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Effect of Homogeneous Reaction on the Dispersion of a Solute on the Laminar Flow of a Slowly Rotating Channel

by

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ABSTRACT

The effect of homogeneous reaction on the dispersion of a solute in a viscous liquid flowing in a channel under the action of a uniform pressure gradient is studied, when in addition the channel is rotated with a uniform angular velocity about an axis perpendicular to the flow. It has been shown that, for small values of angular velocity, the Taylor diffusion coefficient decreases with an increase in the reaction rate constant. Further, the Taylor diffusion coefficient is also found to decrease with increase in rotation for fixed reaction rate constant corresponding to the bulk reaction.

1. Introduction. Gupta and Gupta [1] considered the unsteady dispersion of solutes with simultaneous chemical reaction in a liquid flowing in a channel. They found that the effective dispersion coefficient decreases with an increase in the reaction rate in diffusion with both homogeneous and combined heterogeneous chemical reactions. The object of this paper is to extend their problem in a rotating frame of reference.

2. Basic flow problem. Consider the laminar flow of a viscous liquid between two parallel walls, rotating at a constant angular velocity 'Ω' perpendicular to the direction of the flow and distant '$2d$' apart under the action of a uniform pressure gradient dP/dx where $P = p - \frac{1}{2}|\vec{\Omega} \times \vec{r}|^2$. Taking the x-axis along the midsection of the channel and the z-axis perpendicular to the walls, the governing equations of motion and the boundary conditions as in Vidyanidhi and Nigam [2] are

$$-2\Omega\, v=-\frac{1}{\rho}\frac{dP}{dx}+\nu\frac{d^2u}{dz^2}, \tag{1}$$

$$2\Omega\, u=\nu\frac{d^2v}{dz^2}, \tag{2}$$

$$u(\pm d)=0,\ v(\pm d)=0. \tag{3}$$

In terms of the non-dimensional quantities, $W_0=-\dfrac{d^2}{3\rho\nu}\dfrac{dP}{dx}$,

$$U=u/W_0,\ V=v/W_0,\ T^2\ (\text{Taylor number})=\Omega\, d^2/\nu,\ \eta=z/d, \tag{4}$$

we solve eqs. (1) and (2) subject to (3) and obtain, for small values of T^2, the following forms

$$U=U_0+T^4U_4,\ V=T^2V_2, \tag{5}$$

where $$U_0=\tfrac{3}{2}(1-\eta^2),\ U_4=\tfrac{1}{60}(-61+75\eta^2-15\eta^4+\eta^6), \tag{6}$$

and V_2 is not needed in the evaluation of the distribution of concentration.

The mean axial velocity is given by

$$W=\frac{1}{2}W_0\int_{-1}^{1}U\,d\eta=W_0\left(1-\frac{68}{105}T^4\right). \tag{7}$$

3. Diffusion with First Order Homogeneous Reaction. If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity is given by

$$W_x=W_0U-W. \tag{8}$$

Following the analysis of Gupta and Gupta [1] and their notation, the variation of 'C' with 'η' is calculated from their eq. (7) *i.e.*

$$\frac{\partial^2C}{\partial\eta^2}-\alpha^2C=\frac{d^2}{DL}\frac{\partial C}{\partial\xi}W_x. \tag{9}$$

In deriving the above equation, it is further assumed that for small values of angular velocity, C is independent of y. Substituting

$$C=\frac{W_0d^2}{DL}\frac{\partial C}{\partial\xi}(C_0+T^4C_4) \tag{10}$$

in (9) and solving subject to the boundary conditions,

$$\frac{\partial C_i}{\partial\eta}=0\ \text{ at }\ \eta=\pm1,\ (i=0,4), \tag{11}$$

we obtain $$C_0=-\frac{1}{2\alpha^2}\left(1-\frac{6}{\alpha^2}-3\eta^2\right)-3\,\frac{\cosh(\alpha\eta)}{\alpha^3\sinh(\alpha)}, \tag{12}$$

$$C_4=\frac{1}{\alpha^2}\left(\frac{31}{84}-\frac{5}{2\alpha^2}+\frac{6}{\alpha^4}-\frac{12}{\alpha^6}\right)+\frac{\eta^2}{\alpha^2}\left(-\frac{5}{4}+\frac{3}{\alpha^2}-\frac{6}{\alpha^4}\right)$$
$$+\frac{\eta^4}{\alpha^2}\left(\frac{1}{4}-\frac{1}{2\alpha^2}\right)-\frac{\eta^6}{60\alpha^2}+4\,\frac{\cosh(\alpha\eta)}{\alpha^3\sinh(\alpha)}\left(\frac{2}{5}-\frac{1}{\alpha^2}+\frac{3}{\alpha^4}\right). \quad (13)$$

The volumetric rate at which the solute is transported across a section of the channel of unit breadth is,

$$Q=\int_{-d}^{d} CW_x\, dz. \quad (14)$$

Integrating, we get

$$Q=\frac{2d^5\,(dP/d\xi)^2}{\rho^2\,\nu^2 L^3\,K_1}\,(Q_0+T^4Q_4), \quad (15)$$

where $$Q_0=\frac{1}{3\alpha^2}-\frac{\coth(\alpha)}{\alpha^3}+\frac{1}{\alpha^4}-\frac{1}{45}, \quad (16)$$

$$Q_4=\frac{88}{2835}-\frac{136}{315\alpha^2}-\frac{8}{\alpha^8}+\frac{8\coth(\alpha)}{3\alpha^3}\left(\frac{2}{5}-\frac{1}{\alpha^2}+\frac{3}{\alpha^4}\right) \quad (17)$$

Comparing with Fick's law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^*

$$D^*=\frac{d^6\,(dP/dx)^2}{D\rho^2\,\nu^2}\,F(\alpha), \quad (18)$$

where $$F(\alpha)=\frac{1}{\alpha^2}\left(\frac{1}{45}-\frac{1}{3\alpha^2}+\frac{\coth(\alpha)}{\alpha^3}-\frac{1}{\alpha^4}\right)$$
$$-\frac{T^4}{9\alpha^2}\left(\frac{88}{315}-\frac{136}{35\alpha^2}-\frac{72}{\alpha^8}+\frac{24\coth(\alpha)}{\alpha^5}\left\{\frac{2}{5}-\frac{1}{\alpha^2}+\frac{2}{\alpha^4}\right\}\right). \quad (19)$$

The term independent of T in the above equation agrees with eq. (12) of Gupta and Gupta [1] and when $\alpha\to 0$, we recover,

$$\lim_{\alpha\to 0} F(\alpha)=\frac{2}{945}-\frac{8T^4}{2673}, \quad (20)$$

so that from eq. (18),

$$D^*=\frac{2}{945}\,\frac{d^6\,(dP/dx)^2}{D\rho^2\,\nu^2}\left(1-\frac{140}{99}T^4\right). \quad (21)$$

The term independent of T in the above equation is the solution obtained by Wooding [3] and the additional term gives the effect of rotation. It can be seen that D^* is reduced by rotation only if

$$T^4<\frac{99}{140}. \quad (22)$$

Figure given below shows the variation of $F(\alpha)$ against the dimensionless reaction rate parameter α for different but values of Taylor number as restricted by Equation (22). It can be seen that the effective dispersion coefficient decreases with increase in the reaction rate constant even in the presence of rotation and that for a fixed α, the effect of increasing rotation is to reduce the effective dispersion coefficient.

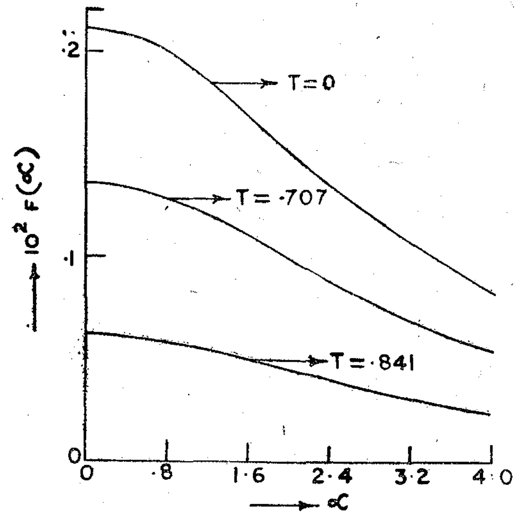

FIG. *Variation of Effective Dispersion Coefficient against the Homogeneous Reaction Parameter.*

Following Gupta and Gupta [1], the effective Taylor dispersion coefficient for diffusion with combined homogeneous and heterogeneous chemical reaction has been obtained but not recorded here in view of cumbersome algebra. It is observed that for a fixed α, an increase in the wall catalytic parameter β causes a decrease in the effective Taylor dispersion coefficient as in the case of homogeneous reaction in the bulk of the liquid.

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