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# THE RAYLEIGH-TAYLOR INSTABILITY THROUGH POROUS MEDIUM OF TWO VISCOELASTIC SUPERPOSED CONDUCTING FLUIDS

*by*

**R. C. SHARMA AND K. N. SHARMA**
**Department of Mathematics, Himachal Pradesh University**
**Simla - 171005, India**

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## ABSTRACT

The instability of the plane interface between two viscoelastic superposed conducting fluids through porous medium is studied when the whole system is immersed in a uniform horizontal magnetic field. The medium permeability is considered to be large and the fluids are considered to be highly viscous and of equal kinematic viscosities, for mathematical simplicity. It is found that the stability criterion is independent of the effects of medium permeability, viscosity and viscoelasticity and is dependent on the orientation and magnitude of the magnetic field. The magnetic field is found to stabilize a certain wave number range of the unstable configuration. The growth rates both increase or decrease with the increase in medium permeability.

## 1. Introduction

A detailed account of the instability of the plane interface between two fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [2]. Bhatia [1] has studied the Rayleigh-Taylor instability of two viscous superposed conducting fluids in the presence of a uniform horizontal magnetic field. Sharma [5] has studied the instability of the plane interface between two viscoelastic (fluids obeying Oldroyd's constitutive equation) superposed conducting fluids in the presence of a uniform magnetic field. The medium has been considered to be non-porous in all the above studies. Lapwood [3] has studied the stability of convective flow in hydrodynamics in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [7]. When the fluid slowly percolates through the pores of the rock, the gross effect is represented by Darcy's law which states that the usual viscous term in the equations of fluid motion is replaced by the resistance term $(\mu/k_1)\,\mathbf{v}$, where $\mu$ is the viscosity of the fluid, $k_1$ is the permeability of the medium and $\mathbf{v}$ is the velocity of the fluid.

In the present paper we study the instability of the plane interface between two viscoelastic superposed conducting fluids through porous medium, when the whole system is immersed in a uniform horizontal magnetic field. The viscoelastic (Oldroyd) fluids explain the rheological behaviour of some polymer solutions at small rates of shear. The instability of such viscoelastic superposed conducting fluids through porous medium may find applications in geophysics. This aspect forms the subject matter of the present study wherein we have carried out the stability analysis, for large medium permeability and for two highly viscous fluids of equal kinematic viscosities.

## 2. Perturbation Equations

Assume that the viscoelastic fluid is described by the constitutive relations

$$(1)\qquad \begin{cases} T_{ij}' = -p\delta_{ij} + T_{ij}\,, \\ \left(1+\lambda\dfrac{d}{dt}\right)T_{ij} = 2\mu\left(1+\lambda_0\dfrac{d}{dt}\right)e_{ij}; \\ e_{ij} = \frac{1}{2}\left(\dfrac{\partial v_i}{\partial x_j}+\dfrac{\partial v_j}{\partial x_i}\right) \end{cases}$$

where $T_{ij}$, $T_{ij}'$, $e_{ij}$, $\mu$, $\lambda$, $\lambda_0$ $(<\lambda)$, $p$, $\delta_{ij}$, $\frac{d}{dt}$, $v_i$ and $x_i$ denote respectively the shear stress tensor, the stress tensor, the rete-of-strain tensor, the viscosity, the stress relaxation time, the strain retardation time, the isotropic pressure, the Kronecker delta, the mobile operator, the velocity vector and the position vector. Relations of the type (1) were first proposed by Jeffreys for earth and studied by Oldroyd [4]. Oldroyd [4] also showed that many rheological equations of state, of general validity, reduce to (1) when linearized.

Consider the motion of an incompressible, infinitely conducting, viscoelastic fluid through porous medium in the presence of a uniform magnetic field $\mathbf{H}$ $(H_x, H_y, 0)$. Let $\mathbf{v}$ $(u, v, w)$, $\mathbf{h}$ $(h_x, h_y, h_z)$, $\delta\rho$ and $\delta p$ denote the perturbations in velocity, magnetic field, density and pressure respectively. Then the linearized hydromagnetic perturbation equations of viscoelastic fluid through porous medium are

(2) $$\left(1+\lambda\frac{\partial}{\partial t}\right)\rho\frac{\partial \mathbf{v}}{\partial t}=\left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p+\mathbf{g}\,\delta\rho+\frac{1}{4\pi}(\nabla\times\mathbf{h})\times\mathbf{H}\right]$$

$$+\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left[\rho\nu\nabla^2\mathbf{v}-\frac{\rho\nu}{k_1}\mathbf{v}+\left(\frac{\partial w}{\partial \mathbf{x}}+\frac{\partial \mathbf{v}}{\partial z}\right)\frac{d\mu}{dz}\right],$$

(3) $$\nabla.\mathbf{v}=0,$$

(4) $$\nabla.\mathbf{h}=0,$$

(5) $$\frac{\partial \mathbf{h}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{H}),$$

(6) $$\frac{\partial}{\partial t}\delta\rho+(\mathbf{v}.\nabla)\rho=0,$$

where $\nu$ $(=\mu/\rho)$, $\mathbf{g}$ $(0, 0, -g)$ and $\bar{\mathbf{x}}$ $(x, y, z)$ denote the kinematic viscosity of the fluid, the acceleration due to gravity and the position vector respectively. Equation (6) ensures that the density of every particle remains unchanged as we follow it with its motion.

Analyzing the disturbances into normal modes, we assume that the perturbed quantities have the $x, y$, $z$ and $t$ dependence of the form

(7) $f(z)\, exp\, (ik_x x + ik_y y + nt)$,

where $f(z)$ is some function of $z$, $n$ is the growth rate of the harmonic disturbance and $k_x$, $k_y$ are the horizontal wave numbers $(k^2 = k_x^2 + k_y^2)$.

For perturbations of the form (7), Eqs. (2)—(6) become

(8) $$(1+\lambda n)\rho n u = (1+\lambda n)\left[ -ik_x \delta p + \frac{H_y}{4\pi}(ik_y h_x - ik_x h_y)\right]$$

$$+ (1+\lambda_0 n)\left[\rho\nu (D^2 - k^2)u - \frac{\rho\nu}{k_1}u + (ik_x w + Du)D\mu\right],$$

(9) $$(1+\lambda n)\rho n v = (1+\lambda n)\left[ -ik_y \delta p + \frac{H_x}{4\pi}(ik_x h_y - ik_y h_x)\right]$$

$$+ (1+\lambda_0 n)\left[\rho\nu (D^2 - k^2)v - \frac{\rho\nu}{k_1}v + (ik_y w + Dv)D\mu\right],$$

(10) $$(1+\lambda n)\rho n w = (1+\lambda n)\left[ -D\delta p - g\delta\rho + \frac{H_x}{4\pi}(ik_x h_z - Dh_x)\right.$$

$$\left. + \frac{H_y}{4\pi}(ik_y h_z - Dh_y)\right] + (1+\lambda_0 n)\left[\rho\nu (D^2 - k^2)w - \frac{\rho\nu}{k_1}w + 2(Dw)(D\mu)\right],$$

(11) $i k_x u + i k_y v + Dw = 0$,

(12) $i k_x h_x + i k_y h_y + Dh_z = 0$,

(13) $n\mathbf{h} = (i k_x H_x + i k_y H_y)\mathbf{v}$,

(14) $n\delta\rho = -wD\rho$,

(15) where $D = d/dz$.

Multiplying Eqs. (8) and (9) by $-ik_x$ and $-ik_y$ respectively and adding, using (11) – (13) in it and finally eliminating $\delta p$ between the resulting equation and Eq. (10) [after substituting for $\delta\rho$ from (14) ], we obtain

$$(16)\quad (1+\lambda n)\left[\{D(\rho Dw)-k^2\rho w\}+\frac{gk^2}{n^2}(D\rho)w+\frac{(k_xH_x+k_yH_y)^2}{4\pi n^2}(D^2-k^2)w\right]$$

$$-\left(\frac{1+\lambda_0 n}{n}\right)\left[D\{\rho\nu(D^2-k^2)Dw\}-k^2\rho\nu(D^2-k^2)w\right]+\left(\frac{1+\lambda_0 n}{nk_1}\right)\left[D(\rho\nu Dw)\right.$$

$$\left.-k^2\rho\nu w\right]-\left(\frac{1+\lambda_0 n}{n}\right)\left[D\{(D\mu)(D^2+k^2)w\}-2k^2(D\mu)(Dw)\right]=0.$$

## 3. Two Uniform Superposed Viscoelastic Fluids Separated by a Horizonial Boundary

we next consider the case when two superposed viscoelastic fluids of uniform densities $\rho_1$ and $\rho_2$ and uniform viscosities $\mu_1$ and $\mu_2$ are separated by a horizontal boundary at $z=0$. The subscripts 1 and 2 distinguish the lower and the upper fluids respeetively. Then, in each region of constant $\rho$ and constant $\mu$, Fq. (14) becomes

$$(17)\quad (D^2-k^2)\,(D^2-q^2)w=0,$$

where

$$(18)\quad q^2=\left[k^2+\frac{1}{k_1}+\frac{n}{\nu}\;\frac{1+\lambda n}{1+\lambda_0 n}\left\{1+\frac{1}{4\pi n^2\rho}(k_x H_x+k_y H_y)^2\right\}\right].$$

Since $w$ must vanish both when $g\to-\infty$ (in the lower fluid) and $z\to+\infty$ (in the upper fluid) we can write the solutions, appropriate to the two regions, as

$$(19)\quad w_1=A_1\,e^{+kz}+B_1\,e^{+q_1 z}\qquad (z<0)\,,$$

$$(20)\quad w_2=A_2\,e^{-kz}+B_2\,e^{-q_2 z}\qquad (z>0)\,,$$

where $A_1, B_1, A_2, B_2,$ are constants,

$$(21)\quad q_1 = \left[ k^2 + \frac{1}{k_1} + \frac{n}{\nu_1} \; \frac{1+\lambda n}{1+\lambda_0 n} \left\{ 1 + \frac{1}{4\pi n^2 \rho_1} \left( k_x H_x + k_y H_y \right)^2 \right\} \right]^{\frac{1}{2}}$$

and

$$(22)\quad q_2 = \left[ k^2 + \frac{1}{k_1} + \frac{n}{\nu_2} \; \frac{1+\lambda n}{1+\lambda_0 n} \left\{ 1 + \frac{1}{4\pi n^2 \rho_2} \left( k_x H_x + k_y H_y \right)^2 \right\} \right]^{\frac{1}{2}}.$$

It may be added that in writing the solutions (19) and (20) it is assumed that $q_1$ and $q_2$ are so defined that their real parts are positive.

## 4. Boundary Conditions

The solutions (19) and (20) must satisfy certain boundary conditions, These conditions ( Chandrasekher [2], p. 432 ) require that at an interface

(23) $w$,

(24) $Dw$,

and

(25) $\mu(D^2+k^2)\, w$ ,

must be continuous.

Integrating Eq. (16) across the interface $z=0$ we obtain another condition

$$(26)\quad (1+\lambda n)\Big[ \rho_2 Dw_2 - \rho_1 Dw_1 \Big]_{z=0} - (1+\lambda_0 n) \Big[ \frac{\mu_2}{n}(D^2-k^2)Dw_2 - \frac{\mu_1}{n}(D^2-k^2)Dw_1 \Big]_{z=0} + (1+\lambda n)\, \frac{(k_x H_x + k_y H_y)^2}{4\pi\, n^2} \left(Dw_2 - Dw_1\right)_{z=0}$$

$$= -\frac{gk^2}{n^2}(1+\lambda n)\ (\rho_2-\rho_1)_{w_0} - \frac{2k^2}{n}(1+\lambda_0 n)(\mu_2-\mu_1)\ (Dw)_0$$

$$-\left(\frac{1+\lambda_0 n}{k_1}\right)\left(\frac{\mu_2}{n}Dw_2 - \frac{\mu_1}{n}Dw_1\right)_{z=0} \quad ,$$

where $w_0$ and $(Dw)_0$ are the common values of $w_1$ , $w_2$ , and $Dw_1$ , $Dw_2$ , respectively at $z=0$

## 5. Dispersion Relation and Discussion

Applying the conditions (23)—(26) to the solutions (19) and (20), we obtain

(27) $A_1+B_1=A_2+B_2$ ,

(28) $kA_1+q_1B_1 = -kA_2-q_2B_2$,

(29) $\mu_1\left[2k^2A_1+(q_1^2+k^2)B_1\right]=\mu_2\left[2k^2A_2+(q_2^2+k^2)B_2\right]$,

(30) $$(1+\lambda n)\Big[(-\rho_2A_2-\rho_1A_1)-(A_1+A_2)\frac{1}{4\pi n^2}\left(k_xH_x+k_yH_y\right)^2+\frac{gk}{2n^2}$$

$$(\rho_2-\rho_1)\ (A_1+B_1+A_2+B_2)\Big] = (1+\lambda_0 n)\ \Big[\frac{k}{n}(\mu_1-\mu_2)\ (kA_1$$

$$+ q_1B_1-kA_2-q_2B_2)+\frac{1}{nk_1}(\mu_2A_2+\mu_1A_1)\Big].$$

Eliminating the constants $A_1$ , $B_1$ , $A_2$ , $B_2$ from (27) – (30), we obtain

$$(31)\quad \begin{vmatrix} 1 & 1 \\ k & q_1 \\ 2k^2\mu_1 & \mu_1(q_1^2+k^2) \\ \left[(1+\lambda n)\left\{\frac{1}{2}R-\alpha_1-\frac{(\mathbf{k},\mathbf{v}_A)^2}{n^2}\right\} - (1+\lambda_0 n)\left(C+\frac{\alpha_1\nu_1}{nk_1}\right)\right] & \left[\frac{R}{2}(1+\lambda n) - C\frac{q_1}{k}(1+\lambda_0 n)\right] \end{vmatrix}$$

$$\begin{vmatrix} -1 & -1 \\ k & q_2 \\ -2k^2\mu_2 & -\mu_2(q_2^2+k^2) \\ \left[(1+\lambda n)\left\{\frac{1}{2}R-\alpha_2-\frac{(\mathbf{k},\mathbf{v}_A)^2}{n^2}\right\} + (1+\lambda_0 n)\left(C-\frac{\alpha_2\nu_2}{nk_1}\right)\right] & \left[\frac{R}{2}(1+\lambda n) + C\frac{q_2}{k}(+\lambda_0 n)\right] \end{vmatrix} = 0,$$

where we have written

$$(32)\quad \alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1+\rho_2}, \quad (\mathbf{k}.\ \mathbf{v}_A)^2 = \frac{(k_xH_x+k_yH_y)^2}{4\pi(\rho_1+\rho_2)},$$

$$R = \frac{g\mathbf{k}}{n^2}(\alpha_2-\alpha_1),\ C=\frac{k^2}{n}\ \frac{\mu_1-\mu_2}{\rho_1+\rho_2} = \frac{k^2}{n}(\alpha_1\nu_1-\alpha_2\nu_2).$$

Here **k** is the wave vector and $\mathbf{v}_A$ is Alfuén velocity vector.

Evaluating the determinant (31), we obtain the following characteristic equation :

$$(33)\quad (q_1-k)\Bigg(2k^2(\alpha_1\nu_1-\alpha_2\nu_2)\Big[\,(1+\lambda n)\Big\{\alpha_2+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{n^2}\Big\}$$

$$+(1+\lambda_0 n)\Big\{\frac{C}{\mathrm{k}}(q_2-k)+\frac{\alpha_2\nu_2}{nk_1}\Big\}\Big]+\Big[\frac{\alpha_2\nu_2}{k_1}$$

$$+\alpha_2 n\left(\frac{1+\lambda n}{1+\lambda_0 n}\right)\Big\{1+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{\alpha_2 n^2}\Big\}\Big]\Big[(1+\lambda n)\Big\{R-1-\frac{\alpha_1\nu_1+\alpha_2\nu_2}{nk_1}$$

$$-2\frac{(\mathbf{k}.\mathbf{V}_A)^2}{n^2}\Big\}\Big]\Bigg)-2k\Bigg(\Big[\frac{\alpha_1\nu_1}{k_1}+\alpha_1 n\left(\frac{1+\lambda n}{1+\lambda_0 n}\right)\Big\{1+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{\alpha_1 n^2}\Big\}\Big].$$

$$\Big[(1+\lambda n)\Big\{\alpha_2+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{n^2}\Big\}+(1+\lambda_0 n)\Big\{\frac{C}{\mathrm{k}}(q_2-k)+\frac{\alpha_2\nu_2}{nk_1}\Big\}\Big]$$

$$+\Big[\frac{\alpha_2\nu_2}{k_1}+\alpha_2 n\left(\frac{1+\lambda n}{1+\lambda_0 n}\right)\Big\{1+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{\alpha_2 n^2}\Big\}\Big]\Big[(1+\lambda n)\Big\{\alpha_1$$

$$+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{n^2}\Big\}+(1+\lambda_0 n)\Big\{-\frac{C}{\mathrm{k}}(q_1-k)+\frac{\alpha_1\nu_1}{nk_1}\Big\}\Big]\Bigg)$$

$$+(q_2-k)\Bigg(\Big[\frac{\alpha_1\nu_1}{k_1}+\alpha_1 n\left(\frac{1+\lambda n}{1+\lambda_0 n}\right)\Big\{1+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{\alpha_1 n^2}\Big\}\Big]\Big[(1+\lambda n)\Big\{R-1$$

$$-\frac{\alpha_1\nu_1+\alpha_2\nu_2}{nk_1}-2\frac{(\mathbf{k}.\mathbf{V}_A)^2}{n^2}\Big\}\Big]-2k^2(\alpha_1\nu_1-\alpha_2\nu_2)\Big[(1+\lambda n)\Big\{\alpha_1+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{n^2}\Big\}$$

$$+(1+\lambda_0 n)\Big\{\frac{\alpha_1\nu_1}{nk_1}-\frac{C}{\mathrm{k}}(q_1-k)\Big\}\Big]\Bigg)=0.$$

Since the values of $q_1$ and $q_2$ involve square roots, the dispersion relation (33) is quite complex. We, therefore, carry out the stability analysis for large medium permeability and highly viscous fluids. For, then we can write

$$(k4)\quad q=3\left[1+\frac{1}{k^2k_1}+\frac{n}{k^2\nu}\left(\frac{1+\lambda n}{1+\lambda_0 n}\right)\left\{1+\frac{(k_xH_x+k_yH_y)^2}{4\pi\rho n^2}\right\}\right]^{\frac{1}{2}},$$

$$=k\left[1+\frac{1}{2k^2k_1}+\frac{n}{2k^2\nu}\left(\frac{1+\lambda n}{1+\lambda_0 n}\right)\left\{1+\frac{(k_xH_x+k_yH_y)^2}{4\pi\rho n^2}\right\}\right],$$

so that

$$(35)\quad q_1-k=\frac{1}{2kk_1}+\frac{n}{2k\nu_1}\left(\frac{1+\lambda n}{1+\lambda_0 n}\right)\left\{1+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{\alpha_1 n^2}\right\},$$

and

$$(36)\quad q_2-k=\frac{1}{2kk_1}+\frac{n}{2k\nu_2}\left(\frac{1+\lambda n}{1+\lambda_0 n}\right)\left\{1+\frac{(\mathbf{k}.\mathbf{V}_A)^2}{\alpha_2 n^2}\right\}.$$

Substituting the values of $q_1-k$ and $q_2-k$ from Eqs. (35) and (36) in Eq. (33) and putting $\nu_1=\nu_2=\nu$ (the case of equal kinematic viscosities, for mathematical simplicity, as in Chandrasekhar [2], as any of the essential features of the problem would not be obscured by this simplifying assumption),

we obtain the following dispersion relation

$$(37)\quad A_9n^9+A_8n^8+A_7n^7+A_6n^6+A_5n^5+A_4n^4$$
$$+A_3n^3+A_2n^2+A_1n+A_0=0,$$

where

$$A_9=\alpha_1\alpha_2\, k_1^3\lambda^3,$$

$$A_8=2\nu\alpha_1\alpha_2k_1^2\lambda_0\lambda^2(2+k^2k_1)+\alpha_1\alpha_2k_1^2\lambda^2(\nu\lambda+3k_1),$$

$$A_7=\alpha_1\alpha_2k_1\nu^2\lambda\lambda_0^2(1+4k^2k_1)+2\alpha_1\alpha_2\nu k_1^2(2\lambda\lambda_0+\lambda^2)\,(2+k^2k_1)$$
$$+2\alpha_1\alpha_2k_1\nu^2\lambda_0\lambda^2-\alpha_1\alpha_2k_1^3\lambda^3L+3\alpha_1\alpha_2k_1^3\lambda+3\alpha_1\alpha_2\nu k^2\lambda^2$$
$$+k_1^3\lambda^3(\mathbf{k}.\mathbf{V}_A)^2,$$

$$(38)\; A_6 = \alpha_1\alpha_2\nu^2k_1(1+4k^2k_1)\;(2\lambda\lambda_0+\lambda_0^2)+\alpha_1\alpha_2\nu^3\lambda_0^2(\lambda+2k^2k_1\lambda_0)$$

$$-2\nu\alpha_1\alpha_2k_1^2\lambda_0\lambda^2M+2\alpha_1\alpha_2\nu k_1^2(2+k^2k_1)\;(2\lambda+\lambda_0)$$

$$+2\alpha_1\alpha_2k_1\nu^2(2\lambda\lambda_0+\lambda^2)\;+\;\nu k_1^2\lambda_0\lambda^2(\mathbf{k}.\mathbf{V}_A)^2G-3\alpha_1\alpha_2k_1^3\lambda^2L$$

$$+\alpha_1\alpha_2k_1^2(k_1+3\nu\lambda)\;+\;3\lambda^2k_1^3(\mathbf{k}.\mathbf{V}_A)^2\;+\;\nu k_1^2\lambda^3\;(\mathbf{k}.\mathbf{V}_A)^2,$$

$$A_5 = 6\alpha_1\alpha_2k_1k^2\nu^3\lambda_0^2+\lambda\lambda_0^2N+\alpha_1\alpha_2\nu^3(2\lambda\lambda_0+\lambda_0^2)-2\alpha_1\alpha_2\nu k_1^2(2\lambda\lambda_0+\lambda^2)M$$

$$+2\alpha_1\alpha_2\nu k_1^2(2+k^2k_1)+2\alpha_1\alpha_2\nu^2k_1(2\lambda+\lambda_0)$$

$$+\nu k_1^2(2\lambda\lambda_0+\lambda^2)\;(\mathbf{k}.\mathbf{V}_A)^2G+\nu^2\lambda_0\lambda^2k_1(\mathbf{k}.\mathbf{V}_A)^2-3\alpha_1\alpha_2k_1^3\lambda L$$

$$+\alpha_1\alpha_2\nu k_1^2-k_1^3\lambda^3(\mathbf{k}.\mathbf{V}_A)^2\{L-(\mathbf{k}.\mathbf{V}_A)^2\}+3k_1^2\lambda(\mathbf{k}.\mathbf{V}_A)^2\{k_1+\nu\lambda\},$$

$$A_4 = \alpha_1\alpha_2k_1\nu^2(1+4k^2k_1)+6\alpha_1\alpha_2k^2k_1\nu^3\lambda_0+(2\lambda\lambda_0+\lambda_0^2)N$$

$$+\;\alpha_1\alpha_2\nu^3(2\lambda+\lambda_0)-2\alpha_1\alpha_2\nu k_1^2(2\lambda+\lambda_0)M+2\alpha_1\alpha_2k_1\nu^2$$

$$-\;\nu kk_1^2g\lambda_0\lambda^2(\alpha_2-\alpha_1)\;(\mathbf{k}.\mathbf{V}_A)^2+\nu k_1^2(\mathbf{k}.\mathbf{V}_A)^2(2\lambda+\lambda_0)\;G$$

$$+\;\nu^2k_1(\mathbf{k}.\mathbf{V}_A)^2(2\lambda\lambda_0+\lambda^2)-\alpha_1\alpha_2k_1^3L+2\nu k_1^2\lambda_0\lambda^2(\mathbf{k}.\mathbf{V}_A)^4(1+k^2k_1)$$

$$-\;3k_1^3\lambda^2(\mathbf{k}.\mathbf{V}_A)^2\{L-(\mathbf{k}.\mathbf{V}_A)^2\}+k_1^2(\mathbf{k}.\mathbf{V}_A)^2\{k_1+3\nu\lambda+\nu\lambda^3(\mathbf{k}.\mathbf{V}_A)^2\},$$

$$A_3 = 2\alpha_1\alpha_2\nu^3k_1k^2+(2\lambda_0+\lambda)N+\alpha_1\alpha_2\nu^3-2\alpha_1\alpha_2\nu k_1^2M$$

$$-gk\nu k_1^2(\mathbf{k}.\mathbf{V}_A)^2(2\lambda\lambda_0+\lambda^2)(\alpha_2-\alpha_1)+\nu k_1^2(\mathbf{k}.\mathbf{V}_A)^2G+\nu^2k_1(\mathbf{k}.\mathbf{V}_A)^2(2\lambda+\lambda_0)$$

$$+2\nu k_1^2(\mathbf{k}.\mathbf{V}_A)^4(1+k^2k_1)(2\lambda\lambda_0+\lambda^2)-3\lambda k_1^3(\mathbf{k}.\mathbf{V}_A)^2\{L-(\mathbf{k}.\mathbf{V}_A)^2\}$$

$$+2k_1^2(\mathbf{k}.\mathbf{V}_A)^2\{1+3\lambda^2(\mathbf{k}\;\mathbf{V}_A)^2\}-k_1^3\lambda^3(\mathbf{k}.\mathbf{V}_A)^4L,$$

$$A_2 = N-gk\nu k_1^2(\mathbf{k}.\mathbf{V}_A)^2(\alpha_2-\alpha_1)(2\lambda+\lambda_0)+\nu^2k_1(\mathbf{k}.\mathbf{V}_A)^2$$

$$+2\nu k_1^2(\mathbf{k}.\mathbf{V}_A)^4(2\lambda+\lambda_0)(1+k^2k_1)-k_1^3(\mathbf{k}.\mathbf{V}_A)^2\{L-(\mathbf{k}.\mathbf{V}_A)^2\}$$

$$+3\lambda\nu k_1^2(\mathbf{k}.\mathbf{V}_A)^4-3k_1^3\lambda^2(\mathbf{k}.\mathbf{V}_A)^4L,$$

$$A_1 = -gk\nu k_1^2(\alpha_2-\alpha_1)(\mathbf{k}.\mathbf{V}_A)^2+\nu k_1^2(\mathbf{k}.\mathbf{V}_A)^4(3+2k^2k_1)-3\lambda k_1^3(\mathbf{k}.\mathbf{V}_A)^4L,$$

$$A_0 = -k_1^3(\mathbf{k}.\mathbf{V}_A)^4L$$

and we have written

$$\{gk(\alpha_2-\alpha_1)-2(\mathbf{k}.\mathbf{V}_A)^2\}=L$$

$$\{gk(\alpha_2-\alpha_1)-1-kk_1(\mathbf{k}.\mathbf{V}_A)^2\}=M$$

$$[2\alpha_1\alpha_2k\nu^2k_1^2-gk\nu^2\alpha_1\alpha_2k_1(\alpha_2-\alpha_1)+\{2\nu^2\alpha_1\alpha_2k_1$$

$$+\ 2k^2\nu^2k_1^2(\alpha_1^2+\alpha_2^2)\}\ (\mathbf{k}.\mathbf{V}_A)^2]\ =N,$$

$$[1+2k^2k_1(\alpha_1^2+\alpha_2^2)]=G.$$

When $\alpha_1>\alpha_2$ (potentially stable arrangement) we find, by applying Hurwitz' criterion to Eq. (37), that (as all the coefficients in (37) are then positive) all the roots $n$ are either real and negative or there are complex roots with negative real parts. The system is therefore stable in each case. Hence the potentially stable configuration remains stable whether the effects of viscosity, viscoelasticity, porosity and magnetic field are included or not.

For the potentially unstable arrangement $\alpha_2 > \alpha_1$, the system is unstable in the hydrodynamic case for all wave numbers $\mathbf{k}$ in the presence of viscosity effects and in the absence of porosity and viscoelastic effects ( Chandrasekhar [2] ). Also the system, tn the present case, is unstable if

(39) $2(\mathbf{k}.\ \mathbf{V}_A)^2 < g\ k\ (\alpha_2-\alpha_1).$

In the present hydromagnetic case we find, by appylying Hurwitz' criterion to Eq. (37), when $\alpha_2 > \alpha_1$' that the system is stable for all wave numbers which satisfy the inequality

(40) $2(\mathbf{k}.\mathbf{V}_A)^2 > g\ k\ (\alpha_2-\alpha_1),$

i. e.

(41) $2k\,(V_1 \cos\theta + V_2 \sin\theta)^2 > g\,(\alpha_2 - \alpha_1)$,

where $V_1$ and $V_2$ are the Alfvén velocities in **x** and y directions and $\theta$ is the angle between **k** and $H_*$.

The stability criterion (41) is independent of the effects of viscosity, viscoelasticity and medium porosity. The magnetic field stabilizes a certain wave number range $k > k^*$, where $k^* = g\,(\alpha_2 - \alpha_1) / 2\,(V_1 \cos\theta + V_2 \sin\theta)^2$, of the unstable configuration even in the presence of the effects of viscosity, medium porosity and viscoelasticity. The critical wave number $k^*$, above which the system is stabilized, is dependent on the magnitudes $V_1$ and $V_2$ of the magnetic field as well as the orientation of the magnetic field $\theta$.

We now examine the behaviour of growth rates with respect to medium permeability analytically. Since for $\alpha_2 > \alpha_1$ and $g\,k(\alpha_2 - \alpha_1) > 2\,(k.V_A)^2$, Eq. (37) has one positive root, let $n_0$ denotes the positive root. Then Eq. (37) holds true for $n_0$, substituted in place of $n$. To study the behaviour of growth rates with respect to medium permeability, we examine the nature of $dn_0/dk_1$ from this resulting equation in $n_0$. It is evident from the form of Eq. (37) that $dn_0/dk_1$ may be both positive or negative. Thus the growth rates both increase or decrease with the increase in medium permeability. A similar argument holds good for growth rates with respect to viscosity, stress relaxation and strain retardation time parameters i. e the growth rates both increase or decrease with the increase in stress relaxation and strain retardation time parameters (Sharma [5] ) and with the increase in kinematic viscosity (Sharma and Thakur [6] ).

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