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Fluctuating Flow of a Visco-Elastic Fluid Past an Infinite Plane, Porous Wall with Constant Suction in Slip Flow Regime

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ABSTRACT

This paper considers the Rivlin-Ericksen fluid flow past an infinite plane porous wall with constant suction in slip flow conditions. We found that

(i) the velocity profile is effected by the rarefaction parameter h_1 in the slip flow, but this is not the case in no slip flow.

(ii) If h_1 increases the velocity profile asymptotically approaches the main stream.

(iii) The main stream fluctuations cause the fluctuations in the skin friction.

(iv) The skin friction has a phase lead (or lag) over the main stream fluctuations as in the no slip flow.

(v) If h_1 is moderately high, the skin friction fluctuates in same phase with the main stream.

(vi) When rarefaction parameter $h_1 \to \infty$, certain members of the class of transient velocity profiles are of the separation type.

INTRODUCTION

Lighthill (1954) has studied the effects of fluctuations of the main stream velocity on the flow of an incompressible fluid past two dimensional bodies. Stuart (1955) has obtained the exact solution of the Navier-Stokes equations for such an oscillatory flow over an infinite plane porous wall with constant suction. Siddappa (1973) has extended Stuart's work for Rivlin-Ericksen visco-elastic fluid. In the present paper, we study how Siddappa's results get modified when his no slip boundary conditions are replaced by the velocity slip conditions.

Basic Equations. We consider the flow due to a fluctuating main stream of Rivlin-Ericksen fluid flow past an infinite plane. porous wall with constant suction at the surface. Let $y=0$ be the wall. Let u and v be the velocity components along and normal to the wall.

The visco-elastic equations to the problem are,

$$v=-|v_w| \qquad \ldots(1)$$

$=$a suction velocity at the wall

$$\frac{\partial u}{\partial t}-|v_w|\frac{\partial u}{\partial y}=\frac{dU}{dt}+\alpha\frac{\partial^2 u}{\partial y^2}+\beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}-v_w\frac{\partial u}{\partial y}\right) \qquad \ldots(2)$$

$$0=-\frac{1}{\rho}\frac{\partial P}{\partial y}+2(2\beta+\gamma)\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad \ldots(3)$$

where α is the kinematic viscosity, β is the kinematic visco-elasticity, γ is the kinematic cross-viscoelasticity.

Boundary Conditions. The first order velocity slip condition is

(i) $$u=\left(\frac{2-f_1}{f_1}\right)L\left(\frac{\partial u}{\partial y}\right)=L_1\left(\frac{\partial u}{\partial y}\right) \text{ at } y=0 \qquad \ldots(4)$$

(ii) $u=U(t)$

(iii) $\frac{\partial u}{\partial y}=0$

$$\left.\begin{aligned} &u=U(t)\\ &\frac{\partial u}{\partial y}=0\end{aligned}\right\} \text{ as } y\to\infty \qquad \ldots(5)$$

which arises because of the symmetry about the axis of the pipe through the point at infinity.

Here f_1 is Maxwell's reflexion coefficient.

$L=\mu\left(\frac{\pi}{2P\rho}\right)^{\frac{1}{2}}$ is the mean free path and is constant for an incompressible fluid, and $L_1=\left(\frac{2-f_1}{f_1}\right)L$.

The free stream boundary condition is

$$u=U(t)=U_0(1+\varepsilon\, e^{i\omega t}) \text{ as } y\to\infty \quad ...(6)$$

Solution of the Problem. We look the solution of the equation (2) of the form

$$u=U_0\,[\phi_0\,(y)+\varepsilon\,\phi_1\,(y)\,e^{i\omega t}] \quad ...(7)$$

Substituting the expressions for u and U into (2) and equating non-harmonic terms in the equation, we get the equations

$$-|\,v_w\,|\,\frac{d\phi_0}{dy}=\alpha\,\frac{d^2\phi_0}{dy^2}-\beta\,|\,v_w\,|\,\frac{d^3\phi_0}{dy^3} \quad ...(8)$$

and

$$i\omega\phi_1-|v_w|\,\frac{d\phi_1}{dy}=i\omega+\alpha^2\frac{d^2\phi_1}{dy^2}+\beta i\omega\frac{d^2\phi_1}{dy^2}-\beta\,|v_w|\,\frac{d^3\phi_1}{dy^3} \quad ...(9)$$

with the boundary conditions

$$\left.\begin{aligned}\phi_0&=L_1\,\frac{d\phi_0}{dy}\\ \phi_1&=L_1\,\frac{d\phi_1}{dy}\end{aligned}\right\}\text{ at } y=0$$

and

$$\left.\begin{aligned}\phi_0&=\phi_1=1\\ \frac{d\phi_0}{dy}&=\frac{d\phi_1}{dy}=0\end{aligned}\right\}\text{ at } y=\infty \quad ...(13)$$

Putting $\eta=\frac{|v_w|\,y}{\alpha}$, equations (8), (9) and the boundary conditions (10) reduce to

$$K\,\frac{d^3\phi_0}{d\eta^3}+\frac{d^2\phi_0}{d\eta^2}+\frac{d\phi_0}{d\eta}=0 \quad ...(11)$$

$$K\,\frac{d^3\phi_1}{d\eta^3}+(1-i\lambda K)\,\frac{d^2\phi_1}{d\eta^2}+\frac{d\phi_1}{d\eta}-i\lambda\phi_1=-i\lambda \quad ...(12)$$

and

$$\left.\begin{aligned}\phi_0 &= h_1 \frac{d\phi_0}{d\eta}\\ \phi_1 &= h_1 \frac{d\phi_1}{d\eta}\end{aligned}\right\} \text{ at } \eta=0$$

$$\left.\begin{aligned}\phi_0 = \phi_1 &= 1\\ \frac{d\phi_0}{d\eta} = \frac{d\phi_1}{d\eta} &= 0\end{aligned}\right\} \text{ as } \eta\to\infty \qquad \ldots(13)$$

where

$$\lambda = \frac{\omega\alpha}{|v_w|^2},\ K = -\beta\,\frac{|v_w|^2}{\alpha^2},$$

$$h_1 = \frac{L_1\ |v_w|}{\alpha} = \text{Rarefaction parameter}$$

The solution of eqn. (11) subject to the conditions (13) is

$$\phi_0 = 1 - \frac{e^{-a\eta}}{1+ah_1} \qquad \ldots(14)$$

where $$a = \frac{1-\sqrt{1-4K}}{2K} \quad \text{for } K\leqslant 0$$

and the other solution is

$$\phi_0 = 1 - \frac{e^{-b\eta}}{1+bh_1} \qquad \ldots(15)$$

where $$b = \frac{1+\sqrt{1-4K}}{2K} \quad \text{for } 0<K\leqslant\tfrac{1}{4}$$

The solution of (12) subject to the conditions (13) is

$$\phi_1(\eta) = 1 - \frac{e^{-h\eta}}{1+hh_1} \qquad \ldots(16)$$

where $h=h_r+ih_i$ is given by the equation

$$Kh^3-(1-i\lambda K)\,h^2+h+i\lambda=0 \qquad \ldots(17)$$

This is solved numerically. The tabulation of h for various values of K and λ is given in table 1.

Table 1

k	λ	h_r	h_i
0	0	1	0
0	·25	1·0149	·22754
0	1	1·3002	·62981
0	5	2·1211	1·5421
0	25	4·0533	3·5199
−1	0	·61803	0
−1	·25	·65473	−·091307
−1	1	·79526	·15962
−1	25	·99861	−·019885
−1	100	·99991	·0049982
−10	0	·27016	0
−10	·25	·28830	−·20960
−10	1	·31105	·013619
−10	25	·31622	$6{\cdot}3228\times10^{-4}$
−10	100	·31623	$1{\cdot}5811\times10^{-4}$
−10	1000	·31623	$1{\cdot}5819\times10^{-5}$

The total velocity component parallel to the wall is given by

$$u=U_0\left[1-\frac{e^{-a\eta}}{1+ah_1}+\varepsilon\left(1-\frac{e^{-h\eta}}{1+hh_1}\right)e^{iwt}\right] \qquad \ldots(18)$$

Thus, the velocity profile is effected by the rarefaction parameter h_1 in the slip flow. This is not the case in no slip flow since $h_1=0$.

If h_1 increases then the velocity (18) becomes $u=U_0[1+\varepsilon e^{iwt}]$ which is the main stream velocity. This is not the case in the no slip flow. In slip flow, because of the boundary conditions, the shear stress at the wall τ_0 is proportional to the slip velocity at the wall and is given by

$$\tau_0=\rho\alpha\left(\frac{\partial u}{\partial y}\right)_{y=0}=\frac{\rho\alpha}{L_1}(u)_{y=0}$$

$$=\rho\,|v_w|\,U_0\left[\frac{a}{1+ah_1}+\varepsilon\left(\frac{h}{1+hh_1}\right)e^{i\omega t}\right]$$

$$=\rho\,|v_w|\,U_0\left[\frac{a}{1+ah_1}+\varepsilon\,|H|\,e^{i(\omega t+\theta)}\right] \qquad \text{...(19)}$$

where

$$|H|=\left|\frac{h}{1+hh_1}\right|=\left|\frac{h_r+ih_i}{1+(h_r+ih_i)h_1}\right|$$

$$=\frac{[h_i^2+\{h_r+h_1\,(h_i^2+h_r^2)\}^2]^{\frac{1}{2}}}{(1+h_1h_r)^2+h_1^2\,h_i^2} \qquad \text{...(20)}$$

and

$$\theta=\tan^{-1}\left[\frac{h_i}{h_r+h_1\,(h_r^2+h_i^2)}\right] \qquad \text{...(21)}$$

It is clear from (19) that the main stream fluctuations cause the fluctuations in the skin friction. From (20) and (21), for moderately high h_1, $|H|$ and θ decrease irrespective of λ (small or large). Thus, the skin friction τ_0 decreases and ultimately$\rightarrow 0$ as $h_1\rightarrow\infty$. Hence, certain member of the class of transient velocity profiles are of the separation type.

From the equations (19), (21) and table I, the skin friction has a phase lead (or lag) according as θ is positive or negative over the main stream fluctuation (for certain values of the viscoelastic parameter) as in the no slip flow. This becomes insignificant, if h_1 is moderately high and the skin friction fluctuates in same phase with the main stream.

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