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MHD SLIP FLOW IN RECTANGULAR POROUS CHANNELS OF DIFFERENT PERMEABILLITY

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Abstract. The steady laminar flow of a viscous incompressible fluid in the presence of a transverse magnetic field in a channel of rectangular cross section with two porous walls of different permeability has been examined, using the slip flow conditions at the walls. A numerical estimate for the velocity profile, pressure rise and the skin-friction at the walls has been made and conclusions drawn.

1. Introduction. The steady laminar flow of a viscous incompressible fluid between two parallel and porous walls of different permeability has been studied by Terrill and Shrestha [1]. This problem has been extended in the frame work of MHD by Reddy [2]. In the present analysis, the effect of slip velocity at the walls is taken into consideration, following the notation of Reddy [2].

2. Basic Equations and Solutions. We seek the solutions of eqs. (3.1) and (3.2) of Reddy's work subject to the boundary conditions :

$$\lambda=-1 : u(x, \lambda)=\epsilon \frac{\partial u}{\partial \lambda}, \; V(x, \lambda)=V_1,$$

$$\lambda=1 : u(x, \lambda)=-\epsilon \frac{\partial u}{\partial \lambda}, \; V(x, \lambda)=V_2, \tag{2.1}$$

where $\epsilon=\xi_u/h$, the non-dimensional first order slip as in Schaff and Chambre [3].

when $|V_2|\geqslant|V_1|$. following Reddy's analysis we solve eq. (3.8) of his paper, *i.e.*,

$$f'''+R_2\,(f'^2-ff'')-M^2 f'=K, \tag{2.2}$$

subject to the above boundary conditions, *i.e.*

$$f(-1)=1-\alpha_2, f(1)=1, f'(\pm 1)=\mp\epsilon f''(\pm 1). \tag{2.3}$$

When R_2 is small, we solve eqs. (4.2) of his paper subject to :

$$f_0(-1)=1-\alpha_2,\ f_0(1)=1,\ f_n(\pm 1)=0,\ n\geqslant 1\ ;$$
$$f_n'(\pm 1)=\mp\epsilon f_n''(\pm 1)\ ;\ n\geqslant 0. \tag{2.4}$$

The first-order perturbation solution of eq. (2.2) subject to eq. (2.4) is obtained as :

$$f^{(1)}(\lambda)=f_0(\lambda)+R_2 f_1(\lambda),$$
$$K^{(1)}=K_0+R_2 K_1, \tag{2.5}$$

where $f_0(\lambda)$, $f_1(\lambda)$, K_0 and K_1 are given by

$$f_0(\lambda)=1-\frac{\alpha_2}{2}+2A_1(\sinh M\lambda-A_2\lambda), \tag{2.6}$$

$$f_1(\lambda)=2\beta_1\{\sinh M\lambda-\lambda\sinh M\}+\frac{A_1(2-\alpha_2)}{M}\left\{1-e^{-M\lambda}\right\}$$
$$-\lambda\left\{\frac{A_1}{M}(2-\alpha_2)(1-e^{-M})+\frac{7A_1^2A_2\cosh M}{M}\right.$$
$$\left.+\left(\frac{2-\alpha_2}{2}-A_1A_2\right)A_1\sinh M\right\}+\frac{7A_1^2A_2}{M}\lambda\cosh M\lambda$$
$$+A_1\left\{\frac{(2-\alpha_2)\lambda}{2}-A_1A_2\lambda^2\right\}\sinh M\lambda, \tag{2.7}$$

$$A_1=\frac{\alpha_2}{4\{\sinh M-M(\cosh M+\epsilon M\sinh M)\}}, \tag{2.8}$$

$$A_2=M(\cosh M+\epsilon M\sinh M). \tag{2.9}$$

$$B_1=\{M\cosh M+(\epsilon M^2-1)\sinh M\}^{-1}\left[\left\{\frac{A_1^2A_2M}{2}-\frac{(2-\alpha_2)}{4}A_1M\right\}\right.$$
$$\cosh M-3A_1^2A_2\sinh M+\frac{A_1(2-\alpha_2)}{2}\left\{\frac{1-e^{-M}}{M}-e^{-M}\right\}$$
$$+\frac{\epsilon}{2}\left\{(3MA_1^2A_2+2A_1M(2-\alpha_2))\cosh M-(12A_1^2A_2+\right.$$
$$\left.\left.A_1M(2-\alpha_2)+\frac{A_1M^2}{2}(2-\alpha_2)-A_1^2A_2M^2)\sinh M\right\}\right], \tag{2.10}$$

$$K_0=2A_1M^3(\cosh M+\epsilon M\sinh M), \tag{2.11}$$

$$K_1=4A_1^2(M^2+A_2^2)+7A_1^2A_2M\cosh M$$
$$+A_1M^2\sinh M\left(\frac{2-\alpha_2}{2}-A_1A_2\right)+M^2(1-\alpha_2)$$
$$+A_1M(2-\alpha_2)(e^{-M}-1)+2B_1M^2\sinh M. \tag{2.12}$$

In the absence of magnetic field, the above reduce to :

$$f_0(\lambda)=1-\frac{\alpha_2}{2}+\frac{3\alpha_2(1+2\epsilon)\lambda}{4(1+3\epsilon)}-\frac{\alpha_2\lambda^3}{4(1+3\epsilon)} \tag{2.13}$$

$$f_1(\lambda)=-\frac{\alpha_2^2(1+9\epsilon)\lambda}{560(1+3\epsilon)^3}+\frac{3\alpha_2^2(1+7\epsilon)\lambda^5}{1120(1+3\epsilon)^3}-\frac{\alpha_2^2\lambda^7}{1120(1+3\epsilon)^2}, \tag{2.14}$$

$$K_0=-\frac{3\alpha_2}{2(1+3\epsilon)}, \tag{2.15}$$

$$K_1=\frac{9(36+252\epsilon+560\epsilon^2+420\epsilon^3)\alpha_2^2}{560(1+3\epsilon)^3}-\frac{3\alpha_2(2-\alpha_2)}{4(1+3\epsilon)}. \tag{2.16}$$

When $\alpha_2=2$ and $\epsilon=0$, we recover the solutions of Berman [4] from equations (2.13 through 2.16).

When $|V_1|\geqslant|V_2|$, a perturbation solution for small R_1 can be obtained in a similar manner but this will not present any new physical features.

3. Discussion

(i) Velocity distribution. The velocity components in the axial and perpendicular directions have been obtained from equations (3.5) of Reddy's work, they are

$$\frac{u}{U(0)}=\left[\frac{1}{\alpha_2}-\frac{4R_2}{R^*}\frac{x}{h}\right]f'(\lambda)\ ;\ \frac{V}{U(0)}=\frac{4R_2}{R^*}f(\lambda), \tag{3.1}$$

where $f(\lambda)$ is given by eq. (2.5), $R^*=4h\,U(0)/\nu$ is the entrance Reynolds number and

$$\begin{aligned} f'(\lambda)=&2A_1(M\cosh M\lambda-A_2)+R_2\Big[2B_1(M\cosh M\lambda-\sinh M)\\ &+A_1(2-\alpha_2)e^{-M\lambda}-\Big\{\frac{A_1}{M}(2-\alpha_2)(1-e^{-M})+\frac{7A_1^2A_2\cosh M}{M}\\ &+\left(\frac{2-\alpha_2}{2}-A_1A_2\right)A_1\sinh M\Big\}+\frac{7A_1^2A_2}{M}\{\lambda M\sinh M\lambda+\cosh M\lambda\}+A_1\\ &\Big\{\left(\frac{(2-\alpha_2)\lambda}{2}-A_1A_2\lambda^2\right)M\cosh M\lambda+\left(\frac{2-\alpha_2}{2}-2A_1A_2\lambda\right)\sinh M\lambda\Big\}\Big] \end{aligned} \tag{3.2}$$

The function $f'(\lambda)$ has been plotted in Fig. 1 for $R_2=0{\cdot}5$ and for various values of M, α_2 and ϵ. Increase in the value of α_2 increases the velocity distribution in the channel. It is seen that the dimensionless slip velocity $f'(\pm1)$ increases as M increases. We note, when $\alpha_2=2$, $R_2=0$ and $M=0$, the effect of slip decreases the centre line velocity $f'(0)$ from its maximum value of 1·5 (continuum flow) towards a value of 1·0. It is further observed that the flow at the centre of the channel gets retarded in the presence of the slip as well as the magnetic field.

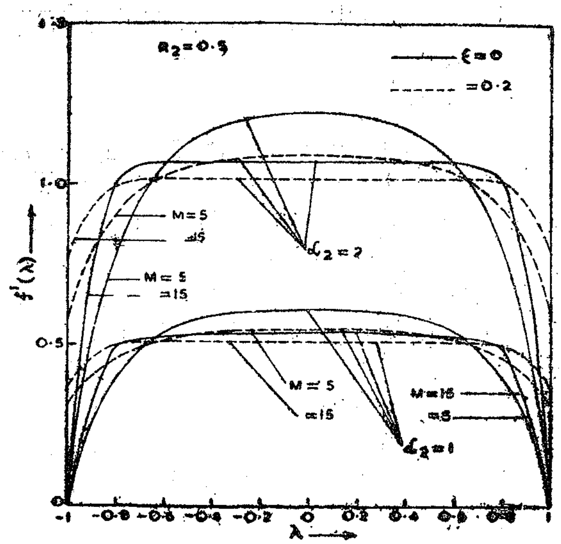

Fig. 1. Plot of velocity distribution in the axial direction

(ii) Pressure distribution. The non-dimensional pressure drop in the axial direction is given by eq. (5·10) of his paper, *i.e.*

$$P_x=\frac{1}{R^*}\left(\frac{x}{h}\right)\left[\frac{1}{\alpha_2}-\frac{2R_2}{R^*}\left(\frac{x}{h}\right)\right](-8\,K), \tag{3.3}$$

where K is given by eq. (2.5)

The variation of the pressure drop in this direction for $R^*=1000$, $R_2=0·5$ has been plotted in Fig. 2 for various values of M, α_2 and ϵ. As in Reddy's work, the pressure drop increases with increase in the Hartmann number M. By increasing the values of ϵ and α_2, it decreases rapidly, more so along the axial direction.

(iii) Friction-coefficient : From eq. (5.13) of his paper, the wall friction in non-dimensional form is given by

$$c_f^{(\mp 1)}=\frac{8}{R^*}\left(\frac{1}{\alpha_2}-\frac{4R_2}{R^*}\,\frac{x}{h}\right)\Big[f''(\lambda)\Big]_{\lambda=-1 \text{ and } 1} \tag{3.4}$$

where

$$f''(-1)=-[2A_1 M^2 \sinh M+R_2 \{(2B_1-A_1^2 A_2)M^2 \sinh M$$

$$+\frac{A_1 M (2-\alpha_2)}{2} (2-M) \sinh M$$

$$+A_1^2 A_2 (12 \sinh M+3M \cosh M)\}], \qquad (3.5)$$

$$f''(1)=2 A_1 M^2 \sinh M+ R_2 \{(2B_1-A_1^2 A_2) M^2 \sinh M$$

$$+\frac{A_1 M(2-\alpha_2)}{2} (2+M) \sinh M$$

$$+A_1^2 A_2 (12 \sinh M+3M \cosh M)\} \qquad (3.6)$$

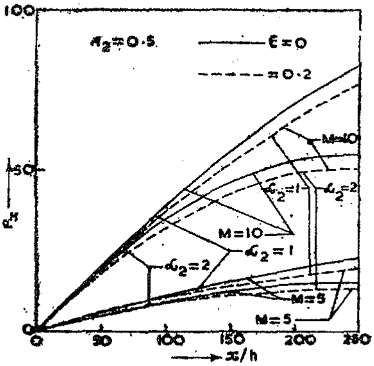

Fig. 2. Plot of pressure drop in the axial direction.

From eqs. (3.5) and (3·6) we note, for $\alpha_2=2$, the skin friction at the walls is equal and opposite. From Fig 3, we observe that this coefficient at the lower wall increases with an increase in the Hartmann number in agreement with Terrill and Shrestha [5]. It is also observed that the skin friction increases with increase in α_2 only upto a certain value of x/h, beyond which it decreases. The effect of the slip coefficient is found, in general, to decrease the skin-friction at the lower wall. From Fig. 4, similar features are also found to hold good for the skin-friction at the upper wall.

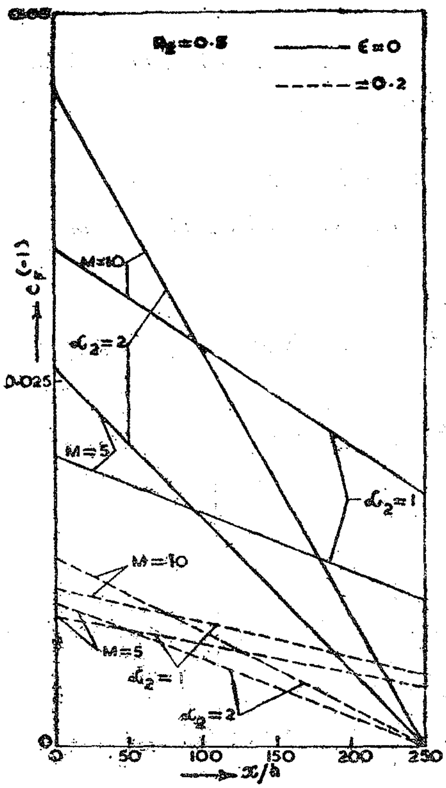

Fig 3. Plot of skin-friction at the lower wall.

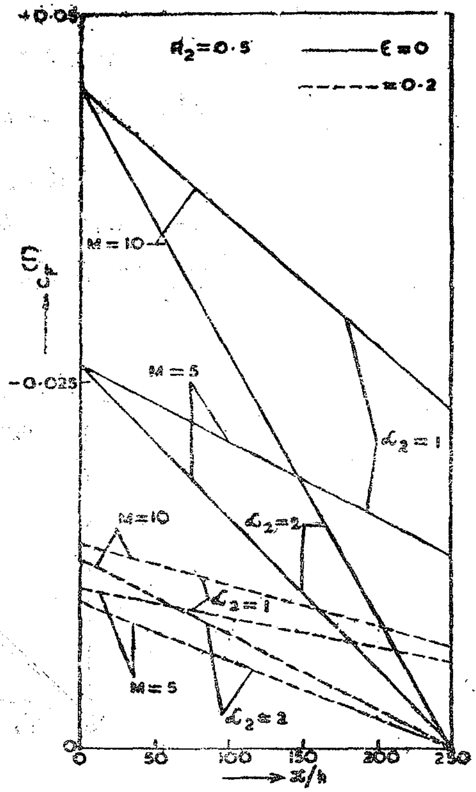

Fig. 4. Plot of skin-friction at the upper wall.

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