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ON HEAT TRANSFER IN MAGNETOHYDRODYNAMIC CHANNEL FLOW UNDER TRANSVERSE MAGNETIC FIELD

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Abstract. The influnce of magnetic field on heat transfer in an electrically conducting viscous fluid flow along a rectangular channel with non-conducting walls is considered. The magnetic field is transversely applied to the fluid flow and wall temperature is assumed to vary linearly in the direction of flow. Expressions are derived for temperature and Nusselt number. Temperature profiles are shown graphically for different values of Hartmann number M, magnetic Eckert number E_c, and a non-dimensional number S^*.

1. Introduction. The problem on heat transfer in magnetohydrodynamic channel flow have been discussed by many authors. Erickson, Wang and Fan (1961) studied the effect of magnetic field, electric field and viscous dissipation on heat transfer in the entrance region of a channel with non-conducting walls. Heat flux at the channel walls was not considered in their paper. Soundalgekar (1968) discussed heat transfer in a fully developed channel flow with conducting as well as non-conducting walls in the absence of externally imposed heat flux at the channel walls.

In the present paper heat transfer in magnetohydrodynamic channel flow under transverse magnetic field with non-conducting walls is discussed. The expression for the velocity profile derived by Shercliff (1953) in the case of steady motion of conducting fluid in a rectangular pipe under transverse magnetic field is untilised to find expressions for temperature and Nusselt number. Numerical calculations for temperature and Nusselt number are carried out under different conditions. The temperature profiles are shown graphically and numerical results for Nusselt number are entered in a table for different values of Hartmann number M, magnetic Eckert number E_c and a non-dimensional number S^*.

2. Mathematical Analysis. Take right-handed axes such that the z-axis is parallel to the fluid velocity V_z and the x-axis is parallel to the imposed uniform magnetic field H_0 existing outside the fluid.

Let the origin be on the centre line of the channel. We take $2a$ as the channel width, V_0 as the mean velocity and σ and η as the fluid conductivity and viscosity respectively ; μ, the permeability of the fluid and the walls, is assumed to be unity and the magnetic field is then continuous at the fluid boundary ; ρ, the fluid density, is of no concern as the flow is unaccelerated.

It has been shown by Shercliff (1953) that when M is large compared with unity, the solutions of the magnetohydrodynamic equations governing the present problem take a boundary layer character as in simple Hartmann case and then assuming that

$$\frac{\partial p}{\partial x}=-K_1,$$

we have

$$V_z=\frac{K_1\, a^2}{\eta M}\left\{1-2e^{-M}\cosh\frac{Mx}{a}\right\} \tag{1}$$

and

$$H_z=4\pi\left(\frac{\sigma}{\eta}\right)^{\frac{1}{2}}\frac{K_1\, a^2}{M}\left\{-\frac{x}{a}+2e^{-M}\sinh\frac{Mx}{a}\right\}. \tag{2}$$

Introducing the non-dimensional variables

$$V=\frac{V_0}{b},\ \xi=\frac{x}{a},\ H_z=H\,H_0,$$

we get

$$V=\frac{K_1\, a^2}{\eta bM}\left\{1-2e^{-M}\cosh M\xi\right\} \tag{3}$$

and

$$H=4\pi\left(\frac{\sigma}{\eta}\right)^{\frac{1}{2}}\frac{K_1 a^2}{H_0\, M}\left\{-\xi+2e^{-M}\sinh M\xi\right\}, \tag{4}$$

where $b=(a\mu H_0^2/4\pi\rho)^{\frac{1}{2}}$, Alfven wave velocity ; $M=abc(\lambda\nu)^{\frac{1}{2}}$, Hartmann number ; ν, the viscous diffusivity ; $\lambda=c^2/4\pi\mu\sigma$, the magnetic diffusivity ; c, the velocity of light assumed here unity.

Now, for steady one-dimensional flow of an incompressible, viscous, electrically conducting fluid with constant properties, the energy equation is given by

$$\rho c_p\, V_z\frac{\partial T}{\partial z}=\chi\,\frac{d^2T}{dx^2}+\eta\left(\frac{dV_z}{dx}\right)^2+\frac{1}{16\pi^2\sigma}\left(\frac{dH_z}{dx}\right)^2. \tag{5}$$

Also, for linearly varying wall temperature, the temperature distribution in the fluid, following Siegel (1960) is assumed as

$$T=\tau z+\bar{\theta}\,(x). \tag{6}$$

Substituting (3), (4) and (6) in equation (5), we get

$$\frac{d^2\theta^*}{d\xi^2}=-\frac{P_r\,E_c}{M^2}\left\{4M^2\,e^{-2M}\cosh 2M\,\xi+1-4Me^{-M}\cosh M\xi\right\}$$
$$+\frac{P_r\,R\,S^*}{M}\left\{1-2e^{-M}\cosh M\xi\right\} \tag{7}$$

where

$$P_r=\frac{\eta C_p}{\lambda},\ \text{Prandtl number},$$

$$E_c=\frac{b^2}{C_p\,\theta_1},\ \text{Magnetic Eckert number},$$

$$R=\frac{ab}{\nu},\ \text{Reynolds number},$$

$$S^*=\frac{S}{K},\quad \theta^*=\frac{\theta}{k},\ S=\frac{\tau a}{\theta_1},\ \theta=\frac{\bar{\theta}}{\theta_1}$$

$$K=\frac{K_1\,a^2}{\eta b},\ \nu=\frac{\eta}{\rho}.$$

Now, integrating equation (7), we get

$$\theta^*=-\frac{P_rE_c}{M^2}\left\{e^{-2M}\cosh 2M\xi+\tfrac{1}{2}\xi^2-\frac{4}{M}\,e^{-M}\cosh M\xi\right\}$$
$$+\frac{P_r\,RS^*}{M}\left\{\tfrac{1}{2}\xi^2-\frac{2}{M^2}\,e^{-M}\cosh M\xi\right\}+A\xi+B, \tag{8}$$

where A and B are constants of integration.

Applying the boundary condition

$$\theta^*=0\ \text{ at }\ \xi=\pm 1$$

we get the values of constants as $A=0$,

$$B=\frac{P_rE_c}{M^2}\left\{e^{-2M}\cosh 2M+\frac{1}{2}-\frac{4}{M}\,e^{-M}\cosh M\right\}$$
$$-\frac{P_r\,R\,S^*}{M}\left\{\frac{1}{2}-\frac{2}{M^2}\,e^{-M}\cosh M\right\}.$$

The Nusselt number N_u is defined as

$$N_u=-\frac{1}{\theta^*_{(0)}}\left(\frac{d\theta^*}{d\xi}\right)_{\xi=1}. \tag{10}$$

From equations (8) and (10), we get

$$N_u = \frac{-\dfrac{P_r E_c}{M^2}\left\{2M\, e^{-2M} \sinh 2M + 1 - 4e^{-M} \sinh M\right\} + \dfrac{P_r R\, S^*}{M}\left\{1 - \dfrac{2}{M}\, e^{-M} \sinh M\right\}}{\dfrac{P_r E_c}{M^2}\left\{e^{-2M} - \dfrac{4}{M}\, e^{-M}\right\} + \dfrac{2P_r R\, S^*}{M^3}\, e^{-M} - B} \tag{11}$$

3. Numerical Calculations and Discussion. Numerical calculations for θ^* are carried out for $P_r = 1$, $R = 0{\cdot}1$, $S^* = 0, 1, 2, 3$; $M = 2, 4, 5$; $E_c = 0{\cdot}3, 0{\cdot}6, 0{\cdot}9$. The results for θ^* are shown graphically in Figs. 1 through 3.

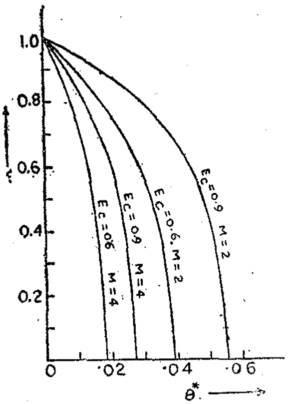

Fig. 1. Tempreature Profile for $S^* = 0$, $M = 2, 4$, $E_c = 0{\cdot}6, 0{\cdot}9$

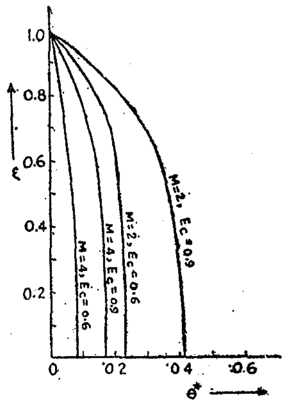

Fig. 2. Temperature Profile for $S^* = 1$, $M = 2, 4$, $E_c = 0{\cdot}6, 0{\cdot}9$

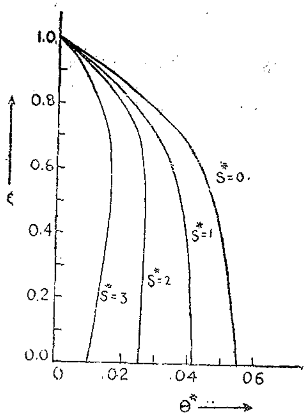

Fig. 3. Variation of θ^* with S^*, $M = 2$, $E_c = 0{\cdot}9$

Fig. 1 shows the temperature profiles in the case of channel with non-conducting walls for $M=2, 4$; $E_c=0{\cdot}6, 0{\cdot}9$ and $S^*=0$. In Fig. 2 temperature profiles are drawn for $M=2, 4$; $E_c=0{\cdot}6, 0{\cdot}9$ and $S^*=1$. It is interesting to see from these figures that the temperature decreases in magnitude with increase in the Hartmann number. Fig 3 shows the variation of θ^* with S^* for $M=2$ and $E_c=0{\cdot}9$. We see that temperature decreases as S^* increases.

Nusselt Number. The numerical results for Nusselt number are entered in the following table for $P_r=1{\cdot}0$ and $R=0{\cdot}1$.

S^*	M	E_c	N_u
0	2	0·3	3·646
		0·6	3·646
		0·9	3·646
	4	0·3	5·524
		0·6	5·524
		0·9	5·524
	5	0·3	6·909
		0·6	6·909
		0·9	6·909
1	2	0·3	24·500
		0·6	6·200
		0·9	5·528
	4	0·3	−15·500
		0·6	11·429
		0·9	8·125
	5	0·3	−37·000
		0·6	15·000
		0·9	10·214

Influence of E_c. For $S=0$, $M=2, 4, 5$ the value N_u remains unchanged when E_c increases. Thus the rate of heat transfer at the wall is not affected by the heat due to viscous dissipation. But for $S^*=1$, the value of N_u is found to decrease with increase in E_c at $M=2$ and at $M \geqslant 4$ it attains negative value for $E_c=0{\cdot}3$ and after that it attians positive value in dereasing order as E_c increases.

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