi^2}\right)}\right\},$$

where $x = a^2/\pi^2$, $R_1 = g\alpha\beta d^4/\nu\kappa\pi^4$, $S_1 = g\alpha'\beta' d^4/\nu\kappa'\pi^4$ and $P = \pi^2 k_1/d^2$. For the stationary convection $\sigma=0$ and equation (18) reduces to

$$(19)\quad R_1 = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{x} + \frac{(1+x)^2}{xp} + S_1\right].$$

Observing a perfect analogy between equation (19) and Veronis [6] (cf. equation (22)), we find that

$$(20)\quad \overline{R}_c = \left(\frac{G}{G-1}\right) R_c',$$

where R_c' and $\overline{R}_c$ stand respectively for the critical Rayleigh numbers in the absence and presence of compressibility and the nondimensional numbers G, P and S_1 accounting for the compressibility, medium Permeability and solute gradient effects be kept as fixed. It is evident from equation (20) that the effect of compressibility is to postpone the onset of thermal instability. Hence we obtain a stabilizing effect of compressibility. The cases $G < 1$ and $G = 1$ correspond to negative and infinite values of critical Rayleigh numbers in the presence of compressibility which are not relevant in the present study. Equation (19) yields

$$(21) \quad \frac{dR_1}{dP} = -\left(\frac{G}{G-1}\right)\frac{(1+x)^2}{x\,P^2} \quad ,$$

and

$$(22) \quad \frac{dR_1}{d\,S_1} = \left(\frac{G}{G-1}\right) \quad ,$$

which imply that medium permeability and stable solute gtradient have destabilizing and stabilizing effects respectively for $G > 1$. Multiplying equation (13) by W^*, the complex conjugate of W, integrating over the range of z, and making use of equations (14) and (15), we obtain

$$(23) \quad I_1 + \sigma I_2 - \frac{C_p \alpha \varkappa a^2}{\nu(G-1)}\,(I_3 + p_1 \sigma^* I_4) + \frac{g\alpha' \varkappa' a^2}{\nu\beta'}\,(I_5 + q\sigma^* I_6) = 0,$$

where

$$(24) \quad I_1 = \int_0^1 \left(|D^2W|^2 + \left(2a^2 + \frac{d^2}{k_1}\right)|DW|^2 + \left(a^4 + \frac{a^2 d^2}{k_1}\right)|W|^2\right)dz,$$

$$I_2 = \int_0^1 \left(|DW|^2 + a^2\,|W|^2\right)dz,$$

$$I_3 = \int_0^1 \left(|D\Theta|^2 + a^2\,|\Theta|^2\right)dz,$$

$$I_4 = \int_0^1 |\mathfrak{B}|^2 \, dz,$$

$$I_5 = \int_0^1 (|D\Gamma|^2 + a^2 |\Gamma|^2) \, dz, \quad I_6 = \int_0^1 |\Gamma|^2 \, dz,$$

which are all positive definite.

Putting $\sigma = \sigma_r + i\sigma_i$ and then equating real and imaginary parts of equation (23), we obtain

$$(25) \quad \sigma_r \left[I_2 + \frac{C_p \alpha \varkappa a^2}{\nu(1-G)} \, \mathfrak{b}_1 I_4 + \frac{g \alpha' \varkappa' a^2}{\nu \beta'} \, q I_6 \right]$$

$$= -\left[I_1 + \frac{C_p \alpha \varkappa a^2}{\nu(1-G)} \, I_3 + \frac{g \alpha' \varkappa' a^2}{\nu \beta'} \, I_5 \right],$$

and

$$(26) \quad i\sigma_i \left[I_2 + \frac{C_p \alpha \varkappa a^2}{\nu(G-1)} \, \mathfrak{b}_1 I_4 - \frac{g \alpha' \varkappa' a^2}{\nu \beta'} \, q \, I_6 \right] = 0.$$

It follows from equation (25) that σ_r is negative if $G < 1$. The system is therefore stable for $G < 1$. Equation (26) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that the modes may be nonoscillatory or oscillatory. In the absence of solute gradient, equation (26) reduces to

$$(27) \quad \sigma_i \left[I_2 + \frac{C_p \alpha \varkappa a^2}{\nu(G-1)} \, \mathfrak{b}_1 I_4 \right] = 0,$$

and the term is brackets is positive definite if $G > 1$. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed in the absence of solute gradient if $G > 1$. The presence of solute gradient, thus, introduces oscillatory modes in thermohaline convection of compressible fluids, for $C_p \beta / g > 1$.

For the case of overstability put $\sigma/\pi^2 = i\sigma_1$ where σ_1 is real in equation

(18) and equate real and imaginary parts of the resulting equation. Solving the imaginary part for σ_1^2, we obtain.

$$(28)\quad \sigma_1^2 = -\frac{(1+x)^3(1+\mathfrak{b}_1)+(1+x)^2\frac{\mathfrak{b}_1}{P}+S_1x(\mathfrak{b}_1-q)}{q^2\left[(1+x)(1+\mathfrak{b}_1)+\frac{\mathfrak{b}_1}{P}\right]}.$$

Since σ_1 is real, σ_1^2 must be positive for overstability. This is impossible if $\mathfrak{b}_1 > q$. Hence $\mathfrak{b}_1 > q$ or $x < x'$ is a sufficient condition for the nonexistence of overstability, the violation of which does not necessarily imply occurrence of overstability.

4. Effect of Rotation

In this section, the problem and the configuration is considered to be the same except that the fluid is now also acted on by a uniform rotation $\vec{\Omega}\,(0, 0, \Omega)$. Then the linearized perturbation equations of momentum become

$$(29)\quad \frac{\partial \mathbf{v}}{\partial t} = -\left(\frac{1}{\rho_m}\right)\nabla\,\delta p + \nu\,\nabla^2\mathbf{v} - \left(\frac{\nu}{k_1}\right)\mathbf{v} + \mathbf{g}\left(\frac{\delta\rho}{\rho_m}\right) + 2\,(\mathbf{v}\times\vec{\Omega}),$$

and the boundary conditions in addition to (8) are

$$(30)\quad \frac{\partial\zeta}{\partial z} = 0 \text{ at } z=0 \text{ and } d.$$

The appropriate nondimensional forms of equations become

$$(31)\quad (D^2-a^2)\,(D^2-a^2-\sigma-\frac{d^2}{k_1})W-\frac{g\alpha d^2}{\nu}a^2\Theta+\frac{g\alpha' d^2}{\nu}a^2\Gamma-\frac{2\Omega d^3}{\nu}D\zeta=0,$$

$$(32)\quad (D^2-a^2-\sigma)\,\zeta = -\left(\frac{2\Omega d}{\nu}\right)DW,$$

together with equations (14) and (15). Eliminating z, Θ and Γ from equations (14), (15), (31) and (32) and substituting in the resulting equation, the proper solution characterizing the lowest mode (17), we get the dispersion relation

$$(33)\quad R_1=\left(\frac{G}{G-1}\right)\left\{\frac{(1+x)(1+x+\frac{\sigma}{\pi^2}+\frac{1}{P})\,(1+x+b_1\frac{\sigma}{\pi^2})}{x}\right.$$

$$\left.+S_1\frac{(1+x+b_1\frac{\sigma}{\pi^2})}{(1+x+q\,\frac{\sigma}{\pi^2})}+\frac{T_1(1+x+b_1\frac{\sigma}{\pi^2})}{x(1+x+\frac{\sigma}{\pi^2}+\frac{1}{P})}\right\},$$

where $\quad T_1 = 4\Omega^2\, d^4/\pi^4\, \nu^2.$

For the stationary convection $\sigma=0$ and equation (33) reduces to

$$(34)\quad R_1=\left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{x}+\frac{(1+x)^2}{xP}+S_1+\frac{T_1(1+x)}{x(1+x+\frac{1}{P})}\right].$$

If the nondimensional numbers G, P, T_1 and S_1 accounting for the compressibility, the medium permeability, the rotation and the stable solute gradient effects be kept as fixed, then we find that

$$(35)\quad \overline{\overline{R}}_c=\left(\frac{G}{G-1}\right)R_c'',$$

where R_c'' and $\overline{\overline{R}}_c$ stand for the critical Rayleigh numbers in the absence and presence of compressibility. Thus we obtain the stabilizing effect of compressibility as its effect is to postpone the onset of thermohaline instability. To find the roles of rotation, stable solute gradient and medium permeability, we examine the natures of $\frac{dR_1}{dT_1}$, $\frac{dR_1}{dS_1}$ and $\frac{dR_1}{dP}$. It is clear from equation (33) that the rotation as well as the stable solute gradient have the stabilizing effects on the system, for the relevent case $G > 1$. It is evident from equation (33) that the medium permeability has stabilizing effect if $T_1 > (1+x)(1+x+1/p)^2$ and destabilizing effect if $T_1 < (1+x)(1+x+1/p)^2$. Thus the effect of medium permeability was destabilizing in the absence of rotation but its effect is stabilizing as well as destabilizing in the presence of rotation.

Multiplying equation (31) by W^*, the complex conjugate of W, integrating over the range of z and making use of equations (14), (15) and (32), we obtain

$$(36)\quad I_1+\sigma I_2-\frac{C_p\alpha\varkappa a^2}{\nu(G-1)}(I_3+\mathfrak{b}_1\sigma^* I_4)+\frac{g\alpha'\varkappa' a^2}{\nu\beta'}(I_5+q\sigma^* I_6) + d^2(I_7+\sigma^* I_8)=0,$$

where $I_1 - I_6$ are given in equation (24) and

$$(37)\quad \left.\begin{aligned} I_7 &= \int_0^1 (|Dz|^2+a^2|z|^2)\,dz, \\ I_8 &= \int_0^1 (|z|^2\,dz, \end{aligned}\right\}$$

which are all positive. Putting $\sigma = \sigma_r + i\sigma_i$ and equating real and imaginary parts of equation (36), we obtain

$$(38)\quad \sigma_r\left[I_2+\frac{C_p\alpha\varkappa a^2}{\nu(1-G)}\mathfrak{b}_1 I_4+\frac{g\alpha'\varkappa' a^2}{\nu\beta'}qI_6+d^2 I_8\right] = -\left[I_1+\frac{C_p\alpha\varkappa a^2}{\nu(1-G)}I_3+\frac{g\alpha'\varkappa' a^2}{\nu\beta'}I_5+d^2 I_7\right],$$

and

$$(39)\quad \sigma_i\left[I_2+\frac{C_p\alpha\varkappa a^2}{\nu(G-1)}\mathfrak{b}_1 I_4-\frac{g\alpha'\varkappa' a^2}{\nu\beta'}q I_6-d^2 I_8\right]=0.$$

It is evident from equation (38) that σ_r is negative if $G < 1$, meaning thereby that the system is stable. Equation (39) yields that the oscillatory modes are introduced due to the presence of rotation and stable solute gradient in thermohaline convection of compressible fluids, for $C_p \beta / g > 1$, which were completely missing in their absence. For the case of overstability put $\sigma/\pi^2 = i\sigma_1$, where σ_1 is real, in equation (33). The imaginary part of the resulting equation implies that σ_1^2 is negative if $\mathfrak{p}_1 > 1$ and $\mathfrak{p}_1 > q$. This is impossible since σ_1^2 must be positive. Thus $\mathfrak{p}_1 > 1$ and $\mathfrak{p}_1 > q$ or $\varkappa < \nu$ and $\varkappa < \varkappa'$ are the sufficient conditions for the nonexistence of overstability, the violation of which do not necessarily imply occurrence of overstability.

5. Effect of Magnetic Field

Here we consider an infinite horizontal, compressible, viscous and finitely conducting fluid of depth d flowing through porous medium and which is heated from below and subjected to a stable solute gradient. The uniform magnetic field $\mathbf{H}$ (0, 0, H) and gravity force $\mathbf{g}$ (0, 0, – g) pervade the system. Then the linearized perturbation equations of motion and Maxwell's equations are

$$(40) \quad \frac{\partial \mathbf{v}}{\partial t} = -\left(\frac{1}{\rho_m}\right) \nabla \delta p + \nu \nabla^2 \mathbf{v} - \left(\frac{\nu}{k_1}\right) \mathbf{v} + \mathbf{g}\left(\frac{\delta\rho}{\rho_m}\right)$$

$$\left(\frac{\mu_e}{4\pi\rho_m}\right)(\triangle \times \mathbf{h}) \times \mathbf{H} ,$$

$$(41) \quad \triangle \cdot \mathbf{h} = 0,$$

$$(42) \quad \frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H}.\nabla)\mathbf{v} + \eta \nabla^2 \mathbf{h} ,$$

where μ_e, η and $\mathbf{h}$ (h_x, h_y, h_z) denote respectively the magnetic permeability, the resistivity and the perturbation in magnetic field $\mathbf{H}$. Equations (3) – (5) and (40) – (42), in non-dimensional from, with the help of equation (7), give

$$(43)\quad (D^2-a^2)\left(D^2-a^2-\sigma\frac{d^2}{k_1}\right)W+\left(\frac{\mu_e Hd}{4\pi\rho_m \nu}\right)(D^2-a^2)\,DK$$

$$-\frac{gd^2}{\nu}\,a^2\,(\alpha\Theta-\alpha\,\Gamma)=0,$$

$$(44)\quad (D^2-a^2-\flat_2\,\sigma)=-\left(\frac{Hd}{\eta}\right)DW,$$

together with equations (14) and (15) and $\flat_2=\nu/\eta$. Here the medium adjoining the fluid is also assumed to be perfect conductor of electricity. So the boundary conditions, in addition to (8), are

$$(45)\quad \frac{\partial h_z}{\partial z}=0.$$

Eliminating Θ, Γ and K from equations (14), (15), (43) and (44) and substituting in the resulting equation, the proper solution (17), we get

$$(46)\quad R_1=\left(\frac{G}{G-1}\right)\left[\frac{(1+x)(1+x+\flat_1\frac{\sigma}{\pi^2})(1+x+\frac{\sigma}{\pi^2}+\frac{1}{P})}{x}\right.$$

$$\left.+\frac{Q_1(1+x)(1+x+\flat_1\frac{\sigma}{\pi^2})}{x(1+x+\flat_2\frac{\sigma}{\pi^2})}+\frac{S_1(1+x+\flat_1\frac{\sigma}{\pi^2})}{(1+x+q\frac{\sigma}{\pi^2})}\right],$$

where $Q_1=\mu_e H^2 a^2/4\pi\rho\nu\eta\pi^2$. For the stationary convection $\sigma=0$ and equation (46) reduces to

$$(47)\quad R_1=\left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{x}+\frac{(1+x)^2}{xP}+Q_1\left(\frac{(1+x)}{x}\right)+S_1\right].$$

If the nondimensional numbers G, P, S_1 and Q_1 accounting for the compressibility. medium permeability, stable solute gradient and magnetic field effects be kept as fixed, then we find that

$$(48)\quad \overline{\overline{R}}_c=\left(\frac{G}{G-1}\right)R_c''' ,$$

where R_c''' and $\overline{\overline{R}}_c$ stand respectively for the critical Rayleigh numbers in the absence and presence of compressibility. Hence we obtain the stabilizing effect of compressibility. It is evident from equation (46) that for $G > 1$, the Rayleigh number increases with the increase in magnetic field as well as stable solute gradient parameters meaning thereby the stabilizing effects of magnetic field and stable solute gradient whereas the medium permeability has a destabilizing effect on the thermohaline convection. Multiplying equation (43) by W^*, integrating over the range of z and making use of equations (14), (15) and (44) we obtain

$$(49)\quad (I_1+\sigma I_2)+\frac{\mu_e\eta}{4\pi\rho_m\nu}(I_9+p_2\sigma^* I_{10})-\frac{C_p\alpha\varkappa a^2}{\nu(G-1)}(I_3+p_1\sigma^* I_4)$$

$$+\frac{g\alpha'\varkappa' a^2}{\nu\beta'}(I_5+q\sigma^* I_6)=0,$$

where I_1-I_6 are given in equations (24) and

$$(50)\quad I_9=\int_0^1\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)dz,$$

$$I_{10}=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$

which are positive definite. Put $\sigma = \sigma_r + i\sigma_i$ in equation (49) and then equate real and imaginary parts. The real part yields that σ_r is negative if $G < 1$ meaning thereby that the system is stable. The imaginary part yields that in the absence of magnetic field and solute gradient, the oscillatory modes are not allowed for $G > 1$ but in the presence of solute gradient and magnetic field, the overstable modes come in to play.

For the case of overstability put $\frac{\sigma}{\pi^2} = i\,\sigma_1$ where σ_1 is real in equation (46). The imaginary part of the resulting equation implies that σ_1^2 is negative if $\mathfrak{p}_1 > q$ and $\mathfrak{p}_1 > \mathfrak{p}_2$. This is impossible since σ_1^2 must be positive. Thus $\mathfrak{p}_1 > q$ and $\mathfrak{p}_1 > \mathfrak{p}_2$ or $\varkappa < \varkappa'$ and $\varkappa < \eta$ are the sufficient conditions for the nonexistence of overstability.

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