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On Conformal Areal Spaces of the Submetric Class

by

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1. Introduction Let $A_n^{(m)}$ and $\bar{A}_n^{(m)}$ be two n-dimensional areal spaces of the submetric class with the fundamental functions $F(x, p)$ and $\bar{F}(x, p)$ respectively having the same system of coordinates. These two spaces are called conformally related, if their respective normalised metric tensors g_{ij} and $\bar{g}_{ij}$ are connected by the relation

$$\bar{g}_{ij} = e^{2\sigma} g_{ij}, \tag{1.1}$$

where σ is at most a point function [4].

Under the change (1.1), we have seen in [1] that

$$(a)\ \bar{g}_{ij,k} = e^{2\sigma}(g_{ij,k} + 2\sigma_{,k} g_{ij}), \qquad (b)\ \bar{g}_{ij;}{}^{\alpha}_{k} = e^{2\sigma} g_{ij;}{}^{\alpha}_{k}, \tag{1.2}$$

$$\bar{\gamma}^h_{ij} = \gamma^h_{ij} + (\sigma_{,j}\,\delta^h_i + \sigma_{,i}\,\delta^h_j - \sigma_{,k}\, g^{hk} g_{ij}), \tag{1.3}$$

$$\overset{0}{\bar{\Gamma}}{}^h_{ij} = \overset{0}{\Gamma}{}^h_{ij} + V^h_{ij} \tag{1.4}$$

$$\bar{C}^{i\nu}_{j,m} = C^{i\nu}_{j,m}. \tag{1.5}$$

The purpose of this paper is to obtain different kinds of curvature tensors and Bianchi identities in our space. Throughout the paper, we use the same notations as in the papers [1], [2], [3] and [4].

2. Covariant Derivatives of Tensors in $\bar{A}_n^{(m)}$. The covariant derivative of any vector X^i in $A_n^{(m)}$ is given by [3]

$$X^i_{|k}=X^i_{,k}-X^i\,;\,\overset{\alpha}{_l}\;\overset{0}{\Gamma}{}^l_{\alpha k}+X^l\,\overset{0}{\Gamma}{}^i_{lk}.$$

Analogous to this, the covariant derivative of any vector $\bar{X}^i$ in $\bar{A}_n^{(m)}$ is defined by

$$(2.1)\qquad \bar{X}^i_{\mathrm{T}k}=\bar{X}^i_{,k}-\bar{X}^i\,;\,\overset{\alpha}{_l}\;\overset{\bar{0}}{\Gamma}{}^l_{\alpha k}+\bar{X}^l\,\overset{\bar{0}}{\Gamma}{}^i_{lk},$$

where the symbol (T) followed by an index denotes the covariant derivative in the space $\bar{A}_n^{(m)}$.

If we replace $\bar{X}^i$ by $\bar{g}_{ij}$ in (2.1), we get

$$(2.2)\qquad \bar{g}_{ij\mathrm{T}k}=\bar{g}_{ij,k}-\bar{g}_{ij}\,;\,\overset{\alpha}{_l}\;\overset{\bar{0}}{\Gamma}{}^l_{\alpha k}-\bar{g}_{lj}\,\overset{\bar{0}}{\Gamma}{}^l_{ik}-\bar{g}_{il}\,\overset{\bar{0}}{\Gamma}{}^l_{jk},$$

then, by help of (1.1), (1.2), (1.4) and the relation $g_{ij|k}=0$ in $A_n^{(m)}$, we conclude

Theorem 2.1. An areal space, satisfying the relation

$$g_{ij,k}+(2\sigma_{,k}\,g_{ij}-g_{ij}\,;\,\overset{\alpha}{_l}\;V^l_{\alpha k}-g_{lj}\,V^l_{ik}-g_{il}\,V^l_{jk})=0,$$

is said to be conformally related to the space $A_n^{(m)}$ whose normalised metric tensor is g_{ij}.

Theorem 2.2. Let $A_n^{(m)}$ and $\bar{A}_n^{(m)}$ be two areal spaces of the sub-metric class, satisfying (1.1). Then, in the space $A_n^{(m)}$, the relation

$$\sigma_{,k}\,g_{ij}=\tfrac{1}{2}\,(g_{ij}\,;\,\overset{\alpha}{_l}\;V^l_{\alpha k}+g_{lj}\,V^l_{ik}+g_{il}\,V^l_{jk})$$

holds good.

Further, from (2.1) if vector $\bar{X}^i$ is replaced by tensor $\bar{C}^{i\;\lambda}_{h,\;\gamma}$ and then relations (1.4) and (1.5) are applied, we get

$$\bar{C}^{i\lambda}_{h,\gamma\mathrm{T}k}=C^{i\lambda}_{h,\gamma\,k}-C^{i\lambda}_{h,\gamma}\,;\,\overset{\beta}{_s}\;V^s_{\beta k}+C^{j\lambda}_{h,\gamma}\,V^i_{jk}-C^{i\lambda}_{j,\gamma}\,V^j_{hk}-C^{i\lambda}_{h,j}\,V^j_{\gamma k}.$$

Therefore, we have

Theorem 2.3. If $C^{i\ \lambda}_{h,\ \gamma|k}$ and $\overline{C}^{i\ \lambda}_{h,\ \gamma T k}$ are the covariant derivatives of $C^{i\ \lambda}_{h,\ \gamma}$ and $\overline{C}^{i\ \lambda}_{h,\ \gamma}$ respectively, then

$$\overline{C}^{i\ \lambda}_{h,\ \gamma T k} = C^{i\ \lambda}_{h,\ \gamma|k} + C^{i\ \lambda}_{h,\ \gamma||k} - \sigma_{,\,k}\, C^{i\ \lambda}_{h,\ \gamma}\,, \tag{2.3}$$

where

$$C^{i\ \lambda}_{h,\ \gamma||k} = \sigma_{,\,k}\, C^{i\ \lambda}_{h,\ \gamma} - C^{i\ \lambda}_{h,\ \gamma\,;}{}^{\beta}_{s}\, V^{s}_{\beta k} + C^{j\ \lambda}_{h,\ \gamma}\, V^{i}_{jk} - C^{i\ \lambda}_{j,\ \gamma}\, V^{j}_{hk} - C^{i\ \lambda}_{h,\ j}\, V^{j}_{\gamma k}.$$

3. The Curvature Tensors in $\overline{A}^{(m)}_n$. In the space $A^{(m)}_n$ the curvature tensor R^i_{jkh} [3] is defined by

$$R^i_{jkh} = \overset{0}{\Gamma}{}^i_{jk,\,h} - \overset{0}{\Gamma}{}^i_{jh,\,k} - \overset{0}{\Gamma}{}^i_{jk\,;}{}^{\alpha}_{l}\, \overset{0}{\Gamma}{}^l_{\alpha h} + \overset{0}{\Gamma}{}^i_{jh\,;}{}^{\alpha}_{l}\, \overset{0}{\Gamma}{}^l_{\alpha k} + \overset{0}{\Gamma}{}^i_{nh}\, \overset{0}{\Gamma}{}^n_{jk} - \overset{0}{\Gamma}{}^i_{nk}\, \overset{0}{\Gamma}{}^n_{jh}.$$

In analogy to the above, we define the curvature tensor $\overline{R}^i_{jkh}$ in $\overline{A}^{(m)}_n$ by

$$\overline{R}^i_{jkh} = \overset{\bar 0}{\Gamma}{}^i_{jk,\,h} - \overset{\bar 0}{\Gamma}{}^i_{jh,\,k} - \overset{\bar 0}{\Gamma}{}^i_{jk\,;}{}^{\alpha}_{l}\, \overset{\bar 0}{\Gamma}{}^l_{\alpha h} + \overset{\bar 0}{\Gamma}{}^i_{jh\,;}{}^{\alpha}_{l}\, \overset{\bar 0}{\Gamma}{}^l_{\alpha k} + \overset{\bar 0}{\Gamma}{}^i_{nh}\, \overset{\bar 0}{\Gamma}{}^n_{jk} - \overset{\bar 0}{\Gamma}{}^i_{nk}\, \overset{\bar 0}{\Gamma}{}^n_{jh}. \tag{3.1}$$

Introducing (1.4) in (3.1), we have

Theorem 3.1. If R^i_{jkh} and $\overline{R}^i_{jkh}$ are the curvature tensors of the spaces $A^{(m)}_n$ and $\overline{A}^{(m)}_n$, respectively, then

$$\overline{R}^i_{jkh} = R^i_{jkh} + H^i_{jkh} - \Theta^i_{jkh}, \tag{3.2}$$

where we have put

$$H^i_{jkh} = V^i_{jk,\,h} - V^i_{jh,\,k} - V^i_{jk\,;}{}^{\alpha}_{l}\, V^l_{\alpha h} + V^i_{jh\,;}{}^{\alpha}_{l}\, V^l_{\alpha k} + V^i_{nh}\, V^n_{jk} - V^i_{nk}\, V^n_{jh}$$

and

$$\Theta^i_{jkh} = \overset{0}{\Gamma}{}^i_{jk\,;}{}^{\alpha}_{l}\, V^l_{\alpha h} + V^i_{jk\,;}{}^{\alpha}_{l}\, \overset{0}{\Gamma}{}^l_{\alpha h} - \overset{0}{\Gamma}{}^i_{jh\,;}{}^{\alpha}_{l}\, V^l_{\alpha k} - V^i_{jh\,;}{}^{\alpha}_{l}\, \overset{0}{\Gamma}{}^l_{\alpha k}$$
$$- \overset{0}{\Gamma}{}^i_{nh}\, V^n_{jk} \quad V^i_{nh}\, \overset{0}{\Gamma}{}^n_{jk} + \overset{0}{\Gamma}{}^i_{nk}\, V^n_{jh} + V^i_{nk}\, \overset{0}{\Gamma}{}^n_{jh}.$$

Now, analogous to the second curvature tensor K^i_{hkl} [3] of $A^{(m)}_n$, we may define the curvature tensor $\overline{K}^i_{hkl}$ of $\overline{A}^{(m)}_n$ by

$$\overline{K}^i_{hkl}=\overline{R}^i_{jkl}+\overline{C}^{i}_{h},{}^{\lambda}_{\gamma}\,\overline{R}^{\gamma}_{\lambda kl}\,.$$

Putting (1.5) and (3.2) in this relation, we conclude

Theorem 3.2. Let K^i_{hkl} be the curvature tensor of $A^{(m)}_n$. Then, under the change (1.1), the tensor K^i_{hkl} is transformed as

$$\overline{K}^i_{hkl}=K^i_{hkl}+\overset{*}{K}{}^i_{hkl}-T^i_{hkl}, \tag{3.3}$$

where
$$\overset{*}{K}{}^i_{hkl}=H^i_{hkl}+C^{i}_{h},{}^{\lambda}_{\gamma}\,H^{\gamma}_{\lambda kl}$$

and
$$T^i_{hkl}=\Theta^i_{hkl}+C^{i}_{h},{}^{\lambda}_{\gamma}\,\Theta^{\gamma}_{\lambda kl}.$$

It is well known that the curvature $P^{i}_{hk},{}^{\lambda}_{\gamma}$ [3] in $A^{(m)}_n$ is given by

$$P^{i}_{hk},{}^{\lambda}_{\gamma}=\overset{0}{\Gamma}{}^{i}_{hk},{}^{\lambda}_{\gamma}-C^{i}_{h},{}^{\lambda}_{\gamma|k}+C^{i}_{h},{}^{\delta}_{n}\,\overset{0}{\Gamma}{}^{n}_{sk}\,;\,{}^{\lambda}_{\gamma}\,p^{s}_{\delta}\,.$$

Corresponding to this, we define the curvature $\overline{P}^{i}_{hk},{}^{\lambda}_{\gamma}$ in $\overline{A}^{(m)}_n$ by

$$\overline{P}^{i}_{hk},{}^{\lambda}_{\gamma}=\overset{\overline{0}}{\Gamma}{}^{i}_{hk},{}^{\lambda}_{\gamma}-\overline{C}^{i}_{h},{}^{\lambda}_{\gamma\mathrm{T}k}+\overline{C}^{i}_{h},{}^{\delta}_{n}\,\overset{\overline{0}}{\Gamma}{}^{n}_{sk}\,;\,{}^{\lambda}_{\gamma}\,p^{s}_{\delta}\,.$$

Using (1.4), (1.5) and (2.3) in the above relation, we get

Theorem 3.3. Under the change (1.1) the curvature $P^{i}_{hk},{}^{\lambda}_{\gamma}$ of $A^{(m)}_n$ is transformed as

$$\overline{P}^{i}_{hk},{}^{\lambda}_{\gamma}=P^{i}_{hk},{}^{\lambda}_{\gamma}+\overset{*}{P}{}^{i}_{hk},{}^{\lambda}_{\gamma}+\sigma,{}_{k}\,C^{i}_{h},{}^{\lambda}_{\gamma}, \tag{3.4}$$

where we have written

$$\overset{*}{P}{}^{i}_{hk},{}^{\lambda}_{\gamma}=V^{i}_{jk}\,;\,{}^{\lambda}_{\gamma}-C^{i}_{h},{}^{\lambda}_{\gamma||k}+C^{i}_{h},{}^{\delta}_{n}\,V^{n}_{sk}\,;\,{}^{\lambda}_{\gamma}\,p^{s}_{\delta}\,.$$

Analogous to the curvature $S^{i}_{h},{}^{\lambda}_{\gamma},{}^{\mu}_{s}$ [3] of $A^{(m)}_n$, we define the curvature $\overline{S}^{i}_{h},{}^{\lambda}_{\gamma},{}^{\mu}_{s}$ of $\overline{A}^{(m)}_n$ by

$$\overline{S}^{i}_{h},{}^{\lambda}_{\gamma},{}^{\mu}_{s}=\overline{C}^{i}_{h},{}^{\lambda}_{\gamma}\,;\,{}^{\mu}_{s}-\overline{C}^{i}_{n},{}^{\lambda}_{\gamma}\,\overline{C}^{n}_{h},{}^{\mu}_{s}-\overline{C}^{i}_{n},{}^{\mu}_{s}\,\overline{C}^{n}_{h},{}^{\lambda}_{\gamma}+\overline{C}^{i}_{h},{}^{\mu}_{s}\,;\,{}^{\lambda}_{\gamma}\,.$$

Applying (1.5) in this relation, we get

Theorem 3.4. Under the conformal change (1.1) the curvature $S_{h,\ \gamma,\ s}^{i\ \lambda\ \mu}$ of $A_n^{(m)}$ remains invariant.

We define the Riemannian curvature $R(x^i, p_\alpha^i\ v^\alpha, X^i)$ [5] of $A_n^{(m)}$, at a point x^i, by means of the formula

$$R(x^i, p_\alpha^i\, v^\alpha, X^i) = \frac{R_{ijkh}\, p_\alpha^j\, p_\beta^k\, X^i\, X^j\, v^\alpha\, v^\beta}{(g_{ih}\, g_{jk} - g_{ik}\, g_{jh})\, p_\alpha^j\, p_\beta^k\, X^i\, X^j\, v^\alpha\, v^\beta},$$

where $R_{ijkh} = g_{is}\, R^s_{jkh}$. Corresponding to this, we can define the Riemannian curvature $\bar{R}(x^i, p_\alpha^i\ v^\alpha, X^i)$ of $\bar{A}_n^{(m)}$ by

$$\bar{R}(x^i, p_\alpha^i\, v^\alpha, X^i) = \frac{\bar{R}_{ijkh}\, p_\alpha^j\, p_\beta^k\, \bar{X}^i\, \bar{X}^j\, \bar{v}^\alpha\, \bar{v}^\beta}{(\bar{g}_{ih}\, \bar{g}_{jk} - \bar{g}_{ik}\, \bar{g}_{jh})\, p_\alpha^j\, p_\beta^k\, \bar{X}^i\, \bar{X}^j\, \bar{v}^\alpha\, \bar{v}^\beta}.$$

Applying (1.1) and (3.2) in this, we conclude

Theorem 3.5. The Riemannian curvature $R(x^i, p_\alpha^i\, v^\alpha, X^i)$ and $\bar{R}(x^i, p_\alpha^i\, v^\alpha, X^i)$ of the spaces $A_n^{(m)}$ and $\bar{A}_n^{(m)}$ are compared by the relation

$$\bar{R}(x^i, p_\alpha^i\, v^\alpha, X^i) = e^{-2\sigma}[R(x^i, p_\alpha^i\, v^\alpha, X^i) + \overset{1}{R}(x^i, p_\alpha^i\, v^\alpha, X^i) - \overset{2}{R}(x^i, p_\alpha^i\, v^\alpha, X^i)],$$

where $$\overset{1}{R}(x^i, p_\alpha^i\, v^\alpha, X^i) = \frac{H_{ijkh}\, p_\alpha^j\, p_\beta^k\, X^i\, X^h\, v^\alpha\, v^\beta}{(g_{ih}\, g_{jk} - g_{ik}\, g_{jh})\, p_\alpha^j\, p_\beta^k\, X^i\, X^h\, v^\alpha\, v^\beta}$$

and $$\overset{2}{R}(x^i, p_\alpha^i\, v^\alpha, X^i) = \frac{\Theta_{ijkh}\, p_\alpha^j\, p_\beta^k\, X^i\, X^h\, v^\alpha\, v^\beta}{(g_{ih}\, g_{jk} - g_{ik}\, g_{jh})\, p_\alpha^j\, p_\beta^k\, X^i\, X^h\, v^\alpha\, v^\beta}.$$

Analogous to the Riemannian curvature $R(x^i, p_\alpha^i)$ [5] in $A_n^{(m)}$, at an isotropic point x^i, the Riemannian curvature $\bar{R}(x^i, p_\alpha^i)$ in $\bar{A}_n^{(m)}$, at the corresponding isotropic point, is given by

$$\bar{R}(x^i, p^i_\alpha) = \frac{\bar{R}^i_{jkh}\, p^j_\alpha\, p^k_\beta}{(\delta^i_h\, \bar{g}_{jk} - \delta^i_k\, \bar{g}_{jh})\, p^j_\alpha\, p^k_\beta}.$$

By the help of (1.1), (3.1) and the above relation, we have

Theorem 3.6. Let $R(x^i, p^i_\alpha)$ and $\bar{R}(x^i, p^i_\alpha)$ be the Riemannian curvatures at the corresponding isotropic points of the spaces $A^{(m)}_n$ and $\bar{A}^{(m)}_n$ respectively, then

$$\bar{R}(x^i, p^i_\alpha) = e^{-2\sigma}[R(x^i, p^i_\alpha) + \overset{1}{R}(x^i, p^i_\alpha) - \overset{2}{R}(x^i, p^i_\alpha)],$$

where $$\overset{1}{R}(x^i, p^i_\alpha) = \frac{H^i_{jkh}\, p^j_\alpha\, p^k_\beta}{(\delta^i_h\, g_{jk} - \delta^i_k\, g_{jh})\, p^j_\alpha\, p^k_\beta}$$

and $$\overset{2}{R}(x^i, p^i_\alpha) = \frac{\Theta^i_{jkh}\, p^j_\alpha\, p^k_\beta}{(\delta^i_h\, g_{jk} - \delta^i_k\, g_{jh})\, p^j_\alpha\, p^k_\beta}.$$

4. The Bianchi Identities. The curvature tensor R^i_{jkh} of the space $A^{(m)}_n$ satisfies the identity [5]

$$R^i_{jkh|\gamma} + R^i_{jh\gamma|k} + R^i_{j\gamma k|h} + \overset{0}{\Gamma}{}^i_{jk}\, ;^{\alpha}_{m}\, R^m_{\alpha h\gamma} + \overset{0}{\Gamma}{}^i_{jh}\, ;^{\alpha}_{m}\, R^m_{\alpha\gamma k} + \overset{0}{\Gamma}{}^i_{j\gamma}\, ;^{\alpha}_{m}\, R^m_{\alpha kh} = 0. \tag{4.1}$$

In the space $\bar{A}^{(m)}_n$ the same identity can be written as

$$\bar{R}^i_{jkh\mathrm{T}\gamma} + \bar{R}^i_{jh\gamma\mathrm{T}k} + \bar{R}^i_{j\gamma k\mathrm{T}h} + \overset{\bar{0}}{\Gamma}{}^i_{jk}\, ;^{\alpha}_{m}\, \bar{R}^m_{\alpha h\gamma} + \overset{\bar{0}}{\Gamma}{}^i_{jh}\, ;^{\alpha}_{m}\, \bar{R}^m_{\alpha\gamma k} + \overset{\bar{0}}{\Gamma}{}^i_{j\gamma}\, ;^{\alpha}_{m}\, \bar{R}^m_{\alpha kh} = 0.$$

Using (1.4), (3.2) and (4.1) in the above relation, we get

Theorem 4.1. The Bianchi identity for the curvature tensor $\overset{1}{\bar{R}}{}^i_{jkh}$ n $\bar{A}^{(m)}_n$ is given by

$$R^{i}_{jkh\mathrm{T}\gamma}+\bar{R}^{i}_{jh\gamma\mathrm{T}k}+\bar{R}^{i}_{j\gamma k\mathrm{T}h}=R^{i}_{jkh|\gamma}+R^{i}_{jh\gamma|k}+R^{i}_{j\gamma k|h}-V^{i}_{jk}\,;\,{}^{\alpha}_{m}\,R^{m}_{\alpha h\gamma}$$

$$-V^{i}_{jh}\,;\,{}^{\alpha}_{m}\,R^{m}_{\alpha\gamma k}-V^{i}_{j\gamma}\,;\,{}^{\alpha}_{m}\,R^{m}_{\alpha kh}-(\overset{0}{\Gamma}{}^{i}_{jk}\,;\,{}^{\alpha}_{m}+V^{i}_{jk}\,;\,{}^{\alpha}_{m})\times$$

$$(H^{m}_{\alpha h\gamma}-\Theta^{m}_{\alpha h\gamma})-(\overset{0}{\Gamma}{}^{i}_{jh}\,;\,{}^{\alpha}_{m}+V^{i}_{jh}\,;\,{}^{\alpha}_{m})\,(H^{m}_{\alpha\gamma k}-\Theta^{m}_{\alpha\gamma k})$$

$$-(\overset{0}{\Gamma}{}^{i}_{j\gamma}\,;\,{}^{\alpha}_{m}+V^{i}_{j\gamma}\,;\,{}^{\alpha}_{m})\,(H^{m}_{\alpha kh}-\Theta^{m}_{\alpha kh}).$$

In $A^{(m)}_{n}$ the curvature tensors R^{i}_{jkl}, K^{i}_{jkl} and $P^{i}_{hk,}{}^{\alpha}_{m}$ satisfy the identity [2]

$$K^{i}_{hkl|j}+K^{i}_{hlj|k}+K^{i}_{hjk|l}+P^{i}_{hk,}{}^{\alpha}_{m}\,R^{m}_{\alpha lj}+P^{i}_{hl,}{}^{\alpha}_{m}\,R^{m}_{\alpha jk}+P^{i}_{hj,}{}^{\alpha}_{m}\,R^{m}_{\alpha kl}=0. \tag{4.2}$$

Under the change (1.1) it transforms to

$$\bar{K}^{i}_{hkl\mathrm{T}j}+\bar{K}^{i}_{hlj\mathrm{T}k}+\bar{K}^{i}_{hjk\mathrm{T}l}+\bar{P}^{i}_{hk,}{}^{\alpha}_{m}\,\bar{R}^{m}_{\alpha lj}+\bar{P}^{i}_{hl,}{}^{\alpha}_{m}\,\bar{R}^{m}_{\alpha jk}+\bar{P}^{i}_{hj,}{}^{\alpha}_{m}\,\bar{R}^{m}_{\alpha kl}=0.$$

Introducing (3.2), (3.4) and (4.2) in this, we have

Theorem 4.2. In the conformal areal space of the submetric class, the Bianchi identity

$$\bar{K}^{i}_{hkl\mathrm{T}j}+\bar{K}^{i}_{hlj\mathrm{T}k}+\bar{K}^{i}_{hjk\mathrm{T}l}=K^{i}_{hkl|j}+K^{i}_{hlj|k}+K^{i}_{hjk|l}$$

$$-P^{i}_{hk,}{}^{\alpha}_{m}\,(H^{m}_{\alpha lj}-\Theta^{m}_{\alpha lj})-P^{i}_{hl,}{}^{\alpha}_{m}\,(H^{m}_{\alpha jk}-\Theta^{m}_{\alpha jk})$$

$$-P^{i}_{hj,}{}^{\alpha}_{m}\,(H^{m}_{\alpha kl}-\Theta^{m}_{\alpha kl})-(\overset{*}{P}{}^{i}_{hk,}{}^{\alpha}_{m}+\sigma_{,k}\,C^{i}_{h,}{}^{\alpha}_{m})\,(R^{m}_{\alpha lj}$$

$$+H^{m}_{\alpha lj}-\Theta^{m}_{\alpha lj})-(\overset{*}{P}{}^{i}_{hl,}{}^{\alpha}_{m}+\sigma_{,l}\,C^{i}_{h,}{}^{\alpha}_{m})\,(R^{m}_{\alpha jk}+H^{m}_{\alpha jk}-\Theta^{m}_{\alpha jk})$$

$$-(\overset{*}{P}{}^{i}_{hj,}{}^{\alpha}_{m}+\sigma_{,j}\,C^{i}_{h,}{}^{\alpha}_{m})\,(R^{m}_{\alpha kl}+H^{m}_{\alpha kl}-\Theta^{m}_{\alpha kl})$$

holds good.

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