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FINITE BUFFER QUEUEING SYSTEM WITH ACCESSIBLE AND NON-ACCESSIBLE BATCHES AND SINGLE VACATION

By

Seema Agarwal[1] and Madhu Jain[2]
[1]Department of Mathematics, I.B.S., Dr B. R. Ambedkar University, Agra-282002
[2]Department of Mathematics, IIT Roorkee, Roorkee-247667(India)
[1]dr.seema1982@gmail.com, [2] madhufma@iitr.ac.in
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Abstract

In this paper, we introduce a finite buffer queueing model with single vacation in which the customers are served in batches. The service is provided according to a bulk service rule in which at least 'a' customers are needed to start a service with maximum serving capacity 'b' customers. The inter-arrival time and service time are assumed to be exponentially and arbitrarily distributed. The server takes a vacation as soon as the queue length falls below 'a' at the completion of service. For analysis purpose we have used supplementary variable technique. Some performance measures such as mean idle period and mean busy period are obtained.
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1 Introduction

Bulk service queues have been discussed extensively over last few decades. At present, their utility continues to expand in telecommunication systems, production and traffic processes. In transportation problems such as bus service and scheduled flights, the situations where service is rendered to a group of customers by a server with variable capacity are naturally common. Borthakur [2] discussed general control strategy of bulk service queueing model. Haridass and Arumuganathan [6] proposed the transient state analysis of a batch service queueing system. Jeyakumar and Senthinathan [8] studied a bulk queue with fast and slow service rates.

Gupta and Goswami [4] and Gupta and Sikdar [5] studied a single server queue with finite buffer space. Sitrarasu *et al.* [12] introduced an $M/M^{ab}/c$ interdependent queueing model with controllable arrival rates. Sikdar and Samanta [11] analysed a finite buffer variable batch service queue with server vacations.

The bulk service rule with non-accessible batches has been developed by Medhi [10]. The concept of accessibility in batches was analyzed by Kleinrock [9] and Gross and Harris [3]. Jain and Sharma [7] gave bulk queueing system with accessible and non-accessible batches. Sikdar [11] provided a batch service queue with multiple vacations in which he has considered a finite capacity waiting room. Baburaj and Rekha [1] discussed an $(a; b)$ policy discrete time bulk service queue under customer's choice.

The present investigation deals with bulk service queueing system with single vacation wherein the inclusion of accessible and non-accessible batches in the work of Gupta and

Sikdar [5] makes our model more versatile to real life congestion situations. The paper is organized in different sections as follows. Section 2 introduces the basic assumptions for the model under study. Section 3 describes the methodology used for the analysis of finite capacity model having accessible and non-accessible batches. Queue size distribution has been provided in section 4. Some performance characteristics are provided in section 5. Conclusions have been drawn in section 6.

2 Model Description

Consider a single server finite buffer $M^X/G^{(adb)}/1/N/SV$ queueing system. The following assumptions are made for formulating the mathematical model:

- Customers arrive at the system according to a Poisson process with rate λ.
- The customers are served in batches according to a general bulk service rule as stated below:
  - The server takes all the customers for batch when $a \leq n \leq d$ and the late arriving customers are allowed to join the ongoing batch; this is known as accessible batch.
  - The late arriving customers are not allowed to join the ongoing service batch whenever $d \leq n \leq b$; these batches are known as the non-accessible batches for late arriving customers. If $n \geq b$, then at a time 'b' customers can be served by the server and remaining (n-b) customers remain in the queue.
- When the server finishes serving a batch and finds less than 'a' customers in the queue, the server leaves for a vacation of random length V. On returning from a vacation if he finds the customers waiting, he starts service.
- The system has finite buffer capacity $N(> b)$ i.e. at most $(N+b)$ customers are allowed to join the system at any time and if more than N customers are tried to join the queue then they have to leave the system without getting service or one can say that those customers are lost customers.
- The service time of the batches are assumed to be arbitrarily distributed. The distribution function $(S_n(x))$, density function $(s_n(x))$ and laplase steiltjes transform $(S_n^*(\theta))$ of accessible and non-accessible batches are given as follows:

$$\text{Distribution function; } S_n(x) = \begin{cases} S_1(x), & \text{for system state } (0, n);\ 0 \leq n \leq d-1 \\ S_2(x), & \text{for system state } (1, n);\ n \geq 0. \end{cases}$$

  In similar manner we can define for density function and LST.

  Also, mean service time and vacation times are

$$E(S_1) = -S_2'^*(0) \text{ and } E(V) = -V'^*(0)$$

  for accessible and non-accessible batches, respectively.
- $g_i^{(1)}, g_i^{(2)}$ and f_i be the probability that i customers arrive during a service time and vacation time of accessible and non-accessible batch i.e.

$$g_i^{(1)} = \int_0^\infty \frac{(\lambda x)^i}{i!} e^{-\lambda x} dS_1(x);\, g_i^{(2)} = \int_0^\infty \frac{(\lambda x)^i}{i!} e^{-\lambda x} dS_2(x);\, f_i = \int_0^\infty \frac{(\lambda x)^i}{i!} e^{-\lambda x} dV(x),\ i \geq 0$$

- The probabilities $\Pi_{n0}(\Pi_{n1})$ and W_n denote the number of customers in the queue when the server is busy and vacation, respectively.
- The probability that 'n' customers in the queue just prior to the service completion and vacation termination epochs is defined by $\Pi_{n0}(0)(\Pi_{n1}(0))$ and $W_n(0)$. Let p_i^* be the steady state probability that i customers are left at a departure epoch.
- At any time t, the state of the system can be characterized by

$$\eta(t) = \begin{cases} 0, & \text{if the server is in dormancy} \\ 1, & \text{if the server is on vacation} \\ 2, & \text{if the server is busy in case when number of customers are less than threshold `}d\text{'} \\ 3, & \text{if the server is busy when number of cumstomers are greater than threshold `}d\text{'}. \end{cases}$$

Let us define

$$\begin{aligned} \Pi_{n1}(x;t)dx &= P(N_q(t) = n,\ x \le \tilde{S}_2(t) \le x + dx\,,\ \eta(t) = 3\,,\quad 0 \le n \le N\,,\, x \ge 0, \\ \Pi_{n0}(x;t)dx &= P(N_q(t) = n,\ x \le \tilde{S}_1(t) \le x + dx\,,\ \eta(t) = 2\,,\quad 0 \le n \le d-1\,,\, x \ge 0, \\ W_n(x;t)dx &= P(N_q(t) = n,\ x \le \tilde{V}(t) \le x + dx\,,\ \eta(t) = 1\,,\quad 0 \le n \le N\,,\, x \ge 0, \\ R_n(t) &= P(N_q(t) = n,\ \eta(t) = 0\,,\quad 0 \le n \le a-1. \end{aligned}$$

3 The Analysis

Now we construct the steady state equations by taking limit as t$\to\ \infty$in the equations obtained using supplementary variable technique in the following way:

$$0 = -\lambda R_0 + W_0(0), \tag{3.1}$$

$$0 = -\lambda R_n + \lambda R_{n-1} + W_n(0)\ ,\ \ 1 \le n \le a-1, \tag{3.2}$$

$$-\frac{d}{dx}\Pi_{00}(x) = -\lambda\Pi_{00}(x) + S_1(x)\left(\sum_{n=a}^{d-1}\Pi_{n0}(0) + \sum_{n=a}^{d-1}W_n(0)\right) + S_2(x)\sum_{n=d}^{b}W_n(0), \tag{3.3}$$

$$-\frac{d}{dx}\Pi_{n0}(x) = -\lambda\Pi_{n0}(x) + \lambda\Pi_{n-1\,0}(x) + S_2(x)\Pi_{n1}(0)\ ,\ \ 1 \le n \le a-1, \tag{3.4}$$

$$-\frac{d}{dx}\Pi_{n0}(x) = -\lambda\Pi_{n0}(x) + \lambda\Pi_{n-1\,0}(x) + S_2(x)\Pi_{n1}(0) - S_1(x)\Pi_{n0}(0)\ ,\ \ a \le n \le d-1, \tag{3.5}$$

$$-\frac{d}{dx}\Pi_{01}(x) = -\lambda\Pi_{01}(x) + \lambda\Pi_{d-1\,0}(x) + S_2(x)\sum_{n=d}^{b}\Pi_{n1}(0) - S_2(x)\Pi_{01}(0), \tag{3.6}$$

$$-\frac{d}{dx}\Pi_{n1}(x) = -\lambda\Pi_{n1}(x) + \lambda\Pi_{n-1\,1}(x) + S_2(x)\left(\Pi_{n+b\,1}(0) + W_{n+b}(0)\right), 1 \le n \le N-b, \tag{3.7}$$

$$-\frac{d}{dx}\Pi_{n1}(x) = -\lambda\Pi_{n1}(x) + \lambda\Pi_{n-1\,1}(x)\ ,\ \ N-b+1 \le n \le N-1, \tag{3.8}$$

$$-\frac{d}{dx}\Pi_{N1}(x) = \lambda\Pi_{N-1\,1}(x), \tag{3.9}$$

$$-\frac{d}{dx}W_0(x) = -\lambda W_0(x) + \Pi_{00}(0)V(x), \tag{3.10}$$

$$-\frac{d}{dx}W_n(x) = -\lambda W_n(x) + \lambda W_{n-1}(x) + \Pi_{n0}(0)V(x)\ ,\ \ 1 \le n \le a-1, \tag{3.11}$$

$$-\frac{d}{dx}W_n(x) = -\lambda W_n(x) + \lambda W_{n-1}(x)\ ,\ \ a \le n \le N-1, \tag{3.12}$$

$$-\frac{d}{dx}W_N(x) = \lambda W_{N-1}(x). \tag{3.13}$$

Now we multiply eqs (3.3)-(3.13) by $e^{-\theta x}$ and integrating with respect to x from 0 to ∞, then after using Laplace transform, we get

$$(\lambda-\theta)\Pi^*_{00}(\theta) = S^*_1(\theta)\left(\sum_{n=a}^{d-1}\Pi_{n0}(0) + \sum_{n=a}^{d-1}W_n(0)\right) + S^*_2(\theta)\sum_{n=d}^{b}W_n(0) + S^*_2(\theta)\Pi_{01}(0)$$
$$+\lambda S^*_1(\theta)R_{a-1} - \Pi_{00}(0) \tag{3.14}$$

$$(\lambda-\theta)\Pi^*_{n0}(\theta) = \lambda\Pi^*_{n-1\,0}(\theta) + S^*_2(\theta)\Pi_{n1}(0) - \Pi_{n0}(0)1 \le n \le a-1, \tag{3.15}$$

$$(\lambda-\theta)\Pi^*_{n0}(\theta) = \lambda\Pi^*_{n-1\ 0}(\theta) + S^*_2(\theta)\Pi_{n1}(0) - S^*_1(\theta)\Pi_{n0}(0) - \Pi_{n\,0}(0)\ ,\ \ a \le n \le d-1, \tag{3.16}$$

$$(\lambda-\theta)\Pi^*_{01}(\theta) = \lambda\Pi^*_{d-1\ 0}(\theta) + S^*_2(\theta)\sum_{n=d}^{b}\Pi_{n1}(0) - S^*_2(\theta)\Pi_{01}(0) - \Pi_{01}(0), \tag{3.17}$$

$$(\lambda-\theta)\Pi^*_{n1}(\theta) = \lambda\Pi^*_{n-1\ 1}(\theta) + S^*_2(\theta)\left(\Pi_{n+b\,1}(0) + W_{n+b}(0)\right) - \Pi_{n1}(0),\ \ 1 \le n \le N-b, \tag{3.18}$$

$$(\lambda-\theta)\Pi^*_{n1}(\theta) = \lambda\Pi^*_{n-1\ 1}(\theta) - \Pi_{n1}(0)\ \ N-b+1 \le n \le N-1, \tag{3.19}$$

$$-\theta\Pi^*_{N1}(\theta) = \lambda\Pi^*_{N-1\ 1}(\theta) - \Pi_{N1}(0), \tag{3.20}$$

$$(\lambda-\theta)W^*_0(\theta) = \Pi_{00}(0)V^*(\theta) - W_0(0), \tag{3.21}$$

$$(\lambda-\theta)W^*_n(\theta) = \lambda W^*_{n-1}(\theta) + \Pi_{n0}(0)V^*(\theta) - W_n(0),\ \ 1 \le n \le a-1, \tag{3.22}$$

$$(\lambda-\theta)W^*_n(\theta) = \lambda W^*_{n-1}(\theta) - W_n(0)\ \ a \le n \le N-1, \tag{3.23}$$

$$-\theta W^*_N(\theta) = \lambda W^*_{N-1}(\theta) - W_N(0). \tag{3.24}$$

4 Queue Size Distribution

In order to evaluate state probabilities$\{R_n\}_0^{a-1}$,$\{\Pi_{n0}\}_0^{d-1}$,$\{\Pi_{n1}\}_0^{N}$ and $TRIALRESTRICTION$ from equations (3.1) and (3.2) and equations (3.14)-(3.24), we will establish the results in the form of propositions and theorems as follows.

Proposition 4.1.

(a)
$$\sum_{n=0}^{j}W_n(0) = \lambda R_j,\ \ 0 \le j \le a-1, \tag{4.1}$$

(b)
$$\sum_{n=0}^{d-1}\Pi_{n0}(0) = \sum_{n=0}^{N}W_n(0). \tag{4.2}$$

Proof. **(a)** Putting $n = 1, 2, ..., a-1$ in equation (3.2), using equation (3.1) and the solving recursively, we get required equation (4.1).
(b) Setting $\theta = 0$ into equations (3.14)-(3.20) and using the same approach as Gupta and Sikdar [5] have applied, we get eq. (4.2). □

Proposition 4.2.

$$E(S_1)\sum_{n=0}^{d-1} W_n(0) + E(S_2)\left(\sum_{n=d}^{N} W_n(0) + \sum_{n=1}^{N} \Pi_{n1}(0)\right) = \sum_{n=0}^{d-1} \Pi_{n0} + \sum_{n=0}^{N} \Pi_{n1}, \tag{4.3}$$

$$E(V)\sum_{n=0}^{a-1} \Pi_{n0}(0) = \sum_{n=0}^{N} W_n. \tag{4.4}$$

Proof. **(a)** Differentiating equations (3.14)-(3.20) with respect to θ and setting $\theta = 0$, then adding these equations by using equations (4.1) and (4.2), we get equation (4.3).
(b) On applying the similar method in equations (3.21)-(3.24) as applied in the proof of (a), we get required equation (4.4). □

Proposition 4.3.

$$E(S_1)\sum_{n=0}^{d-1} W_n(0) + E(S_2)\left(\sum_{n=d}^{N} W_n(0) + \sum_{n=1}^{N} \Pi_{n1}(0)\right) + E(V)\left(\sum_{n=0}^{N} W_n(0) - \sum_{n=a}^{d-1} \Pi_{n0}(0)\right) + \sum_{n=0}^{a-1} R_n = 1, \tag{4.5}$$

Proof. On adding equations (3.14)-(3.24), taking limit as $\theta \to 0$ then apply L'Hospital rule and using the normalizing condition $\sum\limits_{n=0}^{d-1} \Pi_{n0} + \sum\limits_{n=0}^{N} \Pi_{n1} + \sum\limits_{n=0}^{N} W_n + \sum\limits_{n=0}^{a-1} R_n = 1$, we get equation (4.5). □

Remark 3. *Let*

$$E(S_1)\sum_{n=0}^{d-1} W_n(0) + E(S_2)\left(\sum_{n=d}^{N} W_n(0) + \sum_{n=1}^{N} \Pi_{n1}(0)\right) = \rho' = \sum_{n=0}^{d-1} \Pi_{n0} + \sum_{n=0}^{N} \Pi_{n1}. \tag{4.6}$$

Proposition 4.4. *The probability that the server is busy is given by*

$$\rho' = \frac{\lambda E(S_1)}{\lambda E(S_1) + \lambda E(V)\sum\limits_{i=0}^{a-1} p_i^+ + \sum\limits_{n=0}^{a-1} p_n^+ \sum\limits_{j=0}^{a-1-i} A_j}. \tag{4.7}$$

Proof. Let $E(B)$ and $E(I)$ denote the mean length of busy period and idle period , then we get (cf. Takagi, [13], pp. 324)

$$\rho' = \frac{E(B)}{E(B) + E(I)}. \tag{4.8}$$

By using the values of $E(I)$ and $E(B)$ from (5.3) and (5.4) in (4.8), we obtain the equation (4.7). □

Proposition 4.5. *The queue length distribution at service completion epoch* (Π^+_{n0}, Π^+_{n1}) *and at vacation termination epoch* (W^+_n)*are given by*

$$\Pi^+_{no} = \frac{1}{\sigma}\Pi_{n0}(0), \quad \Pi^+_{n1} = \frac{1}{\sigma}\Pi_{n1}(0) \text{ and } W^+_n = \frac{1}{\sigma}W_n(0); 0 \leq n \leq N \tag{4.9}$$

where

$$\sigma = \sum_{n=0}^{d-1} \Pi_{n0}(0) + \sum_{n=0}^{N} (W_n(0) + \Pi_{n1}(0)). \tag{4.10}$$

Proof. The results of equation (4.9) can be obtained as Gupta and Sikdar [5] □

Theorem 4.1 (a). *The probability of 'n' customers in the queue just prior to the vacation termination epoch is given by*

$$\sum_{n=0}^{N} W_n(0) = \frac{1-\rho'}{E(V) + \frac{1}{\lambda \sum\limits_{n=0}^{a-1} p^+_{n0}} \left(\sum\limits_{i=0}^{a-1} p^+_{i0} \sum\limits_{j=0}^{a-1-i} A_j \right)} + \sum_{n=a}^{d-1} \Pi_{n0}(0). \tag{4.11}$$

Proof. P (server is in dormancy) $= (1-\rho')(E(D)/E(1)),$ (a.1.1)

P(server is in dormancy) $= \sum\limits_{j=0}^{a-1} R_j.$ (a.1.2)

Now comparing equations (a.1.1) and (a.1.2) and using equation (30), we get required result. □

(b) The queue length distribution of the number of customers during vacation at arbitrary epoch is given by

$$W_n = \begin{cases} \left(\frac{\sigma}{\lambda}\right) \sum\limits_{j=0}^{n} (\Pi^+_{j0} - W^+_j), & 0 \leq n \leq a-1 \\ \left(\frac{\sigma}{\lambda}\right) \left(\sum\limits_{j=0}^{a-1} \Pi^+_{j0} - \sum\limits_{j=0}^{a} W^+_j \right), & a \leq n \leq N-1. \end{cases} \tag{4.12}$$

Proof. Setting $\theta = 0$ in equations (3.21)-(3.23) and then multiply obtained equations by $(1/\sigma)$ using equation (4.10) and then solving recursively, we get the first part of the required equation (4.12).

For the second case, putting $n = a,\ a+1, ...,\ N-1$ into equation (3.23) when $\theta = 0$, multiply by $(1/\sigma)$ and using equation (4.11), we get the second part of equation (4.12). □

Theorem 4.2. *The queue length distribution immediately after the departure epoch of a batch is given by*

$$p^+_{n0} = \frac{\Pi^+_{n0}}{\sum\limits_{n=0}^{d-1} \Pi^+_{n0} + \sum\limits_{n=0}^{N} \Pi^+_{n1}}, \quad 0 \leq n \leq d-1, \tag{4.13}$$

$$p^+_{n1} = \frac{\Pi^+_{n1}}{\sum\limits_{n=0}^{d-1} \Pi^+_{n0} + \sum\limits_{n=0}^{N} \Pi^+_{n1}}, \quad 0 \leq n \leq N. \tag{4.14}$$

Proof. Applying the steps of Gupta and Sikdar [11] and get the result. □

Remark 4. *Using equation* (4.1) *and divide it by 'σ' and using equation* (4.10), *we get*

$$R_n = \left(\frac{\sigma}{\lambda}\right)\sum_{j=0}^{n} W_j^+ \ , \ \ 0 \le n \le a-1. \tag{4.15}$$

Theorem 4.3. *Relation between* $\left\{\Pi_{n0}^+\right\}_0^{a-1}$, $\left\{\Pi_{n1}^+\right\}_0^{N}$ *and* $\left\{W_n^+\right\}_0^N$ *is given by*

$$\Pi_{n0} = \left(\frac{\sigma}{\lambda}\right)\left(\sum_{i=0}^{b} W_i^+ + \sum_{i=a}^{d-1}\Pi_{i0}^+ + \sum_{i=0}^{n}\Pi_{i1}^+ - \sum_{i=0}^{n}\Pi_{i0}^+\right) \ , \ \ 0 \le n \le a-1, \tag{4.16}$$

$$\Pi_{n0} = \left(\frac{\sigma}{\lambda}\right)\left(\sum_{i=0}^{b} W_i^+ + \sum_{i=a}^{d-1}\Pi_{i0}^+ + \sum_{i=0}^{n}\Pi_{i1}^+ - \sum_{i=0}^{n}\Pi_{i0}^+ - \sum_{i=a}^{n}\Pi_{i0}^+\right), \ \ a \le n \le d-1, \tag{4.17}$$

$$\Pi_{n1} = \left(\frac{\sigma}{\lambda}\right)\left(\sum_{i=0}^{b+n} W_i^+ + \sum_{i=0}^{b+n}\Pi_{i1}^+ - \sum_{i=0}^{d-1}\Pi_{i0}^+ - \sum_{i=0}^{n}\Pi_{i1}^+\right), \ \ 0 \le n \le N-b, \tag{4.18}$$

$$\Pi_{n1} = \left(\frac{\sigma}{\lambda}\right)\left(\sum_{i=0}^{N} W_i^+ + \sum_{i=0}^{N}\Pi_{i1}^+ - \sum_{i=0}^{d-1}\Pi_{i0}^+ - \sum_{i=0}^{n}\Pi_{i1}^+\right), \ \ N-b+1 \le n \le N-1. \tag{4.19}$$

Proof. On Setting $\theta = 0$ into equations (3.14) and (3.17), multiply by $(1/\sigma)$ and using equations (4.10) and (4.15), we get Π_{00} and Π_{01}.

On putting $n = 1, 2, ..., a-1$ at $\theta = 0$ into equation (3.15) and setting $n = a, a+1, ..., d-1$ into equation (3.16), multiply it by $(1/\sigma)$ and using Π_{00}, Π_{01}, we get equation (4.16) and (4.17). Finally apply the same solving procedure into equations (3.18) and (3.19) and using equation (4.17), we get equations (4.18) and (4.19), respectively. □

Theorem 4.4.

$$W_N = (1-\rho') - \left(\frac{\sigma}{\lambda}\right)\left(\sum_{n=0}^{a-1}\sum_{j=0}^{n}\Pi_{j0}^+ + \sum_{n=a}^{N-1}(\sum_{j=0}^{a-1}\Pi_{j0}^+ - \sum_{j=0}^{a} W_j^+)\right), \tag{4.20}$$

$$\Pi_{N1} = \rho' - \sum_{n=0}^{d-1}\Pi_{n0} - \sum_{n=0}^{N-1}\Pi_{n1}. \tag{4.21}$$

Proof. In equation (4.6), substitute the values of W_n and R_n from equations (4.12) and (4.15), we get required equation (4.20).

Again after simplifying equation (4.6), we get results of theorem 4 in the form of equation (4.21). □

5 Performance Measures

In this section, some performance measures in terms of steady state probabilities are formulated to investigate the behaviour of $M/G^{(adb)}/1/N/SV$ queueing system.

I. The mean length of dormant period of the server is given by

$$E(D) = \frac{1}{\lambda\sum_{n=0}^{a-1} p_{n0}^+}\left(\sum_{i=0}^{a-1} p_{i0}^+ \sum_{j=0}^{a-1-i} A_j\right). \tag{5.1}$$

Proof. Applying the concept of accessible and non accessible batches in Gupta and Sikdar [5], we have

$$E(D) = \frac{1}{\sum\limits_{n=0}^{a-1} p_{n0}^{+}} \left(\sum_{i=0}^{a-1} p_{i0}^{+} \sum_{j=0}^{a-1-i} f_j \frac{a-i-j}{\lambda} \right). \tag{5.2}$$

On simplifying equation (5.2), we get equation (5.1).

II. The mean length of idle period of the server is given by

$$E(I) = E(V) + \frac{1}{\lambda \sum\limits_{n=0}^{a-1} p_{n0}^{+}} \left(\sum_{i=0}^{a-1} p_{i0}^{+} \sum_{j=0}^{a-1-i} A_j \right), \text{ where } A_j = \sum_{i=0}^{j} f_i \tag{5.3}$$

(Since $E(I) = E(V) + E(D)$, we get (5.3) by using (5.2).

III. The mean length of busy period of the server is given as

$$E(B) = \frac{E(S_1)}{\sum\limits_{i=0}^{d-1} p_{i0}^{+}} + \frac{E(S_2)}{\sum\limits_{i=0}^{N} p_{i1}^{+}}. \tag{5.4}$$

IV. Average number of customers in the queue at arbitrary epoch when the server is busy

$$L_B = \sum_{n=0}^{d-1} n\, \Pi_{n0} + \sum_{n=0}^{N} n\, \Pi_{n1}. \tag{5.5}$$

V. The average number of customers in the queue at arbitrary epoch when the server is on vacation

$$L_W = \sum_{n=0}^{N} nW_n. \tag{5.6}$$

VI. The average queue length at arbitrary epoch when the server is in dormancy

$$L_D = \sum_{n=0}^{a-1} nR_n. \tag{5.7}$$

VII. The blocking probability of a lost customer is given by

$$P_{loss} = \Pi_{N1} + W_N. \tag{5.8}$$

□

6 Conclusion

In this paper, we have carried out the analysis of a finite buffer bulk service queue with single vacation and accessible and non-accessible batches that has potential applications in manufacturing and transportation systems, etc. In many practical situations, we encounter queueing systems with finite buffer in which if an arrival finds the buffer space full, he is blocked from entering into the system and considered to be lost. Also in finite queues, one of the main concerns of system designer is to provide sufficient buffer space. Therefore, the blocking/loss probabilities play an important role in design and analysis of any queueing system. We have used supplementary variable technique for analysis purpose. The explicit expressions evaluated for various performance indices may be helpful to system designers to evolve optimal control policy of such system at optimum cost. The models MAP/G$^{(adb)}$/1/N and BMAP/G$^{(adb)}$/C/N with multiple vacations will be the topics of our future research.

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