Jñānābha, Vol. 11, 1981

A NOTE ON (-1, 1) RINGS

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Received: *March 3*, 1980

1. A $(-1, 1)$ ring is a non-associative ring in which the following identities are assumed to hold:

(1) $(x,y,z)+(x,z,y) = 0$,

(2) $(x,y,z)+(y,z,x)+(z,x,y)= 0$;

where the associator (x,y,z) is defined by
$$(x,y,z)=(xy)z-x(yz).$$

Let

(3) $U = \{u \in R: (u,x)= 0;$ for all $x \in R\}$,

where the commutator (x,y) is defined by $(x,y)= xy-yx$.

We shall sometimes use the symbol R(*or* U) to represent an element of R (*or* U) When this notation is used, we mean to consider not only all possible expressions where the R's are replaced by elements of R, but the additive subgroup generated by these. Often in statements, both forms will be combined, *e. g.*, $(x,x,(R,R))= 0$ means $(x,x,c)= 0$ for all x $\in$R and all $c \in (R,R)$. In this paper we further assume that R is of characteristic $\neq 2, \neq 3$. The ring R is called semi-prime, if R has no non-zero ideal squaring to zero. With this definition we prove the following theorem.

Theorem. If R is semi-prime and $((R,R),x,y)= 0$, then R is associative.

2. **Proof.** (R,R,R) is an ideal in R and

(4) $(R,R,R)\subseteq\{(x,x,R): x\in R\}$.

This is Lemma 4 in [1]).

The following identities hold in any $(-1,1)$ ring of characteristic $\neq 2, \neq 3$. (See equatiocas $\overline{\text{L}}$ and $\overline{\text{O}}$ in [1]).

(5) $(x,(y,y,z)) - 3(y,(x,z,y))=0$,

(6) $((x,y),z,w)-((z,w),x,y)-(x,(y,z,w))+(y,(x,z,w))=0$.

Since $((R,R),x,y))=0$, the first two terms in (6) vanish and we obtain

(7) $(x,(y,z,w)) - (y,(x,z,w))=0$.

Put $z=w$ in (5) and $z=y$ in (7). We get

(8) $(x,(y,y,w)) - 3(y,(x,w,y))=0$

and

(9) $(x,(y,y,w)) - (y,(x,y,w))=0$

Since $(x,y,w)= - (x,w,y)$ and $(a,-b)= - (a,b)$, (9) reads as

(10) $(x,(y,y,w))+(y,(x,w,y))=0$.

Multiplying (10) by 3 and adding to (8), we get $4(x,(y,y,w))=0$, that is, $(x,(y,y,w))=0$. Using (4) we get $(R,(R,R,R))=0$. In view of (3), we may write this result as

(11) $(R,R,R)\subseteq U$.

Also

(12) $(x,x,y)u=(x,x,uy)$

and

(13) $(x,x,U)=0$.

(These are equations U.6 and U.3 in [1]).

Using (11) and (12), we have

$$(x,x,y)\,(R,R,R) \subseteq (x,x,(R,R,R)y)$$
$$\subseteq (x,x,(R,R,R)), \text{ (since } (R,R,R) \text{ is an ideal in } R)$$
$$\subseteq (x,x,U) \qquad (by\ (11))$$
$$= O \qquad (by\ (13)).$$

Using (4), we get

$$(R,R,R)^2 = O.$$

We have shown that (R,R,R) is an ideal in R and $(R,R,R)^2 = O$. Since R is semi-prime, $(R,R,R) = O$, that is, R is associative.

REFERENCE

[1] I. R. Hentzel, The characterization of $(-1, 1)$ rings, *J. Algebra* **30** (1974), 236-258.