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FLOW BEHAVIOUR OF MICRO-POLAR FLUID BETWEEN CONE-PLATE GEOMETRY

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ABSTRACT

The flow behaviour of micro-polar fluid between cone and plate geometry is discussed. The expressions for velocity and micro-rotation have been obtained. It has been observed that the velocity at any point of micro-polar fluid is larger than the corresponding viscous fluid. The micro-rotations increase with the increase of viscosity constant but decrease with the increase in coupling constant.

1. Introduction

The cone-plate arrangement plays an important role in engineering. This kind of instrument is used to determine the viscosity of the fluids, particularly of non,Newtonian fluids. The cone-plate arrangement consists of a stationary plate with an inverted cone, which is rotated at a known angular velocity Ω with its apex touching the plate. The viscosity is determined by the torque required to turn the cone. The angular distance between the cone and plate is kept very small, say about half of a degree.

Many authors have used cone-plate visco-meter to determine the flow behaviour and heating effects for Newtonian and non-Newtonian fluids. Bird and Turiyan [1] studied the viscous heating in cone-plate viscometer. Turiyan ([4] and [5]) further extended the problem for non-Newtonian and power-law fluids. Eringen [2] introduced the theory of micro-fluids which can support body moments and are influenced by spin inertia. No attempt has so far been made to analyse the flow pattern of a micro-polar fluid in cone and plate geometry. The importance of micro-polar fluids is growing rapidly due to the typical behaviour imparted by

such type of fluids, and scientists are beginning to study the behaviour of these fluids from different angles. In the present paper, we study the flow behaviour of micro-polar fluid in between cone and plate geometry.

2. Formulation of the Problem

The basic equations for micro-polar fluids are given by Eringen ([3], pp. 7–11, Eq. (4.1) to (6.4))

In the present problem, the plate is at rest and the cone rotates about its axis with an angular velocity Ω. Assuming the flow to be entirely tangential, the velocity vector $(\overline{V})$ and the micro-rotational vector $(\overline{V})$ in spherical polar coordinates system may be expressed as follows :

$$\overline{v} = v_{\phi}(r, \theta)\, e_{\phi} \tag{1}$$

and

$$\overline{\nu} = \nu_1(r, \theta)\, e_r + \nu_2(r, \theta)\, e_{\theta}. \tag{2}$$

Under the assumptions of Stokesian flow (i.e. the omission of the flow inertial terms) and neglecting the body forces and couples, the flow is described by the following equations :

$$\frac{\partial p}{\partial r} = \frac{\partial p}{\partial \theta} = 0, \tag{3}$$

$$0 = (\mu_v + k_v)\, D v_{\phi} + k_v g(r, \theta), \tag{4}$$

$$0 = (\alpha_v + \beta_v + \gamma_v)\frac{\partial f}{\partial r} - \frac{\gamma_v}{r}\left(\frac{\partial g}{\partial \theta} + g \cot\theta\right) + \frac{k_v}{r}\left(\frac{\partial v_{\theta}}{\partial \theta} + v_{\phi} \cot\theta\right) - 2k_v \nu_1, \tag{5}$$

$$0 = (\alpha_v + \beta_v + \gamma_v)\frac{\partial f}{2\partial \theta} + \gamma_v\left(\frac{\partial g}{\partial r} + \frac{g}{r}\right) - k_v\left(\frac{\partial v_{\phi}}{\partial r} + \frac{v_{\phi}}{r}\right) - 2k_v \nu_2, \tag{6}$$

where $f(r, \theta)$ and $g(r, \theta)$ are given by

$$f(r, \theta) = \text{div } v = \frac{\partial v_1}{\partial r} + \frac{2v_1}{r} + \frac{1}{r}\frac{\partial v_2}{\partial \theta} + \frac{v_2 \cot \theta}{r} \tag{7}$$

and

$$g(r, \theta) = [\text{curl } v]_\phi = \frac{\partial v_2}{\partial r} + \frac{v_2}{r} + \frac{\partial v_2}{\gamma \partial \theta}, \tag{8}$$

and

$$D = \nabla^2 - \frac{1}{r^2 \sin^2 \theta}, \tag{9}$$

where ∇^2 represents the Laplacian operator in spherical polar coordinates.

From the equations (3), we infer that a constant pressure acts throughout the flow region. From the equations (4), (5) and (6) we see that the velocity satisfies the following equation :

$$D\left(D - \frac{\lambda^2}{R^2}\right) v_\phi = 0, \tag{10}$$

and f and g satisfy the equations

$$\left(\nabla^2 + \frac{c^2}{R^2}\right) f = 0 \tag{11}$$

and

$$\left(D - \frac{\lambda^2}{R^2}\right) g = 0, \tag{12}$$

respectively, where

$$\frac{\lambda^2}{R^2} = \frac{k_v(2\mu_v + k_v)}{\gamma_v(\mu_v + k_v)},$$

$$\frac{c^2}{R^2} = \frac{2k_v}{\alpha_v + \beta_v + \gamma_v}, \tag{13}$$

and R is the slant height of the cone.

3. Solution of the Problem

Solving equations (10), (11) and (12) under the following boundary conditions

$$v_\phi = \Omega R \sin \theta_1 \text{ at } \theta = \theta_1 \text{ and } r = R$$

$$u_\phi = 0 \text{ at } \theta = \pi/2 \tag{14}$$

$$v_1 = v_2 = 0 \text{ at } \theta = \theta_1$$

and $$v_1 = v_2 = 0 \text{ at } \theta = \pi/2,$$

the equation (10) can be written as

$$v = v_{\phi_1} + v_{\phi_2}, \tag{15}$$

where

$$Dv_{\phi_1} = 0, \tag{16}$$

and

$$\left(D - \frac{\lambda^2}{R^2} \right) v_{\phi_2} = 0. \tag{17}$$

The solutions of (16) and (17) are of the forms

$$v_{\phi_1} = \left(Ar^n + \frac{B}{r^{n+1}} \right) \{ A_n P_n^{(1)}(\cos\theta) + B_n Q_n^{(1)}(\cos\theta) \}$$

and (18)

$$v_{\phi_2} = r^{-1/2} \left[(I_{n+1/2} \left(\frac{\lambda r}{R} \right) + Dr^{-1/2} K_{n+1/2} \left(\frac{\lambda r}{R} \right) \right] \times \{ A_n P_n^{(1)}(\cos\theta) + B_n Q_n^{(1)}(\cos\theta) \}. \tag{19}$$

where $I_{n+1/2}\left(\frac{\lambda r}{R}\right)$ and $K_{n+1/2}\left(\frac{\lambda r}{R}\right)$ are the modified Bessel functions, and $P_n^{(1)}(\cos\theta)$ and $Q_n^{(1)}(\cos\theta)$ are the associated Legendre polynomials. From equations (10) and (11) expressions for $f(r, \theta)$ and $g(r, \theta)$ are as follows :

$$f(r, \theta) = \frac{c^*}{r^{1/2}} I_{3/2} \left(\frac{cr}{R} \right) \cos\theta, \tag{20}$$

$$g(r, \theta) = -\left(\frac{\mu_v + k_v}{k_v} \right) Dv$$

$$= -\frac{\lambda^2}{R^2} \left(\frac{\mu_v + k_v}{k_v} \right) v_{\phi_2}. \tag{21}$$

Substituting these values of $f(r, \theta)$ and $g(r, \theta)$ into the equations (5) and (6) we obtain

$$v_1 = \left[A^* + \frac{\mu_v + k_v}{k_v r^{3/2}} \left\{ 3B^* I_{3/2} \left(\frac{\lambda r}{R} \right) \right\} \right.$$

$$+ \frac{R^2 c^*}{c^2 r^{3/2}} \left\{ 2 I_{3/2} \left(\frac{cr}{R} \right) \right. \tag{22}$$

$$+ \frac{cr}{R} I_{1/2}\left(\frac{cr}{R}\right)\Big]\cos\theta,$$

$$v_2 = \Big[-\Lambda^* + \frac{(\mu_v+k_v)B^*}{k_v r^{3/2}}\Big\{ I_{3/2}\left(\frac{\lambda r}{R}\right) + \frac{\lambda r}{R} I_{1/2}\left(\frac{\lambda r}{R}\right)\Big\}$$

$$-\frac{c^* R^2}{c^2 R^{3/2}} I_{3/2}\left(\frac{c}{R}\right)\Big]\sin\theta, \qquad (23)$$

where

$$\Lambda^* = \Omega\,[1 - 2\,I_{3/2}(\lambda)\,\{I_{3/2}(c)\times c\,I_{1/2}(c)\}/D_1]$$

$$B^* = 2\Omega(\{_{3/2}(c) + c\,I_{1/2}(c)\}]/D_1,$$

$$C^* = -\frac{2\Omega c^2}{R^2}\Big[\Big\{\frac{3(\mu_v+k_v)}{k_v} I_{3/2}(\lambda) + \lambda I_{1/2}(\lambda)\Big\}\Big]\Big/D_1, \qquad (24)$$

$$D_1 = \Big[\frac{(\mu_v+k_v)}{k_v}\Big\{\lambda I_{3/2}(c)\,I_{1/2}(c) + \frac{\lambda c}{R^2} I_{1/2}(\lambda) I_{1/2}(c)\Big\}$$

$$-\frac{\mu_v}{k_v}\Big\{I_{3/2}(\lambda)\,I_{3/2}(c) - \frac{c}{R} I_{1/2}(c)\,I_{3/2}(\lambda)\Big\}$$

$$-\frac{(\mu_v+k_v)}{k_v}\Big\{I_{3/2}(\lambda) - \frac{3\lambda}{R} I_{1/2}(\lambda)\Big\}\Big].$$

4. Results and Conctusions

From Table 1, it may be observed that the velocity is larger in micropolar fluid than that in corresponding viscous fluids and it increases with the coupling constant (k_v/γ_v). The velocity decreases as the viscosity coefficient (μ_v/γ_v) increases, and it increases with the distance from the origin. It is also apparent from Table 2, that the value of micro-rotations $\Lambda(r, \theta)$ and $B(r, \theta)$ first suddenly falls, afterwards increases with the distance from the origin and it increases with the viscosity coefficient but decreases when the coupling constant increases.

TABLE I

Velocity Distribution In Cone—Plate Geometry

μ_v/γ_v	r/R	$k_v/\gamma_v=0$	$k_v/\gamma_v=0.5$	$k_v/\gamma_v=1.0$	$k_v/\gamma_v=1.5$
.5	0.0	.00000	0.00000	.00005	.00000
	0.2	.19621	.21739	.22467	·22922
	0.4	.40097	·41231	·41779	·42087
	0.6	.60090	.60825	·61164	·61326
	0.8	.80098	.80450	.80600	.80655
1.0	0.0	.00000	.00000	.00000	.00000
	0.2	.19997	.21042	.21608	.22026
	0.4	.40098	.40747	.41155	.41428
	0.6	.60096	.60513	.60758	.60897
	0.8	.80099	.80298	.80403	.80449
1.5	0.0	.00000	.00000	.00000	.00000
	0.2	.19946	.20758	.31199	.21552
	0.4	.40098	.40551	.40864	.41087
	0.6	.60093	.60387	.60438	.70505
	0.8	.80100	.80237	.80314	.80348

TABLE 2

Variation of Micro-Rotation

		$v_1/\Omega \cos\theta$			$v_2/\Omega \sin\theta$		
μ_ψ/γ_ψ	r	$k_\psi/\gamma_\psi=0.5$	$k_\psi/\gamma_\psi=1$	$k_\psi/\gamma_\psi=1.5$	$k_\psi/\gamma_\psi=0.5$	$k_\psi/\gamma_\psi=1.0$	$k_\psi/\gamma_\psi=1.5$
.5	0.0	1.6845	1.2222	1.0190	-1.5584	-1.1829	-1.0053
	0.2	.6768	.6416	.6151	-0.8000	-0.7755	-0.7639
	0.4	.7014	.6578	.6321	-0.8144	-0.7853	-0.7761
	0.6	.7117	.6697	.6500	-0.8216	-0.7942	-0.7904
	0.8	.7180	.6824	.6707	-0.8263	-0.8039	-0.8064
1.0	0.0	1.4015	1.0783	1.0139	-1.3085	-1.0537	-1.0042
	0.2	.7271	.7016	.6771	-0.8333	-0.8155	-0.7990
	0.4	.7490	.7181	.6961	-0.8469	-0.8263	-0.8123
	0.6	.7580	.7311	.7169	-0.8526	-0.8353	-0.8270
	0.8	.7649	.7452	.7408	-0.8572	-0.8451	-0.8434
1.5	0.0	.7218	1.0809	1.0133	-0.7396	-1.0585	-1.0044
	0.2	.7486	.7314	.7112	-0.8475	-0.8352	-0.8217
	0.4	.7690	.7476	.7311	-0.8601	-0.8458	-0.8353
	0.6	.7776	.7609	.7529	-0.8678	-0.8597	-0.8587
	0.8	.7845	.7756	.7773	-0.8722	-0.8707	-0.8758

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