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FIXED POINT THEOREMS UNDER ASYMPTOTIC REGULARITY AT A POINT. II

by

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Recently, Browder and Petryshyn [1] introduced the notion of asymptotic regularity for a Banach space. Its equivalent form in metric space is given as follows ; A mapping $f : X \to X$ of a metric space (X, d) into itself is said to be *asymptotic regular* at $x \in X$ if lim $d(f^n(x), f^{n+1} (x))$ approaches zero as n tends to infinity, where $f^n (x)$ is the n th iterate of f at $x \in X$. In the present paper we shall generalize the results of [5] and [7] under the asymptotic regularity at a point $x \in X$ for self mappings as taken by Hardy and Rogers [2].

THEOREM 1. *Let f be a self mapping of X into itself of a complete metric space (X,d) satisfying the inequality*

$$d(f (x), f (y)) \leqslant a_1 d(x, f(x)) + a_2 d(y, f(y)) + a_3 d(x, f(y)) + a_4 d(y, f (x)) + a_5 d(x,y), \tag{I}$$

for all $x, y \in X$, where $a_i \geqslant 0$ and $\Sigma a_i \leqslant 1$ for $i=1, 2, 3, 4, 5$.

Then f has a unique fixed point in X, if f is asymptotically regular at some point in X.

PROOF : Consider the sequence $\{ f^n(x_0) \}$ and assume that f is asymptotic regular at some point $x_0 \in X$. Then, for $n, m \geqslant 1$, we have by (I),

$$d(f^n(x_0), f^m(x_0)) \leqslant a_1 d(f^{n-1}(x_0), f^n(x_0)) + a_2 d(f^{m-1} (x_0), f^m(x_0))$$
$$+ a_3 d(f^{n-1}(x_0), f^m(x_0)) + a_4 d(f^{m-1}(x_0), f^n(x_0))$$

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